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ARITHMETIC:

ITS PRINCIPLES AND APPLICATION.

Designed for the Use of Schools.

BY

JAMES HICKIE.

600048433S

ARITHMETIC:

ITS PRINCIPLES AND APPLICATION.

DESIGNED FOR THE USE OF SCHOOLS.

◆

EMBRACING, BESIDES THE ORDINARY MATTER OF A TEXT-BOOK,
SOME USEFUL SUBJECTS AND INFORMATION NOT HITHERTO
INTRODUCED INTO WORKS OF THIS KIND.

THE WHOLE

MODERNISED AND ADAPTED

TO THE PRESENT STATE OF COMMERCE.

BY

JAMES HICKIE,

Author of "Key to Rutherford's Edition of Hutton's Course of Mathematics;"
and formerly, for many years, Mathematical Master
in the Liverpool Institute.

LONDON:
SIMPKIN, MARSHALL, AND CO.
EDINBURGH: W. P. NIMMO; DUBLIN: McGLASHAN & GILL;
LIVERPOOL: PHILIP, SON, & NEPHEW.
1862.

1802. e. 16

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TO

HIS OLD PUPILS,

FOR FRIENDSHIPS FOUNDED,

FOR KIND ATTENTION FREQUENTLY RECEIVED,

AND

IN REMEMBRANCE OF THE PLEASURE EXPERIENCED IN THEIR INSTRUCTION,

THIS WORK

Is Dedicated,

AS A MARK OF ESTEEM AND REGARD,

BY

THEIR SINCERE FRIEND,

AND WELL-WISHER,

THE AUTHOR.

PREFACE.

The Author of this work, for many years a teacher of Mathematics and Arithmetic, in the first commercial town in the British empire, to pupils for the most

ERRATA.

Page 16, line 22 from top, *for* is contained *read* are contained
„ 25, *to* Exercises *annex* (B).
„ 34, *for* chest of tea 84 lb. *read* 84 lb. to 102 lb.
„ 45, Exercises 7 should head the page of Exercises.
„ 100, line 12, *for* rod *read* rood
„ 101, top, *for* ADDITIAL FRACTIONS *read* ADDITION OF FRACTIONS.
„ 156, in note, *for* 70 to 80 *read* 112 to 140
„ 321, *for* (·594 + in.) *read* (·564 + in.)

useful science has been ably handled by several writers, still many of the books produced are so overloaded with explanations as to render them, however admirably fitted for private study, yet but ill-suited for practical use in the class-room. Others, on the contrary, although excellent for the choice and number of their exercises, have been found so deficient in explanation as to have their usefulness greatly impaired. Both these faults are, it is to be hoped, avoided in the composition of the present work; for while it contains an extensive collection of exercises, care has also been taken that the principles should be fully, yet concisely, explained. But in order that the course of the text may be as little encumbered as possible, it has been deemed advisable to throw these explanations into the form of foot-notes. These latter will also, in many instances, be found the vehicles of much valuable information, the correctness of which may be relied upon.

In the new matter introduced, the subjects of Building Societies and Loan Societies have received attention, which, as regards the former, is but deservedly bestowed. For in the success and extension of Building Societies, all classes must feel interested, inasmuch as the habits of industry and economy induced by a connection with them, are not only beneficial to the individual members themselves, but serve also as a wholesome example for the careless and improvident to imitate.

Loan Societies are noticed in order to show the Interest that persons, borrowing money through their medium, will have to pay; and though not a subject of general interest, still, from the fact of thousands repairing annually to such quarters for loans, sufficiently important to deserve some notice here.

The New Tariff, Commercial Allowances, Trade Discount, and the Glossary of Arithmetical and Business Terms, will, it is hoped, be deemed an additional improvement. No subject, indeed, that had not a claim for utility, has been suffered to appear: and for the arrangement of the matter contained, that order has been adopted which the Author, from experience, thought best suited to secure the pupil's gradual progress, and to discipline his mind. A consciousness, however, that no comments offered here could add to the value of the work, precludes any further allusion to its contents, which are left for those engaged and interested in Education to pronounce upon. One favour alone is solicited, viz., that the notice of any errors which may be found in the book, or any suggestion calculated to improve it, may be communicated to the Author or the Publishers.

Before concluding, the Author must take this opportunity of acknowledging the valuable assistance rendered by an old pupil, at present a graduate of the London University, in revising the work in its passage through the press.

Liverpool, 1862.

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ARITHMETIC.

INTRODUCTION.

1. The term *quantity* or *magnitude** is applied to any thing that is susceptible of augmentation or diminution. To give a clear idea of magnitude or quantity by means of language, we take of this quantity, itself an arbitrary, but definite and well understood portion, to serve as a standard of comparison. This portion is termed *unity*,† and when we ascertain how many of such *units* must be combined together to form that quantity, then the mind has a precise idea of the size of that quantity; and the term that expresses how many of these units there are in it, is called a *number*.

2. Hence a *number* may be defined as the expression for one or more units *of the same kind*. Thus *one* is the expression for the unit; *two* represents the union of one unit with another; *three*, the union of two units with another, and so on, the successive augmentation by one *unit* each time generating the numbers *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*. These numbers we represent by the figures or characters called also digits, 1, 2, 3, 4, 5, 6, 7, 8, 9.

3. Besides these nine characters we make use of another (0) which is called nought, cypher, or zero, and is the symbol for *nothing*. These ten characters constitute the alphabet of arithmetic. Now to write 9 and 1 more, we get a *new number*, which we call *ten;* to write 10 and 1 more, we get another new number, which we call *eleven*. But to give every new number a particular name and character, would be to charge the memory with an infinite number of words and characters. To avoid this, the number *ten* is considered a new species of unit, written 10, and we count from *one ten* to

---

* A line admits of lengthening or shortening, a surface admits of increase or diminution, a weight allows of its being made lighter or heavier—hence lines, surfaces, and weights are *quantities*.

† There is but a single elementary idea in the science of numbers—it is the idea of the unit, or one, as it is the base whence all numbers are derived. This unit is either natural or artificial—natural as an apple, a sheep—artificial as a yard for length, a pound for weight.

nine tens, as by simple units; but instead of calling these new units *one ten, two tens, three tens*, &c., we call them ten, twenty, thirty, &c., written 10, 20, 30, 40, &c., making the characters already in use suffice. In continuing our reckoning we call *ten tens* another new unit, which is written *one hundred* (100); and counting from *one hundred to* nine hundreds as by simple units, we have *one hundred, two hundreds*, three hundreds, &c., written 100, 200, 300, &c.

Again, *for ten one hundreds*, we have another new unit, called *one thousand*, written 1000, and, counting by simple units, we read *one thousand, two thousand, three thousand*, &c., written 1000, 2000, 3000, &c. By this happy idea, the ten symbols or characters already adopted are made to be sufficient to express all numbers.

This system arranged—

	Symbol.	Name.
Two units of the first order	2,	Two
Two units of the second order	20,	Twenty
Two units of the third order	200,	Two hundred
Two units of the fourth order	2000,	Two thousand
Two units of the fifth order	20,000,	Twenty thousand
Two units of the sixth order	200,000,	Two hundred thousand
Two units of the seventh order	2,000,000,	Two millions
&c.	&c.	&c.

In this arrangement we have made the character or symbol 2 assume seven different values; the symbol 0, or nothing, being of no value, but merely used to fill up gaps, in order to give the symbol 2 the proper position of its order. From what has been done, it is clearly seen that this symbol (2) has as many different values as it takes positions. And had these different values been united into one group, their sum would be read, *two millions, two hundred and twenty-two thousands, two hundred and twenty-two*, and written, not as above, but in a more compact form, thus, 2,222,222, where the first symbol or figure (2) on the right hand is two units, the second (2) from the right is twenty units, the third (2) from the right is two hundred units, the fourth (2) from the right is two thousand units, &c., the figure (2) holding its value from the place which it occupies from the right towards the left.

4. This tenfold increase, in the value of a figure when removed one place from the right to left, is the reason why our System of Notation in Arithmetic is called the *Decimal System*.

These conventions being once understood, all numbers, however large, can be written and enunciated by means of these ten characters, or figures.

The following table exhibits a general view of this doctrine:—

	Trillions.	Billions.	Millions.	Units.
Hundred thousands.	6	9	3	6
Ten thousands.	5	8	2	5
Thousands.	4	7	1	4
Hundreds.	3	6	9	3
Tens.	2	5	8	2
Units.	1	4	7	1

Which number is read thus: six hundred fifty-four thousand, three hundred twenty-one trillions, nine hundred eighty-seven thousand, six hundred fifty-four billions, three hundred twenty-one thousand, nine hundred eighty-seven millions, six hundred fifty-four thousand three hundred and twenty-one.

This table may be continued to quadrillions, quintillions, sextillions, septillions, octillions, nonillions, &c.*

NOTATION AND NUMERATION.

5. *Notation* is the art of writing numbers by figures.

6. *Numeration* is the art of reading figures correctly in words.

EXERCISES IN NOTATION.

1. Seventy nine. *Ans.* 79.

2. Four hundred and sixteen. *Ans.* 416.

3. Thirteen thousand and sixty-seven. *Ans.* 13,067.

4. Three millions, two hundred and eight thousand, four hundred and sixty-five. *Ans.* 3,208,465.

5. Two billions, three hundred sixty-seven thousand millions, two hundred and thirty-one. *Ans.* 2,367,000,000,231.

6. Eight hundred and fifty-three millions, nine hundred and forty-eight thousand, six hundred and fifty-three. *Ans.* 853,948,653.

7. Two hundred and three millions, four hundred and six thousand, five hundred and eight. *Ans.* 203,406,508.

8. Nine hundred and ninety-three millions. *Ans.* 993,000,000.

---

* A billion or quintillion is easily designated by these characters, although there are not a billion of seconds since the creation of the world, nor a quintillion of cubic inches in the whole earth. For in 6000 years of 365¼ days there are only 189,345,600,000 seconds; and in the earth, supposing it a globe whose diameter is 8000 miles, there are but 68,189,122,953,687,859,200,000,000 cubic inches.

EXERCISES IN NUMERATION.

1.	97.	*Ans.*	Ninety-seven.
2.	2,863.	„	Two thousand, eight hundred and sixty-three.
3.	70,169,049.	„	Seventy millions, one hundred sixty-nine thousand, and forty-nine.
4.	1,489,065,873.	„	One thousand four hundred eighty-nine millions, sixty-five thousand, eight hundred and seventy-three.
5.	30,006,500,875,000.	„	Thirty billions, six thousand five hundred millions, eight hundred and seventy-five thousand.
6.	907,060,206,204.	„	Nine hundred and seven thousand and sixty millions, two hundred and six thousand, two hundred and four.
7.	240,026,100,201.	„	Two hundred and forty thousand and twenty-six millions, one hundred thousand, two hundred and one.
8.	590,960,126,020.	„	Five hundred and ninety thousand nine hundred and sixty millions, one hundred and twenty-six thousand and twenty.

ROMAN NOTATION.*

I.	represents	1	XV.	represents	15
II.	„	2	XVI.	„	16
III.	„	3	XVII.	„	17
IV.	„	4	XVIII.	„	18
V.	„	5	XIX	„	19
VI.	„	6	XX.	„	20
VII.	„	7	XXX.	„	30
VIII.	„	8	XL.	„	40
IX.	„	9	L.	„	50
X.	„	10	LX.	„	60
XI.	„	11	LXX.	„	70
XII.	„	12	LXXX.	„	80
XIII.	„	13	XC.	„	90
XIV.	„	14	XCV.	„	95

---

* Barlow's *Theory of Numbers* treats extensively on the different scales of notation.

C.	represents	100	DCCC.	represents	800
CC.	„	200	DCCCC.	„	900
CCC.	„	300	M.	„	1000
CCCC.	„	400	MM.	„	2000
D.	„	500	MDC.	„	1600
DC.	„	600	MDCL.	„	1650
DCC.	„	700	MDCCCLIX.	„	1859

DEFINITIONS.

7. Arithmetic treats of the properties and laws of numbers, and their application to questions bearing on the business and pursuits of man.

8. An integer is 1, or several reunions of 1.

9. An even number is either 2, or that which is formed by the continual addition of 2, as 4, 6, 8, 10, &c.

10. An odd number is that which cannot be divided into two equal integral numbers, as 1, 3, 5, &c.

11. A prime number is that which has no divisor but itself and unity, as 11, 13, 19, &c.

12. An abstract number is that which has no application to any particular subject, as 3, 5, 7.

13. A concrete or denominate number is that which has reference to a thing, as 3 apples, 4 pounds.

EXPLANATION OF THE SIGNS MADE USE OF IN ARITHMETIC.

$+$ (plus) shews addition, or that the number following is to be added to that before.

$-$ (minus) shews subtraction, and implies that the number following is to be subtracted from that before.

$\times$ (multiplied by) shews multiplication, and indicates that the numbers between which it stands are to be multiplied.

$\div$ (divided by) shews division, and implies that the number going before it is to be divided by the one which follows.

$=$ (equal) shews equality, or that the numbers between which it stands are equal.

: (is to) : : (so is) shews proportion, or equality of ratios.

25^2 denotes that 25 is squared or to be multiplied by itself.

25^3 denotes that 25 is to be cubed.

TABLES.

[illegible]

MULTIPLICATION AND DIVISION.

		[illegible]		4		5		6		7		8		9		10		11		12	
[illegible]	[illegible]	[illegible]	[illegible]	1	4	1	5	1	6	1	7	1	8	1	9	1	10	1	11	1	12
2	[illegible]	2	[illegible]	2	8	2	10	2	12	2	14	2	16	2	18	2	20	2	22	2	24
3	[illegible]	3	[illegible]	3	12	3	15	3	18	3	21	3	24	3	27	3	30	3	33	3	36
4	8	4	12	4	16	4	20	4	24	4	28	4	32	4	36	4	40	4	44	4	48
5	[illegible]	5	15	5	20	5	25	5	30	5	35	5	40	5	45	5	50	5	55	5	60
6	12	6	18	6	24	6	30	6	36	6	42	6	48	6	54	6	60	6	66	6	72
7	14	7	21	7	28	7	35	7	42	7	49	7	56	7	63	7	70	7	77	7	84
8	16	8	24	8	32	8	40	8	48	8	56	8	64	8	72	8	80	8	88	8	96
9	18	9	27	9	36	9	45	9	54	9	63	9	72	9	81	9	90	9	99	9	108
10	20	10	30	10	40	10	50	10	60	10	70	10	80	10	90	10	100	10	110	10	120
11	22	11	33	11	44	11	55	11	66	11	77	11	88	11	99	11	110	11	121	11	132
12	24	12	36	12	48	12	60	12	72	12	84	12	96	12	108	12	120	12	132	12	144

For Multiplication, say "twice one are two, twice two are four," &c.; "three times one are three," &c.

For Division, say "twos in two are one, twos in four are two," &c.; "threes in three are one," &c.

ADDITION.

14. Addition is the operation of uniting two or more numbers of the same kind together, so as to form one single number, called the sum.

Rule.—Place the numbers under each other so that units may stand under units, tens under tens, &c.

Add together the figures in the column of units, and put down the right hand figure of the sum under the unit column, and carry the other to the next column.

Add all the other columns in the same way, and under the last column put down the sum in full contained in it.

EXAMPLES.

									*Proof.**
3786	=	3000	+	700	+	80	+	6	49
987	=			·900	+	80	+	7	2165
2165	=	2000	+	100	+	60	+	5	987
49	=					40	+	9	3786
6987 Sum.		5000		1700		260		27	6987 Sum.

5000	
1700	The sum of the hundreds.
260	„ of the tens.
27	„ of the units.
6987	Sum.

EXAMPLE.†

(1.)	3 4 5 6 7 8 9 0 4 3 5 8 2 7 1
(2.)	6 5 4 3 2 1 0 9 5 6 4 1 7 2 9
(3.)	5 3 7 2 8 6 1 0 8 5 7 3 2 1 8
(4.)	4 6 2 7 1 3 8 9 1 4 2 6 7 8 2
(5.)	5 6 7 3 4 5 8 4 9 3 2 5 8 7 6
(6.)	7 1 9 8 6 4 3 2 8 1 7 3 2 1 5
(7.)	2 8 0 1 3 5 6 7 1 8 2 6 7 8 5
(8.)	7 0 8 4 5 6 1 3 7 9 1 4 5 7 3
(9.)	2 9 1 5 4 3 8 6 2 0 8 5 4 2 7
(10.)	5 6 7 3 4 5 8 4 9 3 2 5 8 7 6
Ans.	5 1 3 4 6 9 1 6 9 8 6 5 1 7 5 2 Sum.

---

* *Proof.*—Add the several columns downwards, and if the sum be the same as before, the work is presumed to be right; or divide the addition sum into two parts, and the sum of the parts added separately will be the amount.

† The rows (1) and (2) added make 1 with ciphers, (3) and (4), (6) and (7), and (8) and (9), do the same; therefore the sum of the eight rows (1), (2), (3), (4), (6), (7), (8), and (9), will be 4 with ciphers, which, added to the sum of row

EXERCISES 1.

1.	2.	3.	4.
56430	7921341	62849765	475693
12798	82345678	98765432	524317
34457	79013265	23456789	285769
21325	7890275	64698705	753812
31792	2857693	75381246	246188
68208	7142307	24618754	285769

5.	6.	7.
2912000	30260736	246783
842240	252172800	753217
2176000	2017382400	357819
1349376	15130368000	467325
1536000	201738240000	532675
2526720	25217280000000	357819

The Revenue Returns for the years 1855 and 1856, are as follows:—

		1856.		1855.
8.	Customs	£23,618,375	9.	£22,534,302
	Excise	18,073,778		17,269,463
	Stamps	7,268,272		7,132,824
	Taxes	3,105,026		3,095,543
	Property Tax	16,028,422		14,121,561
	Post Office	2,869,152		2,704,000
	Crown Lands	284,857		280,516
	Miscellaneous	971,106		1,103,007

The Revenue Returns for the years 1858 and 1859, ending June 30 in each year, are as follows:—

		1859.		1858.
10.	Customs	£24,347,322	11.	£22,838,794
	Excise	18,221.000		17.944,000
	Stamps	7,881,981		7,649,598
	Taxes	3,185,000		3,154,033
	Property Tax	6,266,106		10,330,162
	Post Office	3,220,000		3,010,000
	Crown Lands	280,540		276,654
	Miscellaneous	2,287,624		1.676,475

(5) and (10), will constitute the entire sum; but since the rows (5) and (10) are identical, therefore twice row (5) or (10) with 4, and its due complement of ciphers will be the entire sum. Thus—

567345849325876 × 2 = 1134691698651752
Add 4

Ans. 5134691698651752 entire sum.

Value of the imports of the United Kingdom for 1858 :—

12.	From British Possessions	£38,375,610
	„ United States	34,281,255
	„ France	13,911,401
	„ Russia	11,746,003
	„ South America	11,274,242
	„ Germany	8,641,455
	„ All other Countries	45,505,837

The total amount of Gold raised in Australia up to the end of 1859, was in

13.	New South Wales	£7,253,616
	Victoria	93,810,212
	South Australia	160,000
	Tasmania	8,000
	New Zealand	140,000

Exports from Great Britain and Ireland in 1859 :—

14.	To British America	£5,903,960
	„ Australia	11,225,616
	„ Other possessions	28,995,470
	„ Foreign Countries	84,315,381

State the total of the following losses taken from the Money Market Review of June 9th, 1860 :—

15.	Royal British Bank	£335,000
	Tipperary Bank through J. Sadleir	1,200,000
	The London and Eastern Bank through Colonel Waugh	250,000
	The Western Bank of Scotland	1,500,000
	The Crystal Palace Co. sustained through Robson	100,000
	The Gt. Northern Railway Co. sustained through Redpath	250,000
	The Union Bank of London sustained through Pullinger	263,070

What is the length and tonnage of the following 14 ships :—

16.	Great Eastern		length 680 feet,	tonnage 22000 tons.
	Adriatic		390	5000
	Munster		378	4000
	Leinster	Galway Line, new	375	4000
	Connaught		375	4000
	Ulster		375	4000
	Niagara		375	4580
	Persia		375	3300

Himmàlaya	360	5000
General Admiral	325	6000
Vanderbilt	350	4500
City of Baltimore	340	2367
City of Washington	325	2380
Orlando	337	3727

How many Volumes are there in the following Libraries :—

				Volumes.
17.	Paris	National Library 824,000 vol. Arzenal ,, 180,000 St. Généviève ,, 150,000 Mazarine ,, 100,000		1,254,000
	Munich Royal		Library	600,000
	London British Museum		,,	450,000
	Petersburg Imperial		,,	446,000
	Copenhagen Royal		,,	412,000
	Berlin Royal		,,	410,000
	Vienna Imperial		,,	313,000
	Dresden Royal		,,	300,000
	Madrid National		,,	200,000
	Wolfenbüttel Ducal		,,	200,000
	Stuttgard Royal		,,	187,000
	Milan Brera		,,	170,000
	Darmstadt Grand Ducal		,,	150,000
	Florence Magliabecchian		,,	150,000
	Naples Royal		,,	150,000
	Brussels Royal		,,	133,500
	Rome	Casanata 120,000 Vatican 100,000		220,000
	Hague Royal		,,	100,000
	Parma Ducal		,,	100,000

Income assessed under the Property and Income Tax in 1849 and 1857 :—

	Schedule.	1849-50.		1857-58.
18.	A, Real Property	£94,217,959	19.	£109,978,265
	B, Farmer's Profits	42,516,450		42,777,237
	C, Public Funds	26,310,990		28,083,017
	D, Trades & Professions	54,977,566		77,503,022
	E, Salaries	11,203,964		15,832,511

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* These 14 vessels placed in a line, one after another, extend more than a mile.

SIMPLE SUBTRACTION.

15. Subtraction is the process by which we find the difference between any two numbers of the same denomination.

Rule. — Place the less number under the greater, units under units, tens under tens, &c. Then beginning at the right hand, subtract each lower figure from that which stands over it, and write down the remainder.

If any figure in the lower line be greater than the one above it, add ten to the upper figure before subtracting it, then add one to the next figure on the left in the lower line and so proceed as before.

EXAMPLE. *Proof.*[*]

Minuend	763=700+60+3=700+50+13		328 Subtrahend.
Subtrahend	328=300+20+8=300+20+ 8		435 Remainder.
Remainder	435	400+30+ 5	763 Minuend.

30 the remainder of the tens.
5 „ of the units.

435 the entire remainder.

From	3141600	21025	65450	288750
Take	2269806	7569	62832	176715
Remainder	871794	13456	2618	112035

2201600	300000	8030000	13879125
1893776	282744	7012649	8555625
317824	17256	1017351	5323500

11986671955295600640	74629
2517201110612076135	70686
9469470844683524505	3943

EXERCISES 2.

1. The revenue for the year amounted to £72,218,983 in 1856, and in 1855 to £68,241,216; what is the difference?

2. The revenue for the year 1859, ending June 30th, amounted to £65,689,573, and for the year 1858, ending at the same time, to £66,879,716; what is the difference?

3. The gross revenue of the post office in the United Kingdom was,

---

[*] *Proof.*—Add the remainder and less number together, and the sum, if the work is right, will be equal to the greater.

n 1859, £3,100,939, and the expenditure £1,170,554; what was the net revenue?

4. The paper duty in 1858 amounted to £1,130,653, and in 1859 t £1,153,657, what is the difference?

EXPORTS IN THE YEARS OF

	1858.	1844.		
5. Amount of Exports for the year	£116614331	£58534705	what is the	diff.?
6. Of Apparel and Millinery,	5406190	1485347	”	”
7. Beer and Ale	1851755	436508	”	”
8. Brass & Copper Manufacture	2854551	1735873	”	”
9. Coal and Coke	3043461	664603	”	”
10. Cotton goods	33421843	18814869	”	”
11. Cotton yarn	9579479	6988580	”	”
12. Iron and Steel	11197072	3191641	”	”
13. Leather goods	2012916	463589	”	”
14. Linen goods	4124356	3024353	”	”
15. Linen yarn	1746340	4050675	”	”
16. Machinery	3599352	776115	”	”
17. Woollen goods	9776944	8204133	”	”
18. Woollen yarn	2966923	958204	”	”
19. Hardware and Cutlery	3277607	2178089	”	”
20. Earthen Ware and Porcelain	1153579	766151	”	”
21. Tin and Pewter Wares	1389858	505942	”	”

IMPORTS IN THE YEARS OF

	1858.	1844.	
22. Cocoa imported	10338404 lbs.	3731256 lbs.	”
23. Coffee	60697264 lbs.	46523188 lbs.	”
24. Corn Wheat	4241719 qrs.	1099077 qrs.	”
25. Cotton	1034342176 lbs.	646111304 lbs.	”
26. Flax	1283905 cwt.	1583494 cwt.	”
27. Guano	353541 tons.	104251 tons.	”
28. Hemp, Jute, &c.	1239716 cwt.	913233 cwt.	”
29. Hides	762977 cwt.	642805 cwt.	”
30. Oil, palm	778230 cwt.	414648 cwt.	”
31. Rice	3692023 cwt.	456302 cwt.	”
32. Silk, Raw	6277576 lbs.	4149932 lbs.	”
33. Sugar, Raw	9010796 cwt.	4880075 cwt.	”
34. Tallow	1235789 cwt.	1079486 cwt.	”

35.	Tea	75432578 lbs.	53147078 lbs.	What is the diff.?
36.	Tobacco	62217705 lbs.	38626159 lbs.	,,
37.	Wine	5791656 gals.	8584586 gals.	,,
38.	Wool	126738723 lbs.	65713761 lbs.	,,

MULTIPLICATION.

16. Multiplication is a short way of performing those addition sums in which all the numbers to be added are equal.*

The number to be repeated, or added, is called the *multiplicand.*

The number expressing the number of additions is called the *multiplier.*

The resulting number is called the *product.*

The multiplicand and multiplier are often called *factors.*

Rule (1). When the multiplier is one figure, place the multiplier under the multiplicand; then multiply each successive figure from right to left by the multiplier, writing down the right hand figure of each product, and carrying the rest to the next product, and at the last figure of the multiplicand write the product in full.

(2). When the multiplier consists of several figures, place the multiplicand and multiplier so that units may stand under units, tens under tens, &c.; then multiply as before, writing the respective products in separate lines, and observing to write the right hand figure of each line of products directly under its respective multiplier. The sum of the lines thus obtained will be the required product.

EXAMPLE.

Multiply 78593 by 4.

Multiplicand 78593
Multiplier 4
314372 product

ADDITION.

$$\begin{array}{r} 78593\\ 78593\\ 78593\\ 78593\\ \hline 314372 \end{array}$$

$$\begin{array}{rcrl} 78593 & & & \\ 4 & & & \\ \hline 12 & = & 3 & \times 4\\ 360 & = & 90 & \times 4\\ 2000 & = & 500 & \times 4\\ 32000 & = & 8000 & \times 4\\ 280000 & = & 70000 & \times 4\\ \hline 314372 & = & 78593 & \times 4 \end{array}$$

---

* This definition requires the multiplier to be an abstract number, and the product to be of the same kind as the multiplicand. But in multiplying feet in length by feet in breadth, this rule does not hold, for to the product is assigned a name different to that of the multiplicand, namely, *square* feet.

Multiply 4673549 by 10, and by 100.[*]

Multiplicand 4673549 Multiplicand 4673549
Multiplier 10 Multiplier 100

46735490 product. 467354900 product.

Multiply 2627 by 534.

Multiplicand 2527
Multiplier 534

10508 = product of 2627 by 4 = 10508
7881 = „ 2627 by 30 = 78810
13135 = „ 2627 by 500 = 1313500

1402818 product. 1402818

EXERCISES 3.

1. 24478 × 2[†]
2. 48956 × 3
3. 146868 × 4
4. 587472 × 5
5. 2937360 × 6
6. 17624160 × 7
7. 123369120 × 8
8. 986952960 × 9
9. 8882576640 × 10
10. 88825766400 × 11
11. 977083430400 × 12
12. 385476 × 13
13. 5011188 × 16
14. 3678 × 17
15. 80179008 × 32
16. 1095 × 256
17. 3285 × 294
18. 29565 × 185
19. 18548 × 896
20. 119029 × 392
21. 157428 × 112
22. 9555371 × 839
23. 13848 × 570
24. 49140 × 1148
25. 84970 × 1456
26. 23226 × 1372
27. 5874 × 5418
28. 4025 × 7168
29. 3864 × 3591
30. 8343 × 1624
31. 3128 × 3357
32. 1280475 × 1288
33. 27045 × 18325
34. 495599625 × 305
35. 115501 × 8025
36. 2180 × 2180, or the *square* of 2180[‡]

---

[*] To multiply by 10 or 100 requires no operation but to annex 0 for 10, and two 0s for 100, to the figures to be multiplied, and to multiply by 1000 three 0s are annexed, and so on. For by *Numeration* it is seen that if a cipher be placed at the right hand of a number, every figure in that number has ten times the value it had before, and if two ciphers, a hundred times, &c.

[†] The respective products to these exercises are the dividends in the exercises (answering to the same number) in division.

[‡] A number multiplied by itself produces what is called the *square* of that number; and the square multiplied by the same number gives the *cube*.

37. 4752400 × 2180, which is the cube of 2180
38. 468531729 × 468531729
39. 357814269 × 357814269
40. 837357814269 × 837357814269
41. 8154096372756 × 8154096372756
42. 4563072918 × 4563072918
43. 538169273820954 × 538169273820954
44. 72186354903429 × 72186354903429
45. Cube 468531729
46. 123456789 × 123456789
47. 987654321 × 123456789

EXAMPLE.

Find the product of 4786 by 653.

		Proof.[*]
Multiplicand	4786	653
Multiplier	653	4786
	14358	3918
	23930	5224
	28716	4571
		2612
	3125258 product.	3125258 product.

Multiplicand	2857142857†
Multiplier	483
	8571428571
	22857142856
	11428571428
	1379999999931

---

[*] Let the multiplicand and multiplier change places, and if the product be the same as the former, the work is probably right.

Cast out the 9s as they arise in adding together the digits of the multiplicand and multiplier; then multiply the two remainders together, and if the excess, after casting the 9s out of this product, be equal to the excess after casting the 9s out of the answer, the work is probably correct—and if they differ, the work is presumed to be wrong. Thus the remainder, after casting out the 9s in the multiplicand, is 7, in the multiplier 5, both being multiplied make 35, out of which cast the 9s, which leaves a remainder of 8, which is the same as the remainder after casting the 9s out of the product. This method of proof depends on a property of the number 9.

† In multiplying two numbers, the product will be unaltered if the multiplicand be *divided by any number* for a new multiplicand, and the multiplier

DIVISION.

17. Division is a short method of finding how often a less number can be subtracted from a greater of the same denomination, or of dividing a given number into equal parts.

The number to be divided is called the *dividend.*

The number which divides is called the *divisor.*

The number obtained by the division is called the *quotient.*

The number sometimes left is called the *remainder.*

CASE 1. *When the divisor is not greater than* 12.

Rule.—Place the divisor to the left of the dividend with a mark of separation, and a line beneath the dividend. Then find how many times the divisor is contained in the first figure or first two or three figures on the left of the dividend, and set down the result underneath for the first figure of the quotient, and for every unit that remains carry ten to the next figure of the dividend; divide this number, by the divisor and set down the result as the next figure of the quotient, and thus continue until all the figures in the dividend are exhausted.

CASE 2. *When the divisor consists of several figures.*

Rule.—Place the divisor to the left of the dividend and mark off to its right a place for the quotient. Take as many figures to the left of the dividend as are at least equal to the divisor. Then find how many times the left one or two figures of the divisor is contained in the first one or more figures to the left of the dividend, and write the result as the first figure of the quotient. Multiply the divisor by this quotient figure and subtract the product from the chosen number of the left hand figures in the dividend, and to the remainder annex the next figure of the dividend,—deal with this as a new dividend in a similar way, observing if the divisor be not contained therein, to put a cipher in the quotient, and affix the next figure from the dividend to this *new* dividend. Thus continue until all the figures in the dividend are exhausted, when the quotient, with the remainder, if any, will be ascertained.

Note 1.—If in any case the product of the quotient figure by the

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multiplied by the same for a new multiplier. Thus, 20000000000 × 69 = 20000000000 ÷ 7 and the quotient × ed by 69 × 7, which is precisely what is done with the above question, but here there is a remainder of 1, therefore 69 must be added to the product to yield the latent product, namely, 138 with ciphers. From this example it can be seen how a variety of questions in multiplication may be prepared whose products are known, and the number of such questions can be extended by using in a similar way the numbers, 13, 17, 19, &c.

divisor exceed the number from which it is to be subtracted, the figure in the quotient is too great, and if the remainder be not less than the divisor the quotient figure is too small; in such instances, another quotient figure must be taken.

Note 2.—For every figure of the dividend annexed to the remainder there must be placed a figure in the quotient.

EXAMPLE (1).

Divide 38 by 9?

9 is contained in 38, 4 times, and 2 is left.

That is, 9 can be subtracted from 38, 4 times, leaving a remainder of 2.

Thus 38
9 1st subtraction.
—
29
9 2nd ,,
—
20
9 3rd ,,
—
11
9 4th ,,
—
Remainder 2

EXAMPLE (2).

Divide 7896546 by 7?

Divisor 7)7896546 dividend
1128078 quotient.

EXAMPLE (3).

Divide 756 by 42?

Divisor.	Dividend.	Quotient.
4 2)	7 5 6	(1 8
	4 2	
	3 3 6	
	3 3 6	

EXAMPLE (4).

Divide 785327 by 100?*

1,00)7853,27
7853—27 remainder.

---

* When the divisor is 10, 100, 1000, &c., cut off as many figures from the right of the dividend as there are ciphers in the divisor for the remainder; the

EXAMPLE (5).

Divide 78592 by 400?

Divisor 4,00)785,92 dividend.

Quotient 196—192 remainder.

EXAMPLE (6).

Divide 7698 by 36?*

36) 7698 (213
72
——
49
36
——
138
108
——
30

EXAMPLE.

Divide 4685372 by 769?

769) 4685372 (6092
4614
——
7137
6921
——
2162
1538
——
624

Proof‡

6092 quotient.
769 divisor.
——
54828
36552
42644
——
4684748 product.
624 remainder.
——
4685372 dividend.

---

other part of the dividend will be the quotient. If the divisor be 40, 500, &c., cut off as before from the dividend as many figures as there are ciphers in the divisor, and divide the other part of the dividend by 4, 5, &c., and to the remainder, if any, annex the figures cut off for the real remainder.

* The process, if pursued in its natural and original form, would stand thus,

36) 7698 (200 the 1st quotient.
7200
——
2nd part of dividend 498
360 10 the 2nd „
——
3rd part of dividend 138
108 3 the 3rd „
——
Remainder 30 213 the sum of all the quotients realizing the real quotient.

‡ *Proof.*—1st Method.—Multiply the quotient by the divisor, to the product

EXERCISES 4.

1. 48956 ÷ 2[*]
2. 146868 ÷ 3
3. 587472 ÷ 4
4. 2937360 ÷ 5
5. 17624160 ÷ 6
6. 123369120 ÷ 7
7. 986952960 ÷ 8
8. 8882576640 ÷ 9
9. 88825766400 ÷ 10
10. 977083430400 ÷ 11
11. 11725001164800 ÷ 12
12. 5011188 ÷ 13
13. 80179008 ÷ 16
14. 62526 ÷ 17
15. 2565728256 ÷ 32
16. 280320 ÷ 256
17. 965790 ÷ 294
18. 5469525 ÷ 185
19. 16619008 ÷ 896
20. 46659368 ÷ 392
21. 17631936 ÷ 112
22. 8016956269 ÷ 839
23. 7893360 ÷ 570
24. 56412720 ÷ 1148
25. 123716320 ÷ 1456
26. 31866072 ÷ 1372
27. 31825332 ÷ 5418
28. 28851200 ÷ 7168
29. 13875624 ÷ 3591
30. 13549032 ÷ 1624
31. 7143696 ÷ 3357
32. 1649251800 ÷ 1288
33. 495599625 ÷ 18325
34. 151157885625 ÷ 305
35. 926895525 ÷ 8025
36. 4752400 ÷ 2180
37. 103602000 ÷ 2180
38. 219521981079729441 ÷ 468531729
39. 128031051100004361 ÷ 357814269
40. 701168109117357100004361 ÷ 837357814269
41. 66489287656192556099035536 ÷ 8154096372756
42. 2094783199513034724 ÷ 4563072918
43. 28962616728497296436684947011 6 ÷ 538169273820954
44. 52108698342438080319159580 41 ÷ 72186354903429
45. 102853021448790921843933489 ÷ 468531729.

Ans. Square of Divisor.

46. 152415787501905 21 ÷ 123456789
47. 121932631112635269 ÷ 123456789.

---

add the remainder; then if the sum be equal to the dividend, the work is right.

2nd Method.—Cast the 9s out of the divisor and quotient, and multiply the residues together; from their product cast out the 9s, and add the result to the remainder. Then if the excess over the 9s in the sum be equal to the excess over the 9s in the dividend, the work is probably accurate.

8 dividend
Thus, divisor 4×8 quotient
5+3

[*] The respective quotients to these exercises are the multiplicands in the exercises (answering to the same number) in the multiplication questions.

Divide 423000000 by 987?*

```
987 ) 423000000 ( 428571
      3948
      ────
       2820
       1974
       ────
        8460
        7896
        ────
         5640
         4835
         ────
          8050
          6909
          ────
           1410
            987
           ────
            423
```

PROPOSITIONS.

19. *Propositions inferred from the relations between divisor, dividend, and quotient.*

I. If the divisor and dividend be multiplied by the same number, the quotient will continue unchanged. Thus $36 \div 4 = 9$, and $36 \times 3 \div 4 \times 3 = 9$ also.

II. If the divisor and dividend be divided by the same number, the quotient remains the same.‡ Thus $36 \div 4 = 9$ and $\frac{36}{2} \div \frac{4}{2}$ or $18 \div 2 = 9$ also.

Note.—This is what is called *cancelling.*§

---

* The divisor $987 = 141 \times 7$ and the dividend $423000000 = 141 \times 3000000$. Striking out the common factor 141, the work becomes reduced to 3000000 divided by 7, which gives the quotient 428571 with a remainder of 3, which, if multiplied by the common factor 141, yields the real remainder.

From this example it will be obviously seen how a judicious teacher may prepare a variety of questions on division, the quotient and remainder of each of which, although latent, are readily perceived; and the number of such questions may be multiplied by using the numbers 13, 17, 19, &c., in a similar way to 7.

‡ The fact of the *dividend* and *divisor* being either multiplied by the same number or divided by the same number, and not altering the value of the quotient, is a *truth which cannot too often be impressed on the learner's attention*, in order to prepare him for its early application to the numerator and denominator of fractions, since the term *numerator* is synonomous with *dividend*, and *denominator* with *divisor*. Thus, $\frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Dividend.}}{\text{Divisor.}}$

§ The contraction of arithmetical operations by rejecting equal factors from the dividend and divisor of an unexecuted process is called cancelling.

III. If the divisor remain the same, multiplying the dividend by any number is the same as multiplying the quotient by that number. Thus 36 ÷ 4 = 9 and 36 × 3 ÷ 4 = 9 × 3.

IV. If the dividend remain the same, multiplying the divisor by any number is in effect dividing the quotient by that number. Thus 36 ÷ 4 = 9 and 36 ÷ 4 × 2 = ½ quotient.

SUPPLEMENT TO MULTIPLICATION AND DIVISION.

EXAMPLE 1.

Multiply 2357 by 756. 756 = 9 × 7 × 12.

Multiplicand	2357	2357	2357 × 6
Multiplier	756	9	10
	———	———	———
	14142	21213	23570 × 5
	11785	7	10
	16499	———	———
	———	148491	235700 = 100 times 2357
	1781892	12	7
		———	———
		1781892	1649900 = 700 " "
			117850 = 50 " "
			14142 = 6 " "
			———
			1781892 = 756 times 2357

EXAMPLE 2.

Multiply 6879 by 1512.

6879	Now since 1512 = 4 × 6 × 7 × 9	
1512	Multiplicand 6879	6879 × 2
———	4	10
13758	———	———
6879	27516	68790 × 1
34395	6	10
6879	———	———
———	165096	687900 × 5
Product 10401048	7	10
	———	———
	1155672	6879000
	9	3439500
	———	68790
	10401048 product.	13758
		———
		10401048

EXERCISES.

Multiply 6873 by 270 and $270 = 5 \times 6 \times 9$

„ 3456 by 504 „ $504 = 9 \times 7 \times 8$

63574 by 362880 and $362880 = 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9$

20. *To multiply when the multiplier contains a fraction.*[*]

Rule (1). Multiply the multiplicand by the upper figure of the fraction, and divide the product by the under figure; to this quotient add the product by the integer, and the result is the required product; or,

(2). Multiply the integer of the multiplier by the lower figure of the fraction, and add in the upper; use this as a multiplier, and divide the resulting product by the lower figure of the fraction for the required product.

EXAMPLE.

Multiply 4835 by $9\frac{4}{5}$.

$$\begin{array}{rrr}
\text{By Rule (1).} & \text{By Rule (2).} & \\
4835 & 4835[‡] & 9\frac{4}{5} \\
9\frac{4}{5} & 49 & \overline{} \\
\hline
5)19340 & 43515 & 49 \\
\cline{1-1}
3868 & 19340 & \\
\cline{2-2}
43515 & 5)236915 & \\
\hline
\text{Product } 47383 & 47383 \text{ product.} &
\end{array}$$

EXERCISES. A.

1. $1760 \times 5\frac{1}{2}$.
2. $8756 \times 30\frac{1}{4}$.
3. $3740 \times 9\frac{7}{8}$.
4. $4795 \times 16\frac{3}{4}$.
5. $8146 \times 7\frac{7}{8}$.
6. $94768 \times 23\frac{5}{9}$.

21. *To multiply by* 5, 25, 125, *by short and easy methods.*

EXAMPLE 1.

Multiply 63875 by 5, 25, 125.

To multiply 63875 by 5, is the same as to multiply by 10 and divide by 2.

To multiply 63875 by 25, is the same as to multiply by 100 and divide by 4.

---

[*] A fraction (which is explained at length under the head of "Fractions") is part of a unit, as $\frac{3}{4}$, which means 3 divided by 4 producing a quotient less than unity. This early application of a fraction is essential, inasmuch as fractional quantities enter into the measures of length and surface; as $2\frac{1}{4}$ inches = 1 nail; $5\frac{1}{2}$ yards = 1 perch; and $30\frac{1}{4}$ square yards = 1 square perch.

[‡] In multiplying by 49, the multiplier is 5 times too large; consequently the product is 5 times larger than it should be. Hence, by dividing by 5, the real product is obtained.

To multiply 63875 by 125, is the same as to multiply by 1000 and divide by 8.

DIVISION.

22. EXAMPLE.

Divide 10584 by 108.

By cancelling

$$\frac{10584}{108}=\frac{9\times7\times12\times14}{9\times12}=7\times14=98 \text{ quotient.}$$

$$\begin{array}{r} 108\,)\,10584\,(98 \\ 972 \\ \hline 864 \\ 864 \end{array}$$

DIVISION BY FACTORS.

23. *When the divisor is the exact product of two or more numbers, or is composed of factors none greater than* 12.

Rule.—Divide first by one factor, then the quotient by another, and so on, until all the factors have been used. The last quotient will be the required one.

To find the remainder, if any.

1. For the 2nd factor—multiply the remainder in dividing the 2nd factor by the 1st divisor, and to the product add the first remainder; the sum will be the real remainder to a divisor resolved into two factors.

2. For a 3rd factor and a 3rd remainder—multiply the 3rd remainder and the preceding divisors together, and to this product add the former real remainder to two factors, and the sum is the real remainder to a divisor resolved into three factors, and so on for four or more factors.

EXAMPLE (1).

Divide 47835 by 48 and 48 = 6 × 8.

$$\begin{array}{r} 48\,)\,47835\,(996 \\ 432 \\ \hline 463 \\ 432 \\ \hline 315 \\ 288 \\ \hline 27 \end{array} \qquad 48\left\{\begin{array}{l} 6\,)\,47835 \text{ dividend} \\ \hline 8\,)\;\;7972\text{—}3 \\ \hline \;\;\;\;\;\;996\text{—}4 \end{array}\right. \left.\begin{array}{l} \\ \\ \end{array}\right\} 27$$

EXAMPLE (2).

Divide 378759 by 5184; but 5184 = 4 × 8 × 9 × 6 × 3.

5184) 378759 (73
36288
15879
15552
327 remainder

5184 {
4) 378759
8) 94689—3 } 7 real remainder to a divisor of 32
9) 11836—1
6) 1315—1 } 39 real remainder to a divisor of 288
3) 219—1 } 327 real remainder to a divisor of 1728
quotient 73 } 327 real remainder to a divisor of 5184

24. *To divide when the divisor contains a fraction.*

Rule.—Multiply both the dividend and the integer part of the divisor by the under figure of the fraction, taking in the upper for the new divisor, then divide the new dividend by the new divisor, and the remainder, if any, divide by this multiplier for the true remainder.*

EXAMPLE 1.

Divide 278 by 4.
4)278
69—2

otherwise.
divisor 4) 278 dividend
3 3 multiplier
12) 834 dividend
69—$\frac{6}{3}$ = 2 remainder.

EXAMPLE 2.

Divide 1357 by $5\frac{1}{2}$.
$5\frac{1}{2}$)1357
2
11) 2714
246—$\frac{8}{2}$ = 4

otherwise.
$5\frac{1}{2}$) 1357 (246
11
25
22
37
33
4 remainder.

---

* The principle implied in this rule is, that if both dividend and divisor be multiplied by the same number, the quotient continues the same.

EXAMPLE 3.

Divide 1357 by 30¼.

30¼) 1357 (4
4

121) 5428 (44
484

588
484

4) 104

26 remainder.

otherwise.

30¼) 1357 (44
121

147
121

26

EXAMPLE 4.

Divide 25585 by 30¼.

30¼) 25585
4

121) 102340 (845
968

554
484

700
605

4) 95

23¾

otherwise.

30¼) 25585 (845
342

138
121

175
151¼

23¾

EXERCISES.

1. 5280 ÷ 5⅓
2. 264869 ÷ 30¼
3. 36932½ ÷ 9⅞
4. 80316¼ ÷ 16¾
5. 64149¾ ÷ 7⅞
6. 2232312⅝ ÷ 23⅝

CONCRETE DENOMINATE NUMBERS.

TABLES OF MONEY, WEIGHTS, AND MEASURES.*

25. All operations hitherto have been carried on with abstract numbers, or concrete numbers of one denomination. If concrete numbers had but one denomination,—for instance, if pounds were the only units of weight, yards of length, or if there were concrete numbers of different denominations, and these denominations differed from each other by 10, then all operations with such concrete numbers would be carried on by the rules for abstract numbers. But as the relation between the units of denominations in concrete numbers is by a variable scale, it is therefore necessary to learn their relation from the following tables:—

TABLE OF MONEY.

GOLD.

26. The *standard gold coin* of England is made of 22 parts of pure gold, and 2 parts of *alloy* (a mixture of silver and copper), melted together. From a pound Troy of *standard gold* are coined $46\frac{29}{40}$ sovereigns, or, £46 14s. 6d., making the standard ounce £3 17s. $10\frac{1}{2}$d., and the weight of the sovereign 3 dwt. $5\frac{171}{623}$ grs. But the sovereign, in consideration of the loss of weight by wear, is deemed full weight if it weigh 3 dwts. $2\frac{3}{4}$ grs.

The pound sterling is represented by a gold coin called a *sovereign*, whose standard weight is 123·274 grains, of which 113·001 are pure gold.

SILVER.

The *standard of silver coin* consists of 37 parts of pure silver, and 3 parts of copper. From a pound Troy of standard silver are coined 66 shillings, making the Mint price of silver 5s. 6d. per standard ounce.

Hence standard gold is $14\frac{7}{44}$ times the value of standard silver.

COPPER.

In the old copper coinage, 24 pence were coined from a pound avoirdupois. The new bronze penny issued in 1860, is composed of 94 parts of copper, 4 of zinc, and 2 of tin.

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* There are four principal classes of units in Arithmetic:

1st. Abstract, or simple units.

$$\left.\begin{array}{l}\text{2nd. Units of currency.}\\ \text{3rd. Units of weight.}\\ \text{4th. Units of measure.}\end{array}\right\}\text{Concrete units.}$$

The copper coinage is not a legal tender for more than 12d., nor is the silver for more than 40s. The gold coinage is the standard of value.

STERLING MONEY.

A farthing is expressed (¼d.)
2 farthings = 1 halfpenny, „ (½d.)
4 farthings = 1 penny „ (1d.)
12 pence = 1 shilling „ (1s.)
20 shillings = 1 pound or sovereign (£1)

The following coins are also in daily use.*

A three penny piece in silver = 3d.
A four penny „ „ = 4d.
A six penny „ „ = 6d.
A florin „ „ = 2s.
A half crown „ „ = 2s. 6d.
A crown „ „ = 5s.
A half sovereign (in gold)† = 10s.
A guinea „ „ = 21s. not now in circulation.

WEIGHTS AND MEASURES.

31. An Act of Parliament, for establishing uniformity of weights and measures throughout the kingdom, came into operation in January, 1826. This Act establishes:—

32. *First*,—That the brass standard yard of 1760, then in the custody of the Clerk of the House of Commons, shall be the imperial standard yard, from which all other measures, whether lineal, superficial, or solid, shall be derived.

33. *Secondly*,—That the standard brass weight of one pound Troy, made in the year 1758, then in the custody of the same officer, shall be the *unit* or standard of weight, from which all other weights shall be derived and computed: that 5760 grains shall be contained

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* In addition to these there are 2, 1½, and 1 penny silver coins in circulation. Of 24328526 coins issued from the Mint in 1860, 201088 consisted of these small coins.

By the Coinage Act which came into operation in November, 1861, the defacing of coin by stamping words, &c. on it will subject the offender to imprisonment with hard labour. Further, such coin will not be a legal tender, and will render the person uttering it liable to a penalty of 40s.

† The gold coinage has been much increased since the discovery of the rich mines of Australia, in 1852. From that date to December, 1860 (a period of nine years) £51,719,722 have been coined, which compared with £34,431,958 issued during the nine years prior to 1852, shows an increase of about 50 per cent.

in the standard Troy pound, and that 7000 such grains shall be a pound *avoirdupois*.

34. *Thirdly*,—The standard measure of capacity for liquids and dry goods (not measured by heaped[*] measure) shall be the "Imperial standard gallon" containing 10 pounds, avoirdupois, of distilled water, weighed in air, at the temperature of 62° (Fahrenheit's thermometer), the barometer being at 30 inches. Now, this weight of water fills 277·274 cubic inches, which also constitute an imperial standard gallon.

35. The Imperial bushel, consisting of 8 gallons, will consequently be 8 times 277·274, or 2218·192 cubic inches.

The term *measure* may be divided into seven kinds :—

Measure of	Length	is a yard.
	Surface	a square yard.
	Solidity	a cubic yard.
	Capacity	a gallon.
	Weight	a pound.
	Angles	a degree.
	Time	a second.

Tables of English Weights and Measures of Imperial Standard.

36. MEASURES OF LENGTH.[‡]

12	inches	make	1 foot	marked	1 ft.
3	feet	,,	1 yard	,,	1 yd.
5½	yards[§]	,,	1 pole, rod, or perch		1 po.
40	poles	,,	1 furlong	,,	1 fur.
8	furlongs	,,	1 mile	,,	1 mile.

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[*] The only great fault of the Act of 1826, was the continuance of heaped measure in the "*Imperial Measure*," which was, however, abolished by an Act passed in 1835, directing that all articles, before sold by heaped measure, should be filled only to the level of the brim. By the same Act, also, the use of all local measures were to be discontinued.

[‡] A commission appointed in 1838 to consider the best means of restoring the standards which had been destroyed or rendered useless by the burning of the Houses of Parliament in 1834, reported that their accurate reconstruction could be more securely accomplished by means of material copies, than by reference to the pendulum. This report was confirmed by an Act of Parliament in 1855, which established a standard yard, and a standard pound, constructed from authentic copies of the old standards, to be the standards of reference. Copies of these were lodged at the Exchequer, the Mint, the Royal Observatory, the Palace of Westminster, and with the Royal Society. This commission also recommended that the *avoirdupois* pound should be adopted instead of the *troy* pound as a standard.

[§] Until the year 1826, the perch in Ireland contained 7 yards, but in that year, the English measures and currency were introduced.

The following are other lineal measures sometimes used:—

4 inches make 1 hand.
9 inches ,, 1 span.
6 feet ,, 1 fathom.
22 yards ,, 1 chain.
100 links ,, 1 chain = 4 perches.
3 miles ,, 1 league.
60 geographical or 69½ English miles ,, 1 degree.
39·1393 inches the length of a pendulum vibrating seconds in the latitude of London.

CLOTH MEASURE.

This measure is used for all kinds of cloth, muslin, ribbon, &c.

The *yard* in cloth measure is the same as in long measure, but differs in its divisions and subdivisions.

2¼ inches = 1 nail.
4 nails = 1 quarter.
4 quarters = 1 yard
3 quarters = 1 Flemish Ell.
5 quarters = 1 English Ell.
6 quarters = 1 French Ell.

MEASURE OF SURFACE OR AREA.

This table is formed from the table of lineal measure by multiplying each line in it by itself (as 12 in. × 12 in. = 144 square inches), and is used for measuring all kinds of superficies, such as land, paving, flooring, in short, every thing having length and breadth.

MEASURE OF SURFACE.

144 square inches = 1 square foot.
9 square feet = 1 square yard.
30¼ square yards = 1 square pole.
40 square poles = 1 rood.
4 roods = 1 acre.
640 Acres = 1 square mile.

Note.—A *square* is a four-sided figure, whose sides are equal and perpendicular to one another.

MEASURES OF SOLIDITY OR CUBIC MEASURE.

38. This measure is used to measure all kinds of solids, or figures which have length, breadth, and depth or thickness.

Note.—A *cube* is a solid figure contained by six equal squares. This measure is derived from lineal measure thus:—

12 in. × 12 in. × 12 in. = 1728 cubic or solid inches.
1728 cubic inches = 1 cubic foot.
27 „ feet = 1 „ yard.

The following measures may be added:—

40 cubic feet of rough timber = 1 ton or load.
50 cubic feet of hewn timber = 1 „ „
40 cubic feet = 1 „ of shipping.

MEASURE OF VOLUME OR CAPACITY, OR "IMPERIAL MEASURE."

39. This measure is used for liquids, and all dry goods.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon of 277·274 cubic inches.
2 gallons = 1 peck.
4 pecks = 1 bushel.
8 bushels = 1 quarter.

By the Act of Parliament of 1826, the ale gallon of 282 cubic inches, the wine gallon of 231 cubic inches, and the corn gallon of 268·8 cubic inches, were all abolished, and a uniform measure, called the "*Imperial gallon*," of 277·274 cubic inches, was substituted.

The peck, bushel, and quarter, are used only for dry goods.

IV. MEASURES OF GRAVITY OR WEIGHT.

TROY WEIGHT.

40. This table is supposed to derive its name from *Troyes*, a town in France, where it was first used. It is employed in weighing gold, silver, and jewels, and in philosophical investigations.

24 grains = 1 pennyweight (dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

Diamonds are weighed by *carats*. The *carat* is 3⅙ grains in weight.

But the word *carat*, when used in reference to gold, is employed for the purpose of expressing the degree of fineness or purity of that metal without regard to weight. Thus, if a pound of pure gold be divided into 24 equal parts or *carats*, mint gold must contain 22, watch cases 18, and vessels made of the same metal either 15, 12, or 9, such parts or carats, pure to the pound.

AVOIRDUPOIS WEIGHT.

This weight derives its name from *Avoirs* (goods or chattels) and *poids* (weight). It is used in almost all commercial dealings; such as weighing butter, meat, groceries, &c.

16 drams = 1 ounce. (oz.)
16 ounces = 1 pound. (lb.)
28 pounds = 1 quarter. (qr.)
4 quarters (112 lbs.) = 1 hundred weight (cwt.)
20 cwt. = 1 ton.
1 lb. Avoirdupois = 7000 grs. *Troy.*
14* lbs. = 1 stone.

APOTHECARIES WEIGHT.

41. This weight is established by custom only, and not by Act of Parliament.

Weight.

20 grains = 1 scruple. ℈
3 scruples = 1 dram ʒ
8 drams = 1 ounce ℥
12 ounces = 1 pound ℔

Apothecaries compound their medicines by this weight, but buy and sell their drugs by avoirdupois.

The pound and ounce of the apothecaries' weight are equal to those of troy weight, only differently divided and subdivided.

Fluid Measure.

60 minims = 1 fluid drachm f ʒ
8 drachms = 1 ounce f ℥
20 ounces = 1 pint O
8 pints = 1 gallon gall.

42. MEASURE OF TIME.

60 seconds = 1 minute.
60 minutes = 1 hour.
24 hours = 1 day.
7 days = 1 week.
365 days = 1 year, for three years together.
366 days = 1 leap year, occurring every 4th year.

The common year is divided into twelve calendar months, viz.:—

January	31 days.	May	31 days.	September	30 days.
February	28 —	June	30 —	October	31 —
March	31 —	July	31 —	November	30 —
April	30 —	August	31 —	December	31 —

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* By Act of Parliament of January 1835, the *stone*, in all cases, is to consist of 14 lbs. *avoirdupois.*

The number of days in each month may be easily remembered from the following rhymes:—

30 days hath September,
April, June, and November,
February hath 28 alone,
And all the rest have 31.
But *leap year* coming once in four,
February then has one day more.

The Civil year or *Julian* (from being adjusted by Julius Cæsar, B. C. 45), was estimated to consist of 365 days 6 hours. The 6 hours in 4 years making a whole day, it was established that the year should consist of 365 days for 3 successive years, and the 4th year of 366 days, which was termed Leap-Year, or *Bissextile*. In that year, February has 29 days.

But in the course of years it was found that the time between the sun's departure from, and return to, a certain point in the ecliptic, consisted not of 365 days 6 hours, but of 365 days 5 hours, 48 min. 48 sec. And thus reckoning time according to the Julian Calendar, would place the end of the year 11 minutes 12 seconds after the time the sun had returned to the same point of the ecliptic which it had occupied at the beginning of the year. This error of 11 minutes 12 seconds would, in the course of 400 years, amount to 3 days 2 hours 40 minutes. At the Council of Nice, which was held in the year 325, the vernal equinox fell on the 21st of March; it was decreed, therefore, that Easter should be celebrated on the first Sunday after the first full moon occurring on or next after the vernal equinox, which was supposed to fall always on the 21st of March. But in 1582 (that is 1287 years later) the vernal equinox was found to have occurred on the 11th of March, and to celebrate Easter in relation to the 21st of March, would be doing so 10 days after the time decreed.

To remedy this irregularity, Pope Gregory XIII. reformed the calendar by retrenching 10 days in October—calling the 5th of October the 15th, making, thereby, the year 1582, to consist of 355 days, and ordering that leap-years should occur as before except in centuries where the numbers expressing which are not divisible by 4; thus 1700, 1800, and 1900, would not be leap-years, but 2000 would, because 17, 18, and 19, are not divisible by 4, but 20 is. This is called in consequence, the *Gregorian Calendar* or *New Style*.

This New Style was adopted in England in 1752, and in doing so, 11 days were retrenched by calling the 3rd of September the 14th.

In Russia, they still calculate according to the Julian or old style; but in the other countries of Europe the new style is used.

MEASURE OF ANGLES AND THE CIRCLE.

43. 60 seconds (60″) = 1 minute (1′).
60 minutes = 1 degree (1°).
30 degrees = 1 sign (1^{s}).
360 degrees = 1 circumference (1^{c}).

Note.—An angle is the opening between two lines meeting at a point, and is measured by the portion of the circumference of a circle (whose centre is the meeting point) included between the lines.

COMMERCIAL NUMBERS.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross, or 144 dozen = 1 great gross.
20 units = 1 score.
5 score, or 100 = 1 short hundred.
6 score, or 120 = 1 long hundred.
24 sheets = 1 quire of paper or parchment.
20 quires = 1 ream of ditto.
21½ quires = 1 perfect or printer's ream.
2 reams = 1 bundle of ditto.
12 skins of parchment = 1 roll.
4 pages, or 2 leaves = 1 sheet of folio.
8 pages, or 4 leaves = 1 sheet of quarto (4to).
16 pages, or 8 leaves = 1 sheet of octavo (8vo).
24 pages, or 12 leaves = 1 sheet of duodecimo (12mo).
36 pages, or 18 leaves = 1 sheet of eighteens (18mo).

CUSTOMARY MEASURES AND WEIGHTS OF MISCELLANEOUS ARTICLES.

Bag of Coffee	weighs	from	1¼ cwt.	to	1½	cwt.
„ Hops	„	about			2½	„
„ Cocoa	„	„			1	„
„ Rice (E.I.)„	„	„			1½	„
„ Pepper (white)	„	„			1½	„
„ Pimento	„	„			1	„
„ Sugar (E.I.)	„	from	1	to	1½	„
Bale of Cotton	„		3	to	6½	„
„ Flax (Russia)	„		5	to	6	„

Barrel of Beef weighs about	200	lbs.
,, Flour	196	,,
,, Indian meal	196	,,
,, Soap	256	,,
,, Gunpowder	100	,,
,, Beer or Ale	36	gallons.
Bushel of Wheat	70	lbs.
,, Barley	60	,,
,, Beans	63	,,
,, Oats	45	,,
,, Peas	64	,,
Cask of Clover	from 7 to 9	cwt.
,, Raisins	1 to 2½	,,
,, Rice (American)	about 6	,,
,, Tallow	,, 9	,,
Chest of Tea	84	lbs.
,, Opium (E.I.)	150	,,
,, Sago	196	,,
Firkin of Butter	56	,,
,, Beef	100	,,
Fodder of Lead	19½	cwt.
Hogshead of Beer	54	gallons.
,, Brandy	60	,,
,, Rum	from 45 to 50	,,
,, Claret	about 46	,,
,, Sugar	from 13 to 16	cwt.
,, Tobacco	12 to 18	,,
Load of Bricks	about 500	
,, Tiles	1000	
,, Oatmeal	240	lbs.
,, Hay	18	cwt.
Pack of Wool	240	lbs.
Pipe of Port Wine	115	gallons.
,, Madeira and Cape	92	,,
,, Teneriffe	100	,,
Puncheon of Brandy	100 to 110	,,
,, Rum	90 to 100	,,
,, Whiskey	about 120	,,
Sack of Flour	280	lbs.
,, Clover	from 2 to 3½	cwt.
Tierce of Beef	about 304	lbs.
,, Sugar (W.I.)	from 7 to 9	cwt.

FRANCE.

45. Our commercial intercourse with France, especially since the passing, in 1860, of the late Treaty of Commerce, renders it essential that the weights and measures of that country should have a place in our commercial arithmetics.

By a law of 1795, the *unit* of length is called a *Mètre;* it is the ten-millionth part of the distance from the North Pole to the Equator. The Mètre serves as a basis to the whole system, which is constructed on the decimal scale.

The unit of surface measure is the *Are*, which is a square whose side is 10 Mètres.

The unit of solid measure is the *Stère*, which is a cube, whose side is a Mètre.

The unit of capacity is the *Litre*, which is a cube, whose side is the 10th part of the Mètre.

The unit of weight is the *Gramme*, which is the weight of a cube of water, the side of which is the hundreth part of the Mètre,—the water being distilled and at its greatest density.

The following prefixes derived from the *Greek* language are used to convey multiplication.

Deca	signifies	10	times.
Hecto	„	100	„
Kilo	„	1000	„
Myria	„	10000	„

To imply subdivision, the following prefixes derived from the *Latin* language, are used under the following forms:—

Deci	signifying	10th	part.
Centi	„	100th	„
Milli	„	1000th	„

46. MEASURE OF LENGTH.

The Metre = 39·37079 English inches.

Millimètre = 0·03937 English inches.
Centimètre = 0·39371 „
Decimètre = 3·93708 „
MÈTRE = 39·37079 „
Decamètre = 32·80899 feet.
Hectomètre = 328·08992 „
Kilomètre = 1093·63305 yards.
Myriamètre = 10936·33055 „

47. SURFACE MEASURE.

Are = 1 Square Decamètre = 119·60333 square yards.

Centiare = 1·19603 square yards
ARE = 119·60333 „
Decare = 1196·03328 „
Hectare = 11960·33284 „ = 2·4736 acres.

48. SOLID OR CUBIC MEASURE.

Stère = 1 Cubic Mètre = 35·31658 cubic feet.

Decistère = 3·53166 cubic feet
STÈRE = 35·31658 „
Decastère = 353·16577 „

49. MEASURE OF VOLUME OR CAPACITY.

Litre = 1 Cubic Decimètre = 61·02705 cubic inches = 1·760773 pints (English).

Millilitre = 0·06103 cubic inches.
Centilitre = 0·61027 „
Decilitre = 6·10270 „
LITRE = 61·02705 „ = 1·760773 pints
Decalitre = 610·27050 „ = 2·200966 gallons
Hectolitre = 3·53166 cubic feet = 22·009662 „
Kilolitre = 35·31658 „ = 220·096625 „
Myrialitre = 353·16577 „ = 2200·966250 „

50. MEASURE OF GRAVITY OR WEIGHT.

Gramme = 15·4320 grains

Milligramme = 0·1540 grains
Centigramme = 0·1543 „
Decigramme = 1·5432 „
GRAMME = 15·4320 „
Decagramme = 154·3200 „
Hectogramme = 1543·2000 „ $= \left\{ \begin{array}{l} \text{3·2167 ounces troy; or,} \\ \text{3·5291 ounces avoirdupois.} \end{array} \right.$
Kilogramme = 15432·0000 „ = 2·2057 lbs.
Myriagramme = 154320·0000 „ = 22·57 „

MONEY.

The standard unit of money is a silver coin called a *Franc*, containing $\frac{9}{10}$ of pure silver, and $\frac{1}{10}$ alloy.

10 Centimes = 1 Decime
10 Decimes = 1 Franc, weighing 5 grammes.

REDUCTION.

51. Reduction is the name given to the operation by which a quantity is changed from one denomination to another, without altering its value.

To reduce quantities from higher to lower denominations, called reduction descending.

Rule 1.—Multiply the quantity of the highest denomination, by the number which connects it with the next lower, and to the product add the quantity in that lower denomination, if any; deal with this quantity in a similar way, and so continue till the proposed lower denomination is obtained.

To reduce quantities from lower to higher denominations, called reduction ascending.

Rule 2.—Divide the given quantity by the number which shows how many of its denomination make one of the next higher, treat this quotient in a similar way, and thus continue; the last quotient will be the highest denomination required, and the remainders the successive lower denominations.

EXAMPLE (1).	EXAMPLE (2).
Reduce £4 7s. 3d. to halfpence.*	In 1440 halfpence, how many £?
20	2)1440 halfpence
87 shillings	12)720 pence
12	20)60 shillings
1047 pence	*Ans.* 3 pounds.
2	
Ans. 2094 halfpence.	

EXAMPLE (3).

In £3 7s. 9d. how many farthings?

20

67 shillings

12

813 pence

4

3252 farthings.

---

* Example (1) is called reduction *descending*, and example (2) reduction *ascending*.

EXAMPLE (4).

In 47,253 farthings, how many £?

4)47253

12)11813¼ pence

20)984—5¼

£49 4s. 5¼d.

EXAMPLE (5).

In 29 cwt. 3 qrs. 25 lbs., how many lbs.?

29 cwt. 3 qrs. 25 lb.
4

119 qrs.
28

957
240

3357 lbs,

EXAMPLE (6).

In 2097 lbs., how many cwts.?

$$28\left\{\begin{array}{l}7)2097 \text{ lbs.}\\ \hline 4)299—4\end{array}\right.$$

4)74—25

18 cwt. 2 qrs. 25 lbs.

EXERCISES 5.

A.

1. Reduce £3 16s. 8d. to pence.
2. „ £6 3s. 4d. to pence.
3. „ £1 12s. 8d. to farthings.
4. „ £952 19s. 1½d. to halfpence.
5. „ £47 1s. 7½d. to half-pence.
6. „ £258 11s. 11¼d. to farthings.
7. „ 13s. 9d. to farthings.
8. „ £4 13s. to sixpences.

9. Reduce £3 17s. 9d. to threepences.
10. „ £4 8s. 8d. to fourpences.
11. „ 21 guineas to half-farthings.
12. „ 37 guineas 13s. to £.

B.

1. Convert 920 pence to £.
2. „ 1480 pence to £.
3. „ 1568 farthings to £.
4. „ 457,419 halfpence to £.
5. „ 22,599 halfpence to £.
6. „ 248,253 farthings to £.
7. „ 660 farthings to shillings.
8. „ 186 sixpences to £.
9. „ 311 threepences to £.
10. „ 266 fourpences to £.
11. „ 42,336 half-farthings to guineas.
12. „ £39 10s. to guineas.

TROY WEIGHT.

A.

1. Reduce 3lb. of silver to grains.
2. „ 37lb. of silver to penny weights.
3. „ 4 lb. 6 oz. 22 dwts. to grains.
4. „ 59 lb. 13 dwts. 5 grs. to grains.
5. „ 7 lb. 8 oz. 14 dwts. 10 grs. to grains.

B.

1. In 17280 grs. how many lbs.?
2. „ 8880 dwts. how many lbs.?
3. „ 26448 grs. how many lbs?
4. „ 340157 grs. how many lbs.?
5. „ 44506 grs. how many lbs.?

AVOIRDUPOIS WEIGHT.

A.

1. Reduce 42 tons, 7 cwt. to quarters.
2. „ 3 tons 7 cwt. 3 qrs. 11 lb. to lbs.
3. „ 18 cwt. 2 qrs. 25 lb. to lbs.
4. „ 3 cwt. 2 qrs. 21 lb. to ounces.
5. „ 16 cwt. 2 qrs. 27½ lbs. to half lbs.
6. „ 94 cwt. 3 qrs. 13 lb. to ounces.
7. „ 18 tons 2 cwt. 2 qrs. 25 lb. to drams.
8. „ 3 cwt. 3 qrs. 19 lb. 8 oz. to drams.
9. „ 19 tons. 10 cwt. 2 qrs. 14 lb. to ounces.
10. „ 170 cwt. 1 qr. 17 lb. 10 oz. to drams.

B.

1. In 3388 qrs. how many tons?
2. „ 7599 lb. how many tons?
3. „ 2097 lb., how many cwt.?
4. „ 5264 oz., how many cwt.?
5. „ 3751 half lbs. how many cwt.?
6. „ 170000 oz. how many cwt.?
7. „ 10400000 drams, how many tons?
8. „ 112512 drams, how many cwt.?
9. „ 700000 oz. how many tons?
10. „ 4885920 drams, how many cwt.?

APOTHECARIES WEIGHT.

A.

1. In 12 lb. 6 oz. how many grains?
2. „ 8 lb. 6 oz. how many scruples?
3. „ 3 lb. 7 oz. 2 dr. how many scruples?
4. „ 27 lb. 7 oz. 2 dr. 1 sc. 2 grs. how many grains?
5. „ 14 lb. 8 oz. 4 dr. 2 sc. 12 grs. how many grains?
6. „ 148 lb. 4 oz. 3 dr. 2 sc. 12 grs. how many grains?

B.

1. In 72000 grains, how many lbs.?
2. „ 2448 scruples, how many lbs.?
3. „ 1038 scruples, how many lbs.?
4. „ 159022 grains, how many lbs.?
5. „ 84772 grains, how many lbs.?
6. „ 854632 grains, how many lbs.?

LINEAL MEASURE.

A.

1. Reduce 6219 miles to yards.
2. „ 12 miles to inches.
3. „ 11 m. 7 f. 38 p. 2 yds. 2 ft. to inches.
4. „ 145 m. 2 f. to inches.
5. „ 7960 miles to yards.

B.

1. In 10945440 yards, how many miles?
2. „ 760320 inches, how many miles?
3. „ 760020 inches, how many miles?
4. „ 9203040 inches how many miles?
5. „ 14009600 yards, how many miles?

CLOTH MEASURE.

A.

1. Reduce 273 yds. 1 qr. 3 nls. to nails.
2. „ 18 yds. 2 qrs. 1 nl. to nails.
3. „ 12 ells, Flemish, 1 qr. 2 nls. to nails.
4. „ 88 ells, English, 1 qr. 1 nl. to nails.
5. „ 37481 ells, English, to ells Flemish.
6. „ 36 ells, French, to nails.
7. „ 74962 ells, English, to ells Flemish.
8. „ 34 yds. 3qrs. to inches.

B.

1. In 4375 nails, how many yards?
2. „ 297 nails, how many yards?
3. „ 150 nails, how many ells Flemish?
4. „ 1765 nails to ells English?
5. „ 62468 ells Flemish, how many ells English?
6. „ 864 nails, how many ells French?
7. „ 62468 ells, French, 2 qrs. how many ells English?
8. „ 1251 inches how many yards?

SQUARE OR SURFACE MEASURE.

A.

1. Reduce 9 square yards to square inches.
2. „ 8 acres to square yards.
3. „ 289 ac. 3 r. 27 square perches to square perches.
4. „ 97 ac. 3 r. 33 square perches to square perches.

B.

1. In 11664 square inches, how many square yards?
2. „ 14520 square yards, how many acres?
3. „ 46387 square perches, how many acres?
4. „ 15673 square perches, how many acres?

CUBIC OR SOLID MEASURE.

A.

1. Reduce 10 cubic yards to cubic inches.
2. „ 7 cubic yards to cubic inches.
3. „ 3515 cub. yds. 20 cub. ft. 1199 cub. in. to cubic inches.

B.

1. In 466560 cubic inches, how many cubic yards?
2. „ 326592 cubic inches, how many cubic yards?
3. „ 164031599 cubic inches, how many cubic yards?

MEASURE OF CAPACITY.

A.

1. Reduce 3 quarters of wheat to pecks.
2. „ 15 bush. 2 p. 1 gal. to quarts.
3. „ 4167 qrs. 2 bush. 1 p. 1 qt. to quarts.
4. „ 584 gals. 1 qt. to pints.
5. „ 62 gals. 2 qts. to gills.

B.

1. In 96 pecks, how many quarters?
2. „ 500 quarts, how many bushels?
3. „ 1066825 quarts, how many quarters?
4. „ 4674 pints, how many gallons?
5. „ 2000 gills, how many gallons?

MEASURE OF TIME.

A.

1. Reduce 33 hours 50 min. 12 seconds to seconds.
2. „ 41 weeks to minutes.
3. „ 365 d. 6 h. to seconds.
4. „ 365 d. 5 h. 48 m. 48 sec. to seconds.
5. „ 4000 years of 365 days each, how many seconds.

B.

1. In 121812 seconds, how many hours?
2. „ 413280 minutes, how many weeks?
3. „ 31557600 seconds, how many days?
4. „ 31556928 seconds, how many days?
5. „ 126144000000 seconds how many years?

ANGULAR MEASURE.

A.

1. Reduce 5° to seconds.
2. „ 4° 35′ to seconds.

B.

1. In 18000 seconds, how many degrees?
2. „ 16500 seconds, how many degrees?

EXERCISES 6.

1. In £24, how many half-farthings?
2. In 224 crowns, how many £?
3. In £3 15s. 4$\frac{5}{16}$d., how many 16ths of a penny?
4. In £2 12s. 6$\frac{1}{2}$d., how many 32nds. of a penny?
5. In 1147 half-pence, how many £?

6. In 376 guineas, how many £?
7. In £98 14s., how many guineas?
8. In 7080 8ths of a penny, how many £?
9. In 20613 32nds of a penny, how many £?
10. In 10 guineas, how many farthings ?
11. In 5040 farthings, how many guineas?
12. In 88270 half-pence, how many guineas?
13. In 13680 farthings, how many half-crowns?
14. In £289, how many crowns, half-crowns, and shillings, of each an equal number?
15. In a cubic foot of water there are 1000 ounces, how many lbs. of 16 oz. each?
16. In 875700 ounces, how many tons?
17. How many yards from the pole to the equator, each degree being $69\frac{1}{10}$ miles?
18. In 7960 miles square, how many square inches?
19. In £289, how many half-crowns?
20. How many bars of silver of 23 lb. 5 oz. 7 dwts. each, are there in 945336 grains?
21. How many years of 365 days each, are there in 35040 hours?
22. In 8760 French francs of $9\frac{3}{4}$d. each, how many £?
23. How many steps of $2\frac{1}{2}$ feet each, will be taken in walking 10 miles?
24. How often must the driving-wheel of a locomotive engine, 22 ft. in circumference, turn in passing between Liverpool and London, the distance being $203\frac{1}{2}$ miles?
25. In 18 tons, avoirdupois, of Epsom Salts, how many lbs. troy?
26. In 14 yds. 3 qrs. 3 n. of cloth, how many nails?
27. In 67 bushels of wheat, how many lbs?
28. In 47 barrels of flour, how many lbs.?
29. In 27636 lbs. of flour, how many barrels?
30. In 14070 lbs. of wheat, how many bushels?
31. In £1 12s. $3\frac{17}{32}$d., how many 32nds.?
32. In 100,000,000 square inches, how many acres?
33. In 1,000,000,000 square inches, how many acres?
34. In 15 a. 3r. 30 sq. p. 8 sq. ft. 136 sq. in., how many square inches?

COMPOUND RULES OF ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION.

COMPOUND ADDITION.

52. Compound addition is the method of uniting several quantities of the same kind, but of different denominations, into one sum.

Rule.—Arrange the numbers so that those of the same denomination* may stand directly under each other, the lowest being placed to the right. Add together the numbers of the lowest denomination, and find by reduction how many units of the next higher its sum contains; set down the remainder, if any, under the column added, and carry the quotient to the next column; proceed in the same way with all the denominations to the last, which must be added as in simple addition.

£	s.	d.	
34567890583	13	5½	
65432109416	6	6½	
23456789876	9	5¼	
76543210123	10	6¾	
23876954836	13	7¾	
85457869845	17	8½	
14542130154	2	3½	
38705689037	8	5¾	
61294310962	11	6¼	
23876954836	13	7¾	
447753909673	7	3½	Sum.
413186019089	13	10	
447753909673	7	3½	*Proof*

£	s.	d.
453876895	13	5¾
967340187	13	3
32659813	5	10½
546123104	6	6¼
458765436	18	7
541234570	2	5¾
234567895	13	5½
560789274	0	3
765432104	6	6½
439210817	15	4½
5000000099	15	9¾

---

* The principle of addition implies that units of the same kind only are added together.

TROY WEIGHT.

1.

℔	oz.	dwts.	grs.
732960471	7	5	10
329876543	10	7	17
267039528	4	14	14
670123456	1	12	7
345673759	11	13	15
271054693	5	11	13
345673759	11	13	15
728945306	6	8	11

2.

℔	oz.	dwts.	grs.
268569742	3	7	15
548790425	7	17	16
731430257	8	12	9
645219873	2	7	11
354780126	9	12	13
548790425	7	17	16
815723947	8	14	18
184276052	3	5	6

AVOIRDUPOIS WEIGHT.

3.

tons.	cwt.	qrs.	℔	oz.	drs.
187654	17	2	15	11	14
335678	13	1	13	7	11
812345	2	1	12	4	2
664321	6	2	14	8	5
213547	15	3	19	9	11
415387	10	0	7	13	5
213547	15	3	19	9	11
584612	9	3	20	2	11

4.

tons.	cwt.	qrs.	℔	oz.	drs.
954763	11	1	15	11	10
891763	8	2	11	7	7
567892	15	1	17	12	13
45236	8	2	12	4	6
108236	11	1	16	8	9
567892	15	1	17	12	13
347123	13	0	16	9	7
652876	6	3	11	6	9

APOTHECARIES' WEIGHT.

5.

℔	oz.	dr.	sc.	gr.
2492448	10	7	2	19
1989610	0	6	0	10
7874930	1	2	1	15
8760895	1	2	1	15
9357768	0	5	1	13
6549812	7	4	0	15
3450187	4	3	2	5

6.

℔	oz.	dr.	sc.	gr.
3871453	5	5	1	17
4378192	9	7	2	13
6128546	6	2	1	3
2891573	4	6	0	11
9874036	10	5	2	19
7108426	7	1	2	5
125963	1	2	0	1

LINEAL MEASURE.

7.

miles.	f.	p.	yds.	ft.	in.
246327	5	31	4	1	7
479816	6	23	2	1	10
520183	1	16	2	2	8
753672	2	8	0	2	11
379817	4	37	4	1	4
631472	3	19	1	2	0
368527	4	20	3	2	6
379817	4	37	4	1	4

8.

miles.	f.	p.	yds.	ft.	in.
637819	6	33	3	2	9
235789	7	21	4	0	7
362180	1	6	1	1	9
764210	0	18	1	0	11
435617	5	27	5	1	11
213578	6	30	4	1	4
435617	5	27	5	1	11
786421	1	9	1	0	2

CLOTH MEASURE.

	yards.	qrs.	nl.	in.
9.	167453875	3	3	2
	371345367	2	2	2
	832506123	0	1	1
	628154632	1	2	2
	387594632	3	3	2
	415678143	1	2	0
	584321856	2	2	2
	387594632	3	3	0

	yards.	qrs.	nl.	in.
10.	285710483	2	2	1
	533087654	2	3	1½
	714289516	1	2	2
	470912345	1	1	2
	567873421	3	2	1
	357946820	2	3	1½
	567873421	3	2	0
	642053179	1	1	2

SQUARE MEASURE, OR MEASURE OF SURFACE.

	acres.	r.	s.p.	s.yd.	s.ft.	s.in.
11.	54986	3	37	25	7	110
	97845	1	23	13	5	108
	45013	0	2	5	1	34
	2154	2	11	10	7	108
	65487	3	37	14	3	54
	35789	1	23	13	4	67
	65487	3	37	14	7	126
	64210	2	16	30	4	77

	acres.	r.	s.p.	s.yd.	s.ft.	s.in.
12.	35749	2	23	21	5	92
	24156	1	17	20	7	52
	64250	1	16	8	7	122
	75843	2	22	9	1	94
	71843	0	18	13	7	135
	63105	1	31	10	5	87
	71843	0	18	15	3	63
	36894	2	8	29	3	57

CUBIC, OR SOLID MEASURE.

	c.yds.	c.ft.	c.in.
13.	4178967	23	537
	6807435	18	672
	5821032	4	1190
	3192564	7	1257
	7314598	17	1156
	7654321	9	999
	7314598	17	1157
	2345678	17	728

	c.yds.	c.ft.	c.in.
14.	2345678	16	697
	4567891	17	808
	7654321	10	1030
	5432108	9	921
	3974206	12	587
	5642301	14	1604
	3974206	12	588
	4357698	12	123

MEASURE OF CAPACITY.

	qrs.	b.	p.	gl.	qt.	p.
5.	234987	5	3	1	3	1
	517853	6	2	0	2	1
	765012	2	0	0	0	1
	482146	1	1	1	1	1
	354639	3	1	0	3	0
	123456	4	1	1	1	1
	354639	3	1	0	3	0
	876543	3	2	0	2	1

	qrs.	b.	p.	gl.	qt.	p.
16.	387456	5	2	1	2	1
	438719	4	3	0	3	1
	612543	2	1	0	1	1
	561280	3	0	1	0	1
	987546	6	3	1	3	1
	107689	2	2	0	2	1
	987546	6	3	1	3	1
	892310	5	1	1	1	1

MEASURE OF TIME.

	years.	dys.	hs.	m.	sec.
7.	396784	230	13	25	35
	410725	181	10	27	29
	603215	134	10	34	25
	589274	183	13	32	31
	256739	237	17	53	38
	106053	169	11	21	29
	256739	237	17	53	39
	893946	195	12	38	30

	years.	dys.	hs.	m.	sec.
18.	471205	170	10	27	28
	614532	205	12	32	22
	528794	194	13	32	33
	385467	159	11	27	37
	827109	245	18	39	47
	710490	200	20	30	41
	827109	245	18	39	48
	289509	164	3	29	18

ANGULAR MEASURE.

	circles	°	′	″
19.	169478	230	30	35
	358973	160	29	41
	830521	129	29	25
	641026	199	30	19
	456798	236	39	47
	108569	177	23	23
	456798	236	39	48
	891430	182	36	36

	circles	°	′	″
20.	267149	170	23	33
	807491	152	25	26
	732850	189	36	27
	192508	207	34	34
	768954	281	58	49
	123456	169	37	58
	768954	281	53	49
	876543	190	22	02

Liverpool Water Works Account for the Year ending December 31st, 1859.

21. ORDINARY INCOME.

	£	s.	d.
Waterrents of 1859	63179	7	11
Arrears of 1858	2400	1	10
„ of 1857	56	5	5
Water for shipping	6852	12	1
Water for miscellaneous purposes	1549	11	4
Water rates	9016	11	4
Fire Police Committee	500	0	0
Cutting off and putting on water	6	6	11
Meter rents	227	5	6
Rent of pipes	2	14	11
Rent of Land &c., Soho Square	85	14	5
Chorley Water Works	1523	8	4

22. ORDINARY EXPENDITURE.

	£	s.	d.
Bootle Engine Station	666	19	11
Everton „	50	8	4
Bevington „	100	1	8
Wm. Henry St. „	180	14	5
Hotham St. „	149	9	0
Water St. „	697	12	10
Windsor St. „	822	13	7
Green Lane „	2006	16	6
Audley St. „	612	10	1
Kensington reservoir	173	9	8
Park-hill „	28	0	10
Atherton St. „	13	16	4
Audley St. „	2	11	2
General distribution	4794	3	10
General repairs and expenses	4753	7	6
General charges and disbursements	4854	18	3
Salaries	4912	11	10
Rivington account	6037	19	0
Interest account	72985	15	11
Mortgage stamps	137	11	4
Chorley Water Works	2258	15	2

REVENUE FOR THE YEAR ENDING 31ST OF MARCH, 1860.

		£	s.	d.
23.	Customs	24,391,083	17	4
	Excise	20,240,466	13	6
	Stamps	8,040,090	11	10
	Land tax	1,137,034	1	2
	Assessed taxes	2,100,539	7	9
	Property and Income Tax	9,666,141	19	3
	Post Office	3,310,655	8	0
	Crown Lands	416,530	18	8
	Miscellaneous	1,801,584	3	0

COMPOUND SUBTRACTION.

Compound subtraction is the method of finding the difference
en two compound quantities of the same kind.
'e.—Place the less quantity under the greater, with numbers of
me denomination* under each other. Begin at the right hand
ıbtract if possible each number of the lower line from that above
d enter the remainder underneath. But when the number
is greater than that above it, add to the upper as many as
a unit of the next higher denomination, then subtract the
number from the upper thus increased, and enter the remain-
low, taking care to carry one to the next number of the lower
ınd proceed in the same way with each denomination to the
vhich subtract as in simple subtraction.

	£	s.	d.
	6873940	13	5½
	2698765	10	8¼
	4175175	2	9¼
of	6873940	13	5½

£	s.	d.
68210759	11	10¼
35987654	13	7½
32223104	18	2¾

EXERCISES 8.

	£	s.	d.
1.	5798169	0	8½
	1380927	14	10¾

	£	s.	d.
2.	498706	10	10¾
	376970	14	7¼

TROY WEIGHT.

	lb.	oz.	dwt.	grs.
.	387495	10	13	17
	295769	7	16	11

	lb.	oz.	dwt.	grs.
4.	679348	6	11	15
	497634	10	5	18

AVOIRDUPOIS WEIGHT.

tons.	cwt.	qrs.	lbs.	oz.	dr.
4987	13	2	17	11	10
1639	18	2	23	5	13

	tons.	cwt.	qrs.	lbs.	oz.	dr.
6.	106947	12	1	27	10	14
	56398	16	2	21	13	9

---

subtraction, as in addition, units of the same kind only are sub-

APOTHECARIES WEIGHT.

	lb.	oz.	dr.	sc.	grs.		lb.	oz.	dr.	sc.	grs.
7.	890743	9	5	2	17	8.	698054	7	6	1	11
	430658	8	7	1	10		587698	10	3	2	16

LINEAL MEASURE.

	miles.	f.	po.	yd.	ft.	in.		miles.	f.	po.	yd.	ft.	in.
9.	78874	6	25	5	1	5	10.	89854	5	39	3	2	7
	17088	3	30	3	2	10		14806	7	23	4	0	9

CLOTH MEASURE.

	yards.	qrs.	nl.	in.		yards.	qrs.	nl.	in.
11.	46589567	3	2	1	12.	38541729	2	3	2
	92764588	1	3	2		10690487	3	1	1

SQUARE MEASURE OR MEASURE OF SURFACE.

	ac.	ro.	s.p.	s.yd.	s.ft.	s.in.		ac.	ro.	s.p.	s.yd.	s.ft.	s.in.
13.	7685	1	21	18	7	72	14.	5887	3	37	15	3	137
	3809	2	35	20	4	120		8658	1	19	25	7	93

CUBIC OR SOLID MEASURE.

	c.yards.	c.ft.	c.in.		c.yards.	c.ft.	c.in.
15.	7580897	23	587	16.	6849857	17	1689
	3459860	19	1167		2190863	25	1163

MEASURE OF CAPACITY.

	qrs.	b.	p.	gl.	qt.	pt.		qrs.	b.	p.	gl.	qt.	pt.
17.	54987	5	3	1	2	1	18.	68973	7	2	0	1	0
	37895	6	1	0	3	1		39807	4	0	1	2	1

MEASURE OF TIME.

	years.	da.	ho.	m.	sec.		years.	da.	ho.	m.	sec.
19.	58967	287	18	28	55	20.	76948	169	19	57	31
	56967	189	19	43	28		48695	287	11	25	49

ANGULAR MEASURE.

	circles.					circles.			
21.	7496	188°	58′	27″	22.	85695	359°	17′	59″
	8985	280	27	45		87851	164	42	23

COMPOUND MULTIPLICATION.

ompound multiplication is the method of finding the amount pound quantity, when it is to be repeated a certain number

—Begin to multiply the lowest denomination by the multi- d the number of units of the next higher denomination in luct, which carry to the next product, placing the remainder e number multiplied, and so proceed from one denomination er, entering the entire product of the highest denomination.

en the multiplier is a composite number, or can be resolved ors, multiply by each factor in succession; when not a com- mber, multiply by the nearest composite, then multiply the and by the difference, and add the products if the composite nd subtract them if it be more than the given number.

hen the multiplier consists of several figures, multiply nd the product by 10, and so on till the multiplicand and make as many lines as there are figures in the multiplier; ltiply the last line by the last left-hand figure, the next, by d left hand figure, and so on with the rest, the sum of these will be the answer.

—See Supplement to Multiplication, Example 1.

EXAMPLE (1).

ly £2 16s. 7½d. by 5?*

16s. 7½d.
5
3 1½

By Addition.

£2 16 7½
2 16 7½
2 16 7½
2 16 7½
2 16 7½
£14 3 1½

EXAMPLE (2).

ly £2 16s. 5¾d. by 757?

---

ltiplication the multiplier must be an abstract number, for we cannot hillings by shillings, as such a thing as 7 shillings taken 4 shillings no meaning.

Rule 2.—

$$\begin{array}{rrrl}
\pounds 2 & 16 & 5\frac{3}{4} & \times 1 \\
 & & 12 & \\
\hline
33 & 17 & 9 & = 12 \text{ times.} \\
 & & 7 & \\
\hline
237 & 4 & 3 & = 84 \text{ times.} \\
 & & 9 & \\
\hline
2134 & 18 & 3 & = 756 \text{ times.} \\
2 & 16 & 5\frac{3}{4} & 1 \\
\hline
\pounds 2137 & 14 & 8\frac{3}{4} & \ 757
\end{array}$$

Rule 3.—

$$\begin{array}{rrrl}
\pounds 2 & 16 & 5\frac{3}{4} & \times 7 \\
 & & 10 & \\
\hline
28 & 4 & 9\frac{1}{2} & \times 5 \\
 & & 10 & \\
\hline
282 & 7 & 11 & \\
 & & 7 & \\
\hline
1976 & 15 & 5 & = 700 \text{ times.} \\
141 & 8 & 11\frac{1}{2} & = 50 \quad ,, \\
19 & 15 & 4\frac{1}{4} & = 7 \quad ,, \\
\hline
\pounds 2137 & 14 & 8\frac{3}{4} & \ 757
\end{array}$$

Otherwise.

$$\begin{array}{l}
\pounds 2 \ 16 \ \ 5\frac{3}{4} \\
20 \\
\hline
56 \\
\ \ 12 \\
\hline
677 \\
\quad 4 \\
\hline
2711 \text{ farthings.}
\end{array}
\qquad\qquad
\begin{array}{r}
2711 \text{ farthings.} \\
757 \phantom{\text{ farthings.}} \\
\hline
18977 \\
13555 \\
18977 \\
\hline
4)2052227 \\
\hline
12)513056\frac{3}{4} \\
\hline
20)42754 \quad 8\frac{3}{4} \\
\hline
\pounds 2137 \ \ 14 \ \ 8\frac{3}{4}
\end{array}$$

EXAMPLE (3).

Multiply £2 18s. 7½d. by 3, 5, 9, 7, 6, by 6, 7, 9, 5, 3, and by 5, 9, 6, 7, 3.

£2 18s. 7½d.	£2 18s. 7½d.	£2 18s. 7½d.
3	6	5
8 15 10½	17 11 9	14 13 1½
5	7	9
43 19 4½	123 2 3	131 18 1½
9	9	6
395 14 4½	1108 0 3	791 8 9
7	5	7
2770 0 7½	5540 1 3	5540 1 3
6	3	3
	£16620 3 9	£16620 3 9

£16620 3 9 = 3 × 5 × 9 × 7 × 6;
or, 5670 times £2 18s. 5½d.

EXAMPLE (4).

Multiply £2 13s. 7¾d. by 5, 7, 9, 6, 3, 2, 11, by 11, 2, 3, 6, 9, 7, 5, and by 7, 9, 6, 11, 2, 5, 3.

£2 13s. 7¾d.	£2 13s. 7¾d.	£2 13s. 7¾d.
5	11	7
13 8 2¾	29 10 1¼	18 15 6¼
7	2	9
93 17 7¼	59 0 2½	168 19 8¼
9	3	6
844 18 5¼	177 0 7½	1013 18 1½
6	6	11
5069 10 7½	1062 3 9	11152 19 4½
3	9	2
15208 11 10½	9559 13 9	22305 18 9
2	7	5
30417 3 9	66917 16 3	111529 13 9
11	5	3
£334589 1 3	£334589 1 3	£334589 1 3

EXAMPLE (5).

Multiply 3 tons 13 cwt. 3 qrs. 23 lb. 11 oz. 13 dr. by 3, 5, 7, 9, 8, 10, and by 10, 5, 7, 9, 8, 3.

tons	cwt.	qrs.	lb.	oz.	dr.	tons	cwt.	qrs.	lb.	oz.	dr.
3	13	3	23	11	13	3	13	3	23	11	13
					3						10
11	1	3	15	3	7	36	19	2	13	6	2
					5						5
55	9	1	20	1	3	184	18	0	10	14	10
					7						7
388	6	0	0	8	5	1294	6	2	20	6	6
					9						9
3494	14	0	4	10	13	11649	0	0	15	9	6
					8						8
27957	12	1	9	6	8	93192	1	0	12	11	0
					10						3
279576	3	1	10	1	0	279576	3	1	10	1	0

EXAMPLE (6).

Multiply 3 lb. 11 oz. 17 dwt. 19 grs. by 7, 3, 10, 5, 9, 6, 2, and by 9, 6, 2, 5, 10, 3, 7.

lb.	oz.	dwt.	grs.	lb.	oz.	dwt.	grs.
3	11	17	19	3	11	17	19
			7				9
27	11	4	13	35	11	0	3
			3				6
83	9	13	15	215	6	0	18
			10				2
838	0	16	6	431	0	1	12
			5				5
4190	4	1	6	2155	0	7	12
			9				10
37713	0	11	6	21550	3	15	0
			6				3
226278	3	7	12	64650	11	5	0
			2				7
452556	6	15	0	452556	6	15	0

EXAMPLE (7).

Multiply 2 a. 3 r. 33 sq. p. 27 sq. yds. 8 sq. ft. 99 sq. in. by 4, 3, 5, 6, 7, 9, 10, and by 10, 9, 7, 6, 5, 3, 4.

a.	r.	s.p.	s.yds.	s.ft.	s.in
2	3	33	27	8	99
					4
11	3	15	20¼	7	108
					3
35	2	7	2¼	5	36
					5
177	2	35	15¼	8	36
					6
1066	1	12	24	4	72
					7
7464	1	9	19¾	4	72
					9
67178	3	7	¼	4	72
					10
671787	3	30	7½	0	0

a.	r.	s.p.	s.yds.	s.ft.	s.in.
2	3	33	27	8	99
					10
29	2	19	6¾	5	126
					9
266	2	13	5¼	7	126
					7
1866	0	12	12½	1	18
					6
11196	1	34	14½	6	108
					5
55982	1	12	15	6	108
					3
167946	3	37	16¾	2	36
					4
671787	3	30	7½	0	0

EXERCISES 9.

1. Multiply £2 13s. 7d. by 2.
2. „ £3 14s. 9d. by 3.
3. „ £4 17s. 10½d. by 4.
4. „ £5 18s. 9½d. by 5.
5. „ £3 19s. 10½d. by 6.
6. „ £5 18s. 10½d. by 7.
7. „ £4 14s. 8¾d. by 8.
8. „ £2 16s. 7¼d. by 9.
9. „ £1 13s. 7¾d. by 10.
10. „ * £2 13s. 5d. by 3, 5, 6.
11. „ £2 15s. 7d. by 2, 3, 5, 7.

---

* All the questions that follow, may be made to embrace two, three, or more questions, by varying the arrangement of the factors.

12. „ £2 15s. 3d. by 2, 3, 5, 7, 9.
13. „ £1 18s. 9d. by 2, 3, 5, 7, 9.
14. „ £2 13s. 5½d. by 6, 2, 3, 5, 7, 9.
15. „ £2 17s. 9½d. by 4, 3, 5, 7, 9.
16. „ £2 17s. 7½d. by 6, 7, 5, 2, 9.
17. „ £2 13s. 9¾d. by 6, 5, 3, 7, 4.
18. „ £2 13s. 8¾d. by 10, 5, 3, 7, 6.
19. „ £2 13s. 5¾d. by 10, 6, 9, 8, 5.
20. „ £2 16s. 9¾d. by 2, 3, 5, 6, 7.
21. „ £1 18s. 7¾d. by 6, 2, 3, 5, 7, 9, 11.
22. „ £2 15s. 3¾d. by 3, 2, 6, 5, 7, 9.
23. „ £2 15s. 5¾d. by 10, 7, 9, 11, 5, 8.
24. „ £2 15s. 9¾d. by 10, 11, 9, 8, 7, 6.
25. „ £2 11s. 11¾d. by 5, 3, 11, 7, 9, 6, 2.
26. „ £2 13s. 7¾d. by 2, 5, 7, 6, 9, 3, 11.
27. „ £3 18s. 11¾d. by 5, 7, 3, 6, 9, 10, 5, 11,
28. „ 29 tons 13 cwt. 3 qrs. 25lb. 13 oz. by 6, 7, 4. 5, 8.
29. „ 10 tons 15 cwt. 3 qrs. 19 lb. 7 oz. 8 drs. by 4, 2, 5, 6.
30. „ 13 tons 3 cwt. 3 qrs. 21 lb. 13 oz. by 4, 3, 5, 7, 9.
31. „ 28 lb. 10 oz. 17 dwts. 21 grs. by 2, 3, 4, 5, 6, 7.
32. „ 43 lb. 9 oz. 18 dwts. 22 grs. by 3, 4, 5, 6, 7.
33. „ 5 yrs. 195 dys. 18 hrs. 37 min. 49 sec. by 7, 5, 9, 8, 6.
34. „ 8 yrs. 235 dys. 17 hrs. 39 min. 49½ sec. by 2, 3, 4, 5, 6, 7.
35. „ 18 miles 5 fur. 35 p. 4 yds. 2 ft. 7 in. by 2, 3, 5, 6, 7.
36. „ 2 miles 5 fur. 37 p. 4 yds. 2 ft. 7 in. by 2, 3, 4, 5, 6, 7.
37. „ 38 acres 3 rds. 33 sq. p. 25 sq. yds. 7 sq. ft. 59 sq. in. by 2, 3, 4, 5, 6.
38. „ 89 acres 3 rds. 37 sq. p. 25½ sq. yds. 8 sq. ft. 130 sq. in. by 2, 3, 4, 5, 6.

COMPOUND DIVISION.

55. Compound Division is the method of dividing a compound quantity into as many equal parts, as the divisor contains units, and also of finding how often one compound number is contained in another of the same kind.

Rule.—When the divisor is an abstract number divide the highest denomination of the given dividend by the divisor, reduce the remainder, if there be any, to the next lower denomination, adding in any of the same denomination in the dividend; divide this, as before, and so proceed with each successive denomination to the lowest, or until nothing remains.

When the divisor and dividend are both compound numbers of the same kind.

Rule.—Reduce both numbers to the same denomination, then divide as in simple division, and the quotient will be the answer required.

EXAMPLE (1).

Divide £82 2s. 6d. by 36.

```
        £   s.  d.
  36 ) 82   2   6 (£2 5s. 7½d.
       72
       —
       10
        20
      ———
     ) 202 ( 5
       180
      ———
        22
        12
      ———
     ) 270 ( 7
       252
      ———
        18
         2
        —
     ) 86 ( 1
       86
```

```
              £    s.   d.
     ⎧ 4 )  82    2    6
  36 ⎨      ————————————
     ⎩ 9 )  20   10    7½
            ————————————
            £2    5    7½

              £    s.   d.
     ⎧ 6 )  82    2    6
  36 ⎨      ————————————
     ⎩ 6 )  13   13    9
            ————————————
      Ans.  £2    5    7½
```

EXAMPLE (2).

* Divide £15 18s. 6d. by £1 2s. 9d., which means how often can £1 2s. 9d. be taken out of £15 18s. 6d.

```
  £  s.  d.     £   s.  d.
  1  2   9      15  18  6
  20            20
  —            ———
  22           318
  12            12
 ———           ———
 273        ) 3822 ( 14 times
               273
              ————
              1092
              1092
```

Proof.

```
   £   s.   d.
   1   2    9
            7
  —————————————
   7  19    3
            2
  —————————————
 £15  18    6
```

---

* In the application of division to concrete quantities, either the divisor or quotient must be an abstract number, for in dividing 3822 d. by 273 d. it would be absurd to call the quotient 14 d. times, which would be the same as multiplying 273 d. by 14 d., an expression totally devoid of meaning.

EXAMPLE (3).

Divide £106 16s. 10½d. by 328¾.*

328¾) 106 16 10½ (
4 4

1315) 427 7 6 (
20

1315) 8547 (6s. 6d.
7890

657
12

1315) 7890 (6
7890

	s.	d.
	6	6
		3
4)	19	6
	4	10½

Proof.

6s. 6d. × 8
10

3	5	0	× 2		
		10			
32	10	0			
		3			
97	10	0	= 300	times	6s.6d.
6	10	0	= 20	”	”
2	12	0	= 8	”	”
0	4	10½	= ¾	”	”
£106	16	10½	= 328¾	”	”

EXAMPLE (4).

Divide £16620 3s. 9d. by 3, 5, 9, 7, 6, and by 6, 7, 9, 5, 3.

	£	s.	d.
3)	16620	3	9
5)	5540	1	3
9)	1108	0	3
7)	123	2	3
6)	17	11	9
	2	18	7½

	£	s.	d.
6)	16620	3	9
7)	2770	0	7½
9)	395	14	4½
5)	43	19	4½
3)	8	15	10½
	2	18	7½

EXAMPLE (5).

Divide 70046 tons 19 cwt. 1 qr. 24 lb. 15 oz. by 2, 3, 4, 5, 6, 7. and 7, 6, 5, 4, 3, 2.

	tons	cwt.	qr.	lb.	oz.	
2)	70046	19	1	24	15	
3)	35023	9	2	26	7	8
4)	11674	9	3	18	2	8
5)	2918	12	1	25	8	10
6)	583	14	1	27	8	2
7)	97	5	2	27	14	11
	13	17	3	23	15	13

	tons	cwt.	qr.	lb.	oz.	
7)	70046	19	1	24	15	
6)	10006	14	0	23	9	
5)	1667	15	2	22	9	8
4)	333	11	0	15	11	8
3)	83	7	3	3	14	14
2)	27	15	3	19	15	10
	13	17	3	23	15	13

---

* Here the divisor and dividend being each multiplied by 4 without affecting the quotient, is a fact already spoken of under the head of division.

EXAMPLE (6).

Divide 450665 acres 0 r. 4 sq. p. 4 sq. yds. by 3, 9, 7, 10, 5, 8 and by 8, 5, 10, 7, 9, 3.

	acres	r.	sq.p.	sq.yd.		
3)	450665	0	4	4		
9)	150221	2	28	1	3	
7)	16691	1	7	16¼	6	48
10)	2384	1	35	10¼	7	148
5)	238	1	31	15¼	1	120
8)	47	2	30	8½	7	24
	5	3	33	23	7	93

	acres	r.	sq.p.	sq.yd.		
8)	450665	0	4	4		
5)	56333	0	20	15½	1	18
10)	11266	2	20	2½	3	90
7)	1126	2	25	30	3	9
9)	160	3	32	7¾	8	63
3)	17	3	21	10¼	4	135
	5	3	33	23	7	93

And 5 acres 3 r. 33 sq. p. 23 sq. yds. 7 sq. ft. 93 sq. in. is 3 × 9 × 7 × 10 × 5 × 8, or 75600 times less than 450665 acres 0 r. 4 sq. p. 4 sq. yds., and if 450665 acres 0 r. 4 sq p. 4 sq. yds. be divided by 75600, by long division, the quotient will be 5 acres 3 r. 33 sq. p. 23 sq. yds. 7 sq. ft. 93 sq. in.

EXAMPLE (7).

Divide 20060 miles 6 fur. 18 per. 1 yd. by 2, 3, 4, 5, 6, 7, by 7, 6, 5, 4, 3, 2, and by their product 5040.

	miles	f.	p.	yd.		
2)	20060	6	18	1		
3)	10030	3	9	0	1	6
4)	3343	3	29	3	2	6
5)	835	6	37	2	1	0
6)	167	1	15	2	2	0
7)	27	6	35	4	3	1
	3	7	33	3	2	7

	miles	f.	p.	yd.		
7)	20060	6	18	1		
6)	2865	6	25	2½	0	0
5)	477	5	4	1	1	0
4)	95	4	8	4	2	0
3)	23	7	2	1	0	6
2)	7	7	27	1½	2	2
	3	7	33	3	2	7

```
             miles   f.  p.   yd.  m.  f.  p.  yd. ft. in.
5040 ) 20060   6  18    1 ( 3   7  33   3   2   7
       15120
       -----
        4940
           8
       -----
5040 ) 39526 ( 7
       35280
       -----
        4246
          40
       ------
5040 ) 169858 ( 33
       15120
       ------
        18658
        15120
       ------
         3538
           5½
        ------
        17691
         1769
        ------
 5040 ) 19460 ( 3
        15120
        -----
         4340
            3
        -----
 5040 ) 13020 ( 2
        10080
        -----
         2940
           12
        -----
 5040 ) 35280 ( 7
        35280
        -----
```

EXERCISES 10.

1. Divide £19 13s. 8d. by 2.
2. „ £14 16s. 6d. by 3.
3. „ £15 17s. 8d. by 4.
4. „ £16 17s. 6d. by 5.
5. „ £16 0s. 9d. by 6.
6. „ £116 16s. 3d. by 7.
7. „ £19 13s. 8d. by 8.
8. „ £50 1s. 3d. by 9.
9. „ £26 17s. 1d. by 10.
10. „ *£19207 2s. 6d. by 6, 7, 5, 9, 3.
11. „ £13773 7s. 6d. by 3, 6, 5, 7, 9.
12. „ £7268 12s. 6d. by 4, 3, 5, 9, 7.
13. „ £32106 7s. 6d. by 6, 2, 5, 7, 9, 3.
14. „ £16620 3s. 9d. by 3, 5, 9, 7, 6.
15. „ £2107 17s. 6d. by 6, 3, 2, 5, 7.
16. „ £19207 2s. 6d. by 6, 7, 5, 9, 3.
17. „ £21498 15s. 0d. by 2, 7, 5, 3, 9, 4.
18. „ £42784 17s. 6d. by 5, 3, 6, 2, 7. 9.
19. „ £606217 10s. 0d. by 5, 6, 4, 7, 9, 3, 10.
20. „ £768941 5s. 0. by 10, 7, 9, 11, 5, 8.
21. „ £928273 10s. 0d. by 6, 7, 8, 9, 11, 10.
22. „ £9603 11s. 3d. by 3, 9, 5, 7, 6.
23. „ £44497 18s. 9d. by 2, 9, 7, 5, 3, 6.
24. „ £9757 2s. 6d. by 2, 4, 3, 5, 7, 9.
25. „ £10922 12s. 6d. by 4, 3, 5, 7, 9.
26. „ £3201 8s. 9d. by 9, 7, 5, 6.
27. „ £334589 1s. 3d. by 2, 5, 7, 6, 9, 3, 11.
28. „ £30547 2s. 6d. by 5, 2, 3, 6, 7, 9.
29. „ £324194 1s. 3d. by 5, 3, 11, 7, 9, 6, 2.
30. „ £12314826 11s. 3d. by 5, 7, 3, 6, 9, 10, 5, 11.
31. „ £1706729 1s. 3d. by 10, 11, 7, 9, 5, 3, 6.
32. „ £4634437 10s. 0d. by 10, 11, 9, 8, 7, 5, 3, 2.
33. „ 2950 tons 6 cwt. 1 qr. 1 lb. 9 oz. by 2, 5, 4, 3, 6.
34. „ 106023 tons 11cwt. 0qr. 4lb. 3 oz. 6drs. by 6, 3, 9, 5, 7.
35. „ 165 cwt. 3 qrs. 11 lb. 11 oz. 9 drs, by $6\frac{1}{3}\frac{2}{2}$.
36. „ 100702 ac. 0r. 24sq. p. 18sq.yds. 6sq.ft. 72sq.in. by 2520.

---

* All the questions that follow may be made to embrace two, three, or more questions, by varying the arrangement of the divisors; and further, a long division sum can be educod by using the product of the several divisors as the long division divisor.

THE APPLICATION OF THE FOREGOING RULES TO PRACTICAL PURPOSES.*

EXAMPLE (1).

Bought 7 barrels of flour at £1 13s. 8½d. per barrel, what is th cost?

	£1	13	8½	= price of 1 barre
			7	
Ans.	£11	15	11½	= price of 7 barrels.

Addition.

Or thus	£1	13	8½
	1	13	8½
	1	13	8½
	1	13	8½
	1	13	8½
	1	13	8½
	1	13	8½
Ans.	£11	15	11½

EXAMPLE (2).

Bought 387 cwt. of butter at £3 15s. 7½d. per cwt

	£3	15	7½			
			12			
	45	7	6	= price of 12 cw		
			8			
	363	0	0	=	„	96 cwt.
			4			
	1452	0	0	=	„	384 cwt.
	11	6	10½	=	„	3 cwt.
Ans.	£1463	6	10½	=	„	387 cwt

---

* A truly scientific arrangement would place the applications of the science of numbers last, presenting the arrangement in the following order:—

1st. All the methods of using integer numbers according to the scale of tens and according to varying scales.

2nd. All the methods of treating fractional quantities whether vulgar or decimal.

3rd. The proportion and ratios of numbers.

4th and last, The applications of numbers to practical purposes.

£3 15 7½ × 7
10

37 16 3 × 8 = price of 10 cwt.
10

378 2 6 = „ 100 cwt.
3

1134 7 6 = „ 300 cwt.
302 10 0 = „ 80 cwt.
26 9 4½ = „ 7 cwt.

Ans. £1463 6 10½ = 387 cwt.

EXAMPLE (3).

If 9 cwt. of pork cost £24 2s. 3d., what is the cost of 1 cwt.?

£ s. d.
9) 24 2 3

Ans. £2 13 7

£ s. d.
9 { 3) 24 2 3
3) 8 0 9

Ans. £2 13 7

EXAMPLE (4).

How many lbs. of tea at 3s. 10½d. per lb. can I buy for £7 7s. 3d.?

s. d. £ s. d.
3 10½ 7 7 3
12 20

46 147
2 12

93 1767
2

93) 3534 (38 lbs. *Ans.*
279

744
744

EXAMPLE (5).

Bought 3869 cwt. of lead for £3457 18s. $4\frac{1}{2}$d., what is it a cwt.?

£ s. d.
3869) 3457 18 $4\frac{1}{2}$
20
69158 (17 $10\frac{1}{2}$ *Ans.*
3869
30468
27083
3385
12
) 40624 ($10\frac{1}{2}$
3869
1934
2
) 3869 (1
3869

Proof.
17 $10\frac{1}{2}$ × 9
10
8 18 9 × 6
10
89 7 6 × 8
10
893 15 0
3

£	s.	d.			cwt.
2681	5	0	=	price of	3000
715	0	0	=	,,	800
53	12	6	=	,,	60
8	0	$10\frac{1}{2}$	=	,,	9
£3457	18	$4\frac{1}{2}$	=	,,	3869

EXAMPLE (6).

* If $847\frac{5}{16}$ yards of cloth be sold for £748 9s. $2\frac{1}{4}$d., what is it sold at a yard?

£ s. d.
$847\frac{5}{16}$) 748 9 $2\frac{1}{4}$ (
16 4
13557) 2993 16 9
4
11975 7 0
20
13557) 239507 (17s. 8d. *Ans.*
13557
103937
94899
9038
12
13557) 108456 (8
108456

s. d.
17 8
5
16)4 8 4
5 $6\frac{1}{4}$

Proof.
17 8 × 7
10
8 16 8 × 4
10
88 6 8
8

£	s.	d.		
706	13	4	= price of	800
35	6	8	= ,,	40
6	3	8	= ,,	7
	5	$6\frac{1}{4}$	= ,,	$\frac{5}{16}$
£748	9	$2\frac{1}{4}$	=	$847\frac{5}{16}$

---

* Here the dividend and divisor are both multiplied by 16.

EXAMPLE (7).

Bought 3729$\frac{14}{31}$ yards of cloth for £843 0s. 2$\frac{3}{4}$d., what is it for 1 yard?

```
                         £    s.  d.
 3729 14/31  ⎤         843   0   2¾
   31        ⎥                  10
 ──────      ⎥        ─────────────
   3733      ⎥        8430   2   3½
 11188       ⎥                   3
 ──────      ⎥        ──────────────
 115613      ⎦        25290   6  10½ = 30
                        843   0   2¾ =  1
                      ──────────────
                      26133   7   1¼
                         20
                      ──────────────
  115613 ) 522667 (4  6¼
           462452
           ──────
            60215
               12
           ──────
  115613 ) 722581 (6
           693678
           ──────
            28903
                4
           ──────
  115613 ) 115613 (1
           115613
           ──────
```

Add £$\frac{3}{8}$ + $\frac{3}{8}$s. + $\frac{3}{8}$ guinea.

		s.	d.
£$\frac{3}{8}$ means £3 divided by 8 . . .	=	7	6
$\frac{3}{8}$ shilling, means 3s. divided by 8 .	=	0	4$\frac{1}{2}$
$\frac{3}{8}$ guinea means 3 guineas divided by 8	=	7	10$\frac{1}{2}$
Ans.		15	9

Add $\frac{5}{16}$ guinea + £$\frac{7}{8}$ + $\frac{5}{6}$ shilling.

		£	s.	d.
$\frac{5}{16}$ guinea, means 5 guineas divided by 16	=		6	6$\frac{3}{4}$
£$\frac{7}{8}$ means £7 divided by 8 . . .	=		17	6
$\frac{5}{6}$ shilling, means 5s. divided by 6 . .	=		0	10
Ans.		£1	4	10$\frac{3}{4}$

EXERCISES 11.

MULTIPLICATION AND DIVISION.

1. Bought 73 yards of cloth at 13s. 8d. per yard.
2. Bought 85 cwt. of tallow at £2 18s. 11d. per cwt.
3. Bought 365 cwt. of sugar at £2 18s. 9½d. per cwt.
4. Bought 547 cwt. of pork for £1524 15s. 3d., what is it a cwt.?
5. Bought 693 cwt. of sugar for £1238 14s 9d., what is it a cwt.?
6. Bought 785 cwt. of cheese at £1 13s. 10d. per cwt.
7. Bought 854 cwt. of beef at £3 13s. 10½d. per cwt.
8. If 2853 cwt. of soap cost £4374 19s. 1½d., what is it a cwt.?
9. What is the cost of 6987 yards of cloth at 15s. 9d. per yard?
10. If 853 cwt. of pork cost £2493 4s. 11½d., what is the cost of 1 cwt.?
11. Bought 3 tons of tea at 3s. 5¾d. the lb.
12. What is the price of 795½ cwt. of beef at £2 13s. 7½d. per cwt.?
13. If 963½ barrels of flour cost £2481 0s. 3d., what is the cost of 1 barrel?
14. Bought 7 cwt. 3 qrs. 19 lb. of tea at 3s. 10½d. per lb.
15. Bought 3 tons 13 cwt. 3 qrs. 25 lb of coffee at 1s. 5¾d. per lb.
16. What is the price of 17 cwt. 2 qrs. 19 lb. of tea at 2s. 5¾d. per lb.?
17. If 9675½ cwt. of soap cost £16266 18s. 8¼d., what is the cost of 1 cwt.?
18. If 3 tons 16 cwt. 3 qrs. 17 lb. of tea cost £1453 8s. 10½d., what is the price of a lb.?
19. If 13 cwt. 3 qrs. 27 lb. of tea cost £284 0s. 4½d., what is the price of 1 lb.?
20. Bought 8 tons 17 cwt. 2 qrs. 23 lb. of coffee at 1s. 5¾d. per lb.
21. If 7385½ cwt. of iron cost £6862 7s. 2½d., what is the price of 1 cwt.?
22. If 8 tons of coffee cost £1325 6s. 8d., what is the cost of 1 lb.?
23. Bought 79¾ yards of cloth at £1 2s. 11d. per yard.
24. Bought 69¼ cwt. of sugar at £2 13s. 7d. per cwt.
25. If 69⅝ cwt. of sugar cost £96 6s. 3½d., what is the price of 1 cwt.?
26. If $257\frac{15}{16}$ cwt. of beef cost £1094 11s. 0½d., what is the price of 1 cwt.?
27. If 4 tons 17 cwt. 2 qrs. 19 lb. of coffee cost £797 12s. 8½d., what is the price of 1 lb.?
28. Bought 6 tons 13 cwt. 3 qrs. 24½ lb. of coffee at 1s. 5½d. per lb

29. Bought 879$\frac{11}{16}$ yards of cloth at 13s. 8d. per yard.

30. Rented 138$\frac{31}{32}$ acres of land at £3 13s 4d. per acre.

31. If 124$\frac{5}{8}$ cwt. of rice be bought for £258 11s. 11$\frac{1}{4}$d., what is it for 1 cwt.?

32. If the cost of 79$\frac{15}{16}$ cwt. of butter be £233 3s. 0$\frac{1}{4}$d., what is the cost of 1 cwt.?

33. If the cost of 169$\frac{11}{16}$ quarters of wheat be £554 6s. 3d., what is it a quarter?

34. If 247$\frac{7}{8}$ barrels of flour cost £423 9s. 0$\frac{3}{4}$d., what is the price of 1 barrel?

35. Bought 1041$\frac{15}{16}$ yards of superfine cloth at £1 7s. 4d. per yard, what is the cost?

36. Bought 853$\frac{13}{36}$ lb. of tea at 1s. 5$\frac{1}{2}$d. per lb., what is the cost?

37. If 1736$\frac{9}{16}$ barrels of flour cost £2373 6s. 0$\frac{1}{2}$d., what is the cost of a barrel?

38. How many yards of cloth at 15s. 9d. per yard can I have for £5502 5s. 3d.?

39. How many cwt. of pork at £2 15s. 9d. per cwt. can I have for £1524 15s. 3d.?

40. How many cwt. of butter at £3 13s. 10$\frac{1}{2}$d. per cwt. can I buy for £3154 9s. 3d.?

41. How many yards of cloth at 6s. 10$\frac{1}{2}$ per yard can be bought for £2082 15s. 7$\frac{1}{2}$d?

42. Bought 79$\frac{5}{8}$ lb. of sugar at 4$\frac{1}{2}$d. per lb and 69$\frac{15}{31}$ lb at 7$\frac{3}{4}$ per lb., what is the entire cost?

43. Bought 3 tons 15 cwt. 3 qrs. 17$\frac{1}{2}$ lb. of coffee at 1s. 7$\frac{1}{2}$d. per lb., what is the cost?

44. Find the sale of 3957$\frac{18}{37}$ lb. of tea at 3s. 7$\frac{1}{2}$d. per lb.

45. Gave a £300 note to pay for 1769$\frac{28}{77}$ lb. of tea at 3s. 2$\frac{1}{2}$d. per lb,, what change is returned?

46. Divide 23600 miles 1 fur. 13 p. 3 yds. 1 ft. 6 in. by 1260.

47. Divide 28052 ac. 1 r. 14 p. 19 sq. yds. 2 sq. ft. 72 sq.in. by 720.

48. How much land at 7$\frac{3}{4}$d. per sq. yd. can be had for £156 5s.10d.?

49. Divide 2079647 miles 1 fur. 18 p. 1 yd. by 2, 3, 4, 5, 6, 7, 8, 9.

50. Multiply 2 acres 3 r. 33 sq. p. 30 sq. yds. 5 sq. ft. 93 sq. in. by 2, 3, 4, 5, 6.

51. Divide 20060 miles 6 fur. 18 p. 1 yd. by 2, 3, 4, 5, 6, 7.

52. How many acres of land in 100,000,000 square inches?

53. If 3$\frac{1}{2}$ pieces of road contain 65 miles 4 fur. 26 p., what is the average length of a piece?

54. Multiply 17 acres 2 r. 35 sq. p. 30 sq. yds. 7 sq. ft. 126 sq. in. by 13$\frac{7}{8}$.

55. If 5⅔ brewings of ale contain 39 butts 1 bar. 23 gal. 1 qt., what is the average of 1 brewing?

56. Dug up in Australia 6⅞ lumps of gold, each weighing 4 lb. 9oz. 17dwts. 20 grains, what is the entire weight?

57. Add £⅜ + ⅝ guinea + ⅝ crown + ⅝ shilling + 3½ pence.

58. If 7⅝ lumps of gold ore weigh 44 lb. 10 oz. 6 dwt. 12 grains, what is the average weight of a lump?

EXAMPLE.

What is the weight of $37\frac{11}{18}$ packages of Epsom salts, each weighing 5 lb. 7 oz. 5 dr. 2 sc. 17 grs.?

	lb.	oz.	dr.	sc.	grs.		
	5	7	5	2	17		
					9		
	50	9	5	1	13		
					4		
	203	2	6	0	12	=	36 pack
	5	7	5	2	17	=	1 „
	3	5	3	0	11½	=	$\frac{11}{18}$ „
Ans.	212	3	7	1	0½		

		lb.	oz.	dr.	sc.	grs.
		5	7	5	2	17
						11
18	9)	62	1	1	1	7
	2)	6	10	6	1	3
		3	5	3	0	11½

59. If 37½ barrels of pork weigh 363 cwt. 1 qr. 3½ lb., what is the weight of 1 barrel?

60. What is the contents of 75¼ loads of wheat, each 15qrs. 3b. 2p.?

61. If the weight of 3¾ packages of chlorate of potassa be 35 lb. 1 oz. 6 dr. 1 sc. 10 grs., what is the average weight of a package?

62. If $48\frac{1}{16}$ plots of land contain 1842 sq. yds. 8 sq. ft. 30 sq. in., what is the average contents of a plot?

EXAMPLE.

* What o'clock is it in Aix-la-Chapelle which is in 6° 5′ east longitude, when it is 1 p.m. in London?

---

* Since the sun comes to the meridian of any place every 24 hours, it therefore passes over the entire circumference of the earth, in that time; that is, in 24 hours it passes over 360°, which is at the rate of 15° for 1 hour—hence if the number of degrees that any place is east or west of the first meridian be divided by 15°, it will give the difference of time between that place and the place at the first meridian.

$$\left.\begin{array}{r}15^\circ\\ 60\\ \hline 900\end{array}\right\}\qquad \begin{array}{l}6^\circ\ \ 5'\\ 60\\ \hline \end{array}\qquad \frac{6^\circ\ \ 5'}{15^\circ} = \text{time in hours.}$$

) 365 (0 h
60
) 21906 (24 m. 20 sec.
1800
3900
3600
300
60
) 18000 (20
1800
0

Now since Aix-la-Chapelle is east of London, the hour there must be 1h. 24m. 20sec. p.m., that is 24m. 20sec. past one.

63. What o'clock is it in Liverpool which is in longitude 2° 58′ west, when it is noon in London?

64. The quantity of grain, meal, and flour imported into Great Britain in 1858, amounted to 11,291,713 quarters, which is the largest quantity imported for many years, and cost about £20,000,000, what is the average cost of a quarter?

65. How many moons of 29½ days will there be in 365 days?

66. When it is 12 o'clock in London, what o'clock is it in longitude 149° 30′ west?

67. When it is 9 a.m. in London, what o'clock is it in Buenos-Ayres in latitude 34° 35′ south, and longitude 58° 22′ west?

68. Land sold at the site of the Old Rainbow Hotel, Fleet Street, London, brought at a sale a price equivalent to £900,000 per acre, what is that a square yard?

69. In 600 turns of a wheel 16½ feet in circumference, how many miles did it run over?

70. Bought 3 sugar loaves, each 17½ lb. at 5½d. per lb., what is the cost?

71. The pressure of the atmosphere is 14 lb. 8 oz. on the square inch, what is it on an acre of land?

72. If 6873 lbs. of tea cost £1489 3s., what is it a lb.?

73. How many thousand cubic feet of gas had been burned in the course of a year in a shopkeeper's house, when his bill amounted to £60 8s. 9½d. at 3s. 5¾d. per thousand?

74. When it is 12 o'clock a.m. in Greenwich, what o'clock is it in Cape Roca in Portugal, in longitude 9° 30′ west.

EXERCISES 12.

BILLS OF PARCELS.

A bill of parcels is a written account given by seller to buyer of the quantities, sorts, and prices of goods bought.

TARIFF OF DUTIES PAYABLE IN 1861.

Corn duty 1s. per quarter.

Wheat 79 quarters	at 1s.	. . .	£3	19	0
Barley 67½	,,	,, . . .	3	7	6
Oats 45¼	,,	,, . . .	2	5	3
Rye 67¾	,,	,, . . .	3	7	9
Peas 45¾	,,	,, . . .	2	5	9
Beans 67½	,,	,, . . .	3	7	6
Indian corn 35⅜ qrs.	,,	. . .	1	15	4½
			£20	8	1½

Duty on Fruit 7s. per cwt.

1. Currants 635 cwt.	at 7s.	per cwt.	£
Figs 437 cwt. . . .	,,	,, .	
Fig-cake 624 cwt. . . .	,,	,, .	
Plums, French, 68 cwt. .	,,	,, .	
Prunes and prunellos, 93½ cwt.	,,	,, .	
Raisins 850½ cwt. . . .	,,	,, .	

Duty on Meal 4½d. per cwt.

2. Flour 66 cwt.	at 4½d.	per cwt.	£
Barley meal 56 cwt.	,,	,, .	
Oatmeal 35 cwt.	,,	,, .	
Groats 49 cwt.	,,	,, .	
Rye-meal 56½ cwt.	,,	,, .	
Pea-meal 67 cwt.	,,	,, .	
Bean-meal 54½ cwt.	,,	,, .	
Indian corn meal 63 cwt.	,,	,, .	
Buck-meal 87 cwt.	,,	,, .	
Sago & Arrow-root 64 cwt.	,,	,, .	
Semonila 63 cwt.	,,	,, .	
Starch & Tapioca 79 cwt.	,,	,, .	
Maccaroni 57 cwt.	,,	,, .	
Biscuit-bread 49½ cwt.	,,	,, .	
Powder, hair, 32½ cwt.	,,	,, .	

Duty on

3 Chicory, kiln-dried, 47 cwt. at 6s. per cwt. . £
„ roasted, 67 lb. at 4d. per lb. . .
Cocoa, 639 lb. at 1d. per lb. . . . .
„ husks and shells, 85 cwt. at 2s. per cwt.
„ paste 67 lb. at 2d. per lb. . . .
Chocolate 167 lb. at 2d. per lb. . . .
Coffee, 745 lb. at 3d. per lb. . . . .
„ roasted or ground, 693 lb. at 4d. per lb. .

Duty on

4. Almonds, paste of, 64 lb. at 2d. per lb. . . £
Chloroform, 58½ lb. at 3s. per lb. . . .
Playing Cards, 469 packs at 1s. 3d. per pack .
Pepper, of all sorts, 694 lb. at 6d. per lb. .
Hops, 57 cwt. at 15s. per cwt. . . .

Duty on

5. Sugar to the 1st of July, 1861.
„ yellow, Muscovado, and brown clayed, 15 cwt. at 13s. 10d. per cwt. . . £
„ brown Muscovado, 69 cwt. at 12s. 8d. per cwt. . . . . . .
Molasses, 79 cwt. at 5s per cwt. . . . . .
Tea to the 1st of July, 1861, 693 lb. at 1s. 5d. per lb. . . . . . .
Tobacco, 69½ lb. at 3s per lb. . . . .
„ Snuff, 58 lb. at 6s. per lb. . . .
„ Segars, 72½ lb. at 9s. per lb. . . .
Spirits, Brandy, 58½ gallons, at 10s. 5d. per gal.
„ Geneva, 69 gals. at 10s. 5d. per gal. .
„ Gin, 87 gals. at 10s. per gal. . . .
„ Rum, 63½ gals. at 10s. 2d. per gal. . .
„ Whiskey, 58¼ gals. at 10s. per gal. . .

Duty on Wood.

6 Wood, Hewn and Lathwood, 79 loads at 1s. per load . . . . . . £
„ Sawn or split, 57 loads at 2s. per load .
„ Ship-building, 63 loads at 1s. per load .
„ Staves, 49½ loads at 1s. per load . .
„ Mahogany, 58½ tons at 1s. per ton .

Duty on Wine.

The duty on Wines is either 1s., 1s. 9d., 2s. 5d., or 2s. 9d. per gallon, according as they contain less than 18, 26, 40, or 45 degrees, proof spirit respectively.

7.	Bought	146	yards of	tape	at	¼d.	per yard.
	,,	98	,,	,,	,,	½d.	,,
	,,	107	,,	,,	,,	1d.	,,
	,,	116	,,	linen	,,	1s.	,,
	,,	136	,,	cloth	,,	£1	,,

8.	Bought	118	yards of	tape	at	¾d.	per yard.
	,,	115	,,	ribbon	,,	1½d.	,,
	,,	93	,,	calico	,,	3d.	,,
	,,	56	,,	silk	,,	3s.	,,
	,,	76	,,	lace	,,	£3	,,

9.	Bought	79	yards of	calico	at	1¼d.	per yard.
	,,	136	,,	,,	,,	5d.	,,
	,,	69	,,	,,	,,	2½d.	,,
	,,	73	,,	cloth	,,	5s. 0d.	,,
	,,	35	,,	,,	,,	£5. 0s. 0d.	,,

10.	Bought	119	yards of	calico	at	1¾d.	per yard.
	,,	125	,,	,,	,,	3½d.	,,
	,,	135	,,	,,	,,	7d.	,,
	,,	179	,,	cloth	,,	7s. 0d.	,,
	,,	156	pieces of	cloth	at	£7 0s. 0d.	per piece.

11.	Bought	87¾	lb. of	sugar	at	10d.	per lb.
	,,	42½	,,	,,		7d.	,,
	,,	56¼	,,	,,		8d.	,,
	,,	69¾	,,	,,		6d.	,,
	,,	58¾	,,	,,		5d.	,,

12.	Bought	93	yards of	cloth	at	19s. 5½d	per yard.
	,,	57	,,	,,		10s. 9¾d.	,,
	,,	29	,,	,,		£1 9s. 8½d.	,,
	,,	89	,,	,,		7s. 5¼d.	,,
	,,	65	,,	,,		13s. 9¾d.	,,

13.	Bought	7½	cwt. of	cheese	at	£3 0 8	per cwt.
	,,	19¼	,,	,,	,,	3 7 8	,,
	,,	107¾	,,	sugar	,,	1 10 0	,,
	,,	34⅜	,,	,,	,,	1 6 6	,,
	,,	$29\frac{5}{16}$	,,	,,	,,	1 9 8	,,

14. Bought $157\frac{13}{16}$ yards of cloth at 13s. 8d. per yard.
,, $149\frac{2}{3}$,, ,, ,, 7s. $10\frac{1}{2}$d. ,,
,, $178\frac{18}{33}$,, linen ,, 1s. $4\frac{1}{2}$d. ,,

15. Bought $5\frac{3}{4}$ yards of fine cloth at £1 7s. 6d. the yard.
,, $8\frac{1}{2}$,, ,, ,, £1 4s. $8\frac{1}{2}$d. ,,
,, $12\frac{1}{3}$,, ,, ,, 18s. $6\frac{3}{4}$d. ,,
,, $18\frac{1}{4}$,, ,, ,, 17s. 8d. ,,
,, $26\frac{1}{8}$,, ,, ,, 14s. 6d. ,,
,, $38\frac{1}{2}$,, ,, ,, 12s. $9\frac{1}{2}$d. ,,

16. Bought 1875 lbs. of Indian Meal at $1\frac{1}{4}$d. the lb.
,, 1954 ,, ,, ,, $1\frac{3}{4}$d. ,,
,, 1878 ,, ,, ,, $1\frac{1}{2}$d. ,,
,, $3957\frac{18}{37}$ lbs. of tea ,, 3s. $7\frac{1}{2}$d. the lb.
,, $1769\frac{26}{77}$,, ,, ,, 3s. $2\frac{1}{2}$d. ,,
,, $5879\frac{17}{39}$ lbs. of coffee ,, 1s. $7\frac{1}{2}$d. the lb.

17. Bought $897\frac{11}{18}$ cwt. of pork at £2 18 8 the cwt.
,, $9573\frac{5}{8}$,, ,, £3 12 4 ,,
,, $4637\frac{2}{3}$,, ,, £2 13 9 ,,
,, $735\frac{3}{4}$,, ,, £2 19 7 ,,

18. Bought $6915\frac{1}{4}$ lbs. of cotton at $7\frac{5}{8}$d. the lb.
,, $5963\frac{3}{4}$,, ,, $7\frac{11}{16}$d. ,,
,, $4893\frac{1}{2}$,, ,, $7\frac{13}{16}$d. ,,

19. Bought $73\frac{5}{11}$ lb. of rice at $2\frac{3}{4}$d. per lb.
,, $59\frac{11}{15}$,, ,, ,, $3\frac{3}{4}$d. ,,
,, $67\frac{2}{13}$,, ,, ,, $3\frac{1}{4}$d. ,,
,, $85\frac{10}{17}$,, ,, ,, $4\frac{1}{4}$d. ,,
,, $68\frac{5}{19}$,, ,, ,, $4\frac{3}{4}$d. ,,

20. Bought 6875 lb. of coffee at 1s. $5\frac{3}{4}$d. per lb.
,, $5689\frac{13}{31}$,, ,, ,, 1s. $3\frac{1}{2}$d. ,,
,, $6893\frac{17}{33}$,, ,, ,, 1s. $4\frac{1}{2}$d. ,,
,, $3587\frac{11}{35}$,, ,, ,, 1s. $5\frac{1}{2}$d. ,,
,, $6973\frac{15}{37}$,, ,, ,, 1s. $6\frac{1}{2}$d. ,,

[illegible]

[illegible]

[illegible] two quantities of the
[illegible] arising from dividing
[illegible] same denomination. Thus
[illegible] 12 ÷ 4 = 3. The
[illegible] thus: 4 : 12, the first
[illegible]

[illegible] and as two terms
[illegible] constitute a
[illegible] the same ratio
[illegible] in each case being
[illegible] are in proportion, and

$$\frac{[illegible]}{[illegible]} = \frac{4\text{th term}}{3\text{rd term}}$$

[illegible] the extremes, and 12
[illegible] the extremes.

[illegible] multiplied by 3rd

[illegible] = 4th term, and if
[illegible]

[illegible] 4 × 18 = 12 × 6,
[illegible] is equal to the product of the

[illegible] where the term *ratio* is
[illegible] said that the units of
[illegible] a ten-fold ratio, in which instance
[illegible] divided by the first.
[illegible] by the quotient of the
[illegible] By this mode of stating ratio, the 4th term
[illegible] expressed by saying it is equal to the
[illegible] of the other two.—See DAVIS *On Mathematical*

[illegible]

extremes; and if the product of the *means* be known, the product of the *extremes* is known, and *vice versa;* also if one of these products be divided by either factor of the other product, its associate factor is found.

$$\text{Thus: } \text{4th term} = \frac{\text{2nd term} \times \text{3rd term}}{\text{first term}} \textit{ or } \frac{12 \times 6}{4}$$

$$\text{3rd term} = \frac{\text{1st term} \times \text{4th term}}{\text{2nd term}} \textit{ or } \frac{4 \times 18}{12}$$

$$\text{2nd term} = \frac{\text{1st term} \times \text{4th term}}{\text{3rd term}} \textit{ or } \frac{4 \times 18}{6}$$

$$\text{1st term} = \frac{\text{2nd term} \times \text{3rd term}}{\text{4th term}} \textit{ or } \frac{12 \times 6}{18}$$

Whence it is seen that, if any three of the four terms which constitute a proportion are given, the remaining term can be found.

60. To see how these principles are applied to questions occurring in arithmetic,—suppose the price of a certain number of things be given, and the cost of any other number of the same things at the same rate be required, it is evident from the nature of the case, that the price must increase in proportion to the number of things. If the number of things be doubled, trebled, or increased any number of times, the price will be doubled, trebled, or increased the same number of times, and the quantities will hold the same ratio to each other as their prices. Hence, four such quantities are proportionals, and the rule of arithmetic which deals with such questions is termed *direct* or *simple proportion*, or the *Rule of Three Direct.*

Direct or *simple proportion*, then, is the method by which a fourth number can be found, which will be proportional to three given numbers.

61. *In performing the necessary operations, the three given numbers must* (1) *be arranged*, (2) *must be prepared, and then* (3) *operated with.*

ARRANGEMENT.

Rule.—Place in the third term, that number which is of the same kind as the answer, then if upon considering the question, the answer will be *greater* than this number, the greater of the other two terms must be placed second; but if the answer will be *less*, the less must be placed in the second. Having placed two of the terms, the remaining term place first.

PREPARATION.

Rule.—Reduce the first and second terms to the same denomination

[illegible]

[illegible] in order, and the third term to [illegible]

[illegible]

[illegible] terms together, and divide [illegible] the answer or fourth pro- [illegible] in which the third term was [illegible] a higher name if required.

[illegible]

[illegible] 69 cwt.?

[illegible]

In this question the answer will be money, therefore £10 10s. is the 3rd term. As the number of cwt. to be bought is greater, therefore the price or answer will be greater; hence 69 cwt. must be in the 2nd term, and 3 cwt. the first. Shillings is the lowest name mentioned in the 3rd term, which must therefore be expressed in shillings.

[illegible] in shillings.

[illegible]

cwt. cwt. £ s.

* As 3 : 69 :: 10 10 : x

20

$$4830 = \frac{14490}{3} = \frac{69 \times 210}{3} = 70 \times 69 = 4830 = x$$

---

* The reason of the rule given for direct proportion may be explained also as follows:—After the necessary reductions are made, if the price be divided by the number of things bought, the price of *one* thing is obtained; which multiplied by the required number gives the price wanted, but it can make no difference in the answer whether this course be adopted or that of multiplying first (as in the rule), and dividing afterwards.

Or thus, cwt. cwt. £ s.

As 3 : 69 : : 10 10 : x

20

3) 210

70s. = the price of 1 cwt.

69

2,0) 483,0s. = the price of 69 cwt.

£241 10s.

Or thus, cwt. cwt. £ s.

* As 1 : 23 : : 10 10 : x

23

Ans. £241 10

For if 3 cwt. cost £10 10s., 69 cwt. will cost 23 times £10 10s. *or*, £241 10s.

EXAMPLE (2).

How much will 21 yards of cloth come to, if 9 yards cost £2 10s. 7½d.?

yds. yds. £ s. d.

As 9 : 21 : : 2 10 7½ : x

20

50

12

607

2

$$2835 = \frac{25515}{9} = \frac{1215 \times 21}{9} = 135^{\ddagger} \times 21 = 2835 \text{ halfpence } \textit{Ans.}$$

12) 1417½

20) 118 1½

The price of 21 yards £5 18 1½

---

* An instance of *cancelling*, in which the divisor (3) and factor (69) of the dividend are divided by the same number (3), rendering the process shorter for the attainment of the same result.

‡ The price of 1 yard will be 1215 hfp. ÷ 9 = 135 halfpence, and 135 hfp. × 21 will be the price of 21 yards in halfpence, which is the same as (1215 × 21) ÷ 9 = 2835 hfp. the required price.

Or thus,

As 9 yds. : 21 yds. : : £2 10s. 7½d. : x

£ s. d.
9) 2 10 7½

5 7½ = price of 1 yard.
7 × 3 = 21

1 19 4½
3

5 18 1½ = price of 21 yards.

yds. yds. £ s. d.
As 9 : 21 : : 2 10 7½ : x
3 × 7 = 21

7 11 10½
7

9) 53 3 1½

Ans. 5 18 1½

EXAMPLE (3).

If 11 barrels of flour be purchased for £16 17s. 4d., how many barrels can be bought for £53 13s. 4d.?

£ s. d. £ s. d. bar.
As 16 17 4 : 53 13 4 : : 11 : x
20 20

337 1073
12 12

4048 12880
11

4048) 141680 (35 bar.
12144

20240
20240

£ s. d. £ s. d. b.
* As 16 17 4 : 53 13 4 : : 1 : x
20 20

337 1073
12 12

† 4048 12880
368) 12880 (35
1104

1840
1840

---

* Since 11 barrels cost 4048 pence, a penny would purchase $\frac{11}{4048}$ barrel, and this multiplied by 12880, will be the number of barrels that can be bought for £53 13s. 4d.

† Cancelling the divisor against a factor of the dividend.

EXERCISES 13.

1. If 69 cwt. of tea cost £897, how much may be bought for £2132?
2. If 60 gallons of brandy cost £72, what is the price of 135 gal.?
3. If the rent of a farm of 78 acres be £195, what will be the rent of 182 acres at the same rate?
4. If £34 buy 170 yards of cloth, how much will £39 purchase?
5. 800 cwt. of iron cost £883 6s. 8d.; what is the price of 14 cwt.?
6. What ought to be the rent of 5 acres, when the rent of 198 acres 1 r. is £740 2s. 8d.?
7. If 18 men earn £24 15s., what ought 28 men to earn at the same rate?
8. What is the price of 475 yards 2 qrs. 3 nails of cloth at 14s. 8d. per yard?
9. If 31 cwt. of cheese cost £69 4s. 8d., what cost 15 cwt. 2 qrs.?

62. The foregoing questions, with the observations bearing on them, have reference to quantities which are *directly* proportional; that is, where the price or quantity is increased the result is increased.

But the case is different in the *Example*,—"If 7 men mow a field in 12 days, how long will it take 4 men, working alike, to mow the same field?" Here a longer time than 12 days will, of course, be required; that is, if the number of men be diminished, the time they are employed must be increased, and *vice versâ*.

Now, in considering this question, it is evident that as the work is the same, the labour employed must be equal, in whatever way realized. Hence, the products of the agents and the time in which they act, must give equal results; and since the products are equal, the factors of such products will be the means and extremes of 4 quantities which are proportional. These factors are then so arranged, that if the agent be the antecedent of one ratio, its time must be the consequent of the other ratio.* Hence, because $7 \times 12 = 4 \times x$

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* As all questions under this rule merely imply equality of products, the factors producing those products are arranged in proportion. The same observations apply to barter, where goods are exchanged at a certain price each article, for others at a different price; and, as the *entire* value of both must be the same, therefore the quantities and their prices will form a proportion; *but the quantities themselves must be viewed as abstract numbers.* The same

(where x = the days), the statement will be; as 4m. : 7m. : : 12d. : x = 21 days.

Note.—The rule for the disposition of numbers in direct proportion, will also serve to arrange the numbers in this, and similar questions.

Questions like the above in the older arithmetics, constituted what was called the *Rule of Three Inverse.*

EXAMPLE (2).

How many men would perform in 81 days, what 117 men can do in 63 days?

d. d. m.
As 81 : 63 : : 117 : x = 91
63

351
702

81 { 9) 7371
9) 819

Ans. 91 men.

d. d. m.
As 9 : 7 : : 117 : x.
7

9) 819

91

10. If 8 men do a piece of work in 12 days, how long will it take 4 men?
11. A captain of a ship is supplied with provisions for 40 men for 10 months; how long would the same quantity serve 16 men?
12. If 20 cwt. of goods are carried 84 miles for £1, what distance ought 12 cwt. be carried for the same money?
13. If with £100 in a year I clear £6, what sum must I employ to clear the same in 30 days?
14. How much water must be added to 50 gallons of brandy at 21s. per gallon to reduce it to 17s. 6d. per gallon?
15. If the sixpenny loaf weigh 4 lb. 14 oz. when wheat is £3 per quarter, what ought it to weigh when wheat is £4 10s. per quarter?

---

also applies to equal surfaces formed by rectangles; where, if the side of one rectangle be made the antecedent of one ratio, its other side will be the consequent in the other ratio. The latter is called "*reciprocal proportion*" in the Sixth Book of Euclid.

EXAMPLE (1).

If 37 cwt. 2 qr. 24½ lb. of clover seed cost £111 17s. 11¾d., what is it a cwt.?

cwt. qr. lb. cwt. £ s. d. £ s. d.

As 37 2 24½ : 1 : : 111 17 11¾ : x = 2 19 4

4	4	20
150	4	2237
28	28	12
1224	112	26855
300	2	4
4224	224	107423
2		224
8449		429692
		214846
		214846

8449) 24062752 (2848 farthings

16898

12) 712 pence

71647

67592 20) 59 4

40555 £2 19 4 *Ans.*

33796

67592

67592

EXAMPLE (2).

How many acres of land at £1 14s. 0¾d. per acre can be rented for £434 12s. 9d.

```
        £    s.   d.           £    s.   d.      acre       acre  r  s.p.
As      1    14   0¾     :    434   12   9   ::  1 : x = 255   0  32
        20                     20
        ——                   ————
        34                   8692
        12                     12
      ————                 ——————
       408                 104313
         4                      4
      ————                 ——————
      1635                 417252
                                1
                           ——————
                     1635) 417252 ( 255 a. 0 r. 32 sq. p
                           3270
                           ——————
                            9025
                            8175
                            —————
                             8502
                             8175
                            —————
                              327
                                4
                            —————
                           ) 1308 ( 0
                                40
                            ——————
                           ) 52320 ( 32
                             4905
                             ——————
                              3270
                              3270
                              ————
```

EXERCISES 14.

1. If 56 cwt. of sugar cost £224, what would 65 cwt. cost at the same rate?
2. If 85 cwt. of tallow cost £250 7s. 11d., what is the cost of 81 cwt.?
3. Paid £274 6s. 3d. for 95 cwt. of beef; what would be the cost of 19 cwt.?

4. If 74 yards of cloth cost £59 15s., how many yards can be bought at the same rate for £89 12s. 6d.?
5. If 287 cwt. 2 qrs. of pork cost £559 8s. 6½d., what is the price of 240 cwt.?
6. If 457 cwt. of cheese cost £723 11s. 8d., what is the cost of 93 cwt.?
7. Bought 6 cwt. of tea for £121 16s., what is the price of 7 lb.?
8. If 110 cwt. of iron cost £97 12s. 6d., what will 387 cwt. come to?
9. If 2 acres 2 roods of land be rented for £2 6s. 8d., how much may be rented for £70 10s. 6d.
10. If 6 yards of merino cost £2 17s. 4½d., what cost 42 yards?
11. What is the cost of 56 cwt. 3 qrs. 14 lb. of beef at the rate of 5s. 3d. for 6 lb.?
12. If 257 cwt. 3 qrs. 21 lbs. of butter cost £1094 11s. 0½d., what will be the cost of 7 cwt?
13. What is the cost of 6 cwt. 1 qr. 4 lb. of sugar at 6s. 3d. for 15 lb.?
14. Bought 69 yards 2 qs. 2 nails of cloth for £48 3s. 1¾d.; what is the cost of 5 yards at the same rate?
15. If 16 tons 15 cwt. 3 qrs. 27 lb. of tea cost £6820 12s. 4½d., what is the cost of 1 ton 1 cwt.?
16. If 37,631½ yards of linen cost £6820 14s. 2¼d., what is the price of 7 yards?
17. Bought 6 tons 13 cwt. 3 qrs. 24½ lb. of coffee for £1094 1s. 6¾d.; what is the cost of 5 cwt.?
18. Rented 138 acres 3 roods 35 perches of land for £509 11s. 0½d. what is the rent of 3 acres 2 roods?
19. Bought $879\frac{11}{16}$ yards of cloth for £601 2s. 4¾d.; what is the cost of 4 yards?
20. If 8 tons 16 cwt. 2 qrs. 23 lb. of tea cost £286 7s. 8d., what is the price of 1 ton?
21. Gave a £300 note to pay for 17 cwt. 2 qrs. 24½ lb. of tea at £298 13s. 4d. per ton; what balance should be received?
22. Bought 1041 yards 3 qs. 3 nails of cloth for £1423 19s. 7½d., what is it for 1 yard?
23. If 11 yards of cloth cost £6 4s. 5¼d., what is the price of 45 yards?
24. How many yards of cloth at 6s. 10½d. per yard can be bought for £2082 15s. 7½d.?
25. Find the price of 13 cwt. 2 qrs. 23 lb. of coffee at £128 6s. 8d. the ton.

26. If 47⅝ bags of coffee, each weighing 1 cwt. 2 qrs. 16 lb., cost £310 7s. 1½d., what is the price of 1 cwt.?

27. Gave a £50 note to pay for 13 cwt. 2 qrs. 24½ lbs., of coffee at £54 13s. 4d. the ton; what balance must I get back?

28. The cost of 169 cwt. 3 qrs. 26½ lb. of butter is £364 12s. 4d., and the gain by selling it £79 6s. 6¼d.; at what price per cwt. was it sold?

29. If a clerk's salary be £191 12s. 6d., per annum, in what time would he be entitled to receive £39 7s. 6d.?

30. What is a clerk's salary from the 7th of April to the 29th of October, at £114 1s. 3d. per annum?

31. A bankrupt pays his creditors £687 13s. 9d. which was at the rate of 5s. 3d. in the pound; what sum did he owe?

32. Bought 3 hhds. of tobacco, weighing 7 cwt. 2 qrs. 22 lb.; 8 cwt. 0 qr. 6 lb.; and 10 cwt. 3 qrs. 25 lb. at £1 10s. 4d. per cwt.; what is the cost?

33. If 47 tons 13 cwt. 3 qrs. 14 lb. of steel cost £1001 11s. 4½d., what is the cost of 1 ton?

34. Bought 79⅜ hhds. of sugar, each weighing 5 cwt. 2 qrs. 16 lb. at £1 5s. 8d. per cwt., what is the amount?

35. If a clerk's salary be £60 1s. 5½d., per annum, in what time will he receive £19 8s. 5d.?

36. What sum must a bankrupt pay on £2735 10s., at 5s. 9¾d. in the pound?

37. If I buy cloth at 3s. 4d. per yard, which I sell at 4s. 2d. per yard, what did I gain on the purchase of £100 worth of the cloth?

38. If by paying down £100 I become entitled to £5 a year, what income shall I derive from the disposal of £790 16s. 8d.?

39. What would be the purchase money of a farm producing a rental of £121 10s. at the rate of £3 15s. for every £100?

40. What is a house-agent's commission on £245 12s. 6d. at £5 for every £100?

41. Bought 23⅝ chests of tea, each weighing 13 cwt. 2 qrs. 8 lb., for £5162 1s 3d.; what is that per lb.?

42. If the weight of $6\frac{13}{32}$ hhds. of sugar be 165 cwt. 3 qrs. 11 lb. 11 oz. 9 drams, what is the weight of 1 hogshead?

43. Hired a servant woman on the 3rd of May at £11 8s. 1½d. a year—she leaves on the 26th of September; what wages is she entitled to?

44. If I buy 210 quarters of wheat for £262 10s., at what price per quarter must I sell it to gain 20 guineas on the entire?

45. Gave a £500 note to pay for 27 pieces of cloth, each piece 23 yards, at 7s. $8\frac{1}{2}$d. per yard; what sum is received back?

46. If the cost of conveying by railway 580 tons of goods 157 miles 6 furlongs 30 perches be £289 7s. $7\frac{1}{4}$d., what is it a mile?

47. What poor rates are paid on 79 acres 3 roods 30 perches of land valued at £3 18s. 4d. the acre, at 2s. in the pound?

48 A house-agent sold property for £3079 13s. 9d., what is his commission for the sale at £2 for every £100?

49. If a clerk's salary be £205 6s. 3d. a year, what sum would he be entitled to from the 7th of April to the 9th of November?

50. Gave a £300 note to pay for 17 cwt. 3 qrs. $27\frac{1}{2}$ lb. of tea at £298 13s. 4d. per ton, what change do I get back?

51. Bought $9\frac{3}{4}$ barrels of beef, each weighing 2 cwt. 3 qrs. 20 lb. for £66 12s. 6d.; what is the cost of 1 cwt.?

52. An estate being purchased, pays $3\frac{3}{4}$ per cent. when let for £275; what was the purchase-money?

53. What is the cost of 2 qrs. 23 lb. of tea at £29 17s. 4d. per cwt.?

54. If 18 cwt. 2 qrs. 14 lb. of coffee cost £139 1s. 4d., what is it per ton?

55. I hired a servant-man on the 9th of March at £22 16s. 3d. a year, what does he receive on leaving on the 17th of October?

56. If a clerk receive a salary of £205 6s. 3d., what must he get for a year, and from the 7th of April to the 18th of December?

57. If 1 cwt. 1 qr. 21 lb. of butter cost £8 1s., how much may be bought for £39 10s.?

58. Bought 43 barrels of pork, each of equal weight, for £161 12s. 2d., which was at £2 11s. 4d. per cwt.; what was the weight of èach barrel?

59. If 3319 English ells 4 quarters of cloth cost £3371 15s. $3\frac{1}{4}$d., what is it a yard?

60. At £4 8s. 4d. for one cwt. of butter, what quantity can be bought for £2259 11s. $11\frac{4}{7}$d.?

61. A merchant bought 240 cwt. 1 qr. 14 lb. of tallow for £656 3s., how must he sell it the cwt. to gain £70?

62. The expense of making 31 miles 3 furlongs 35 perches of railway, is £1,023,242 3s. 9d., what is it a mile?

63. If $4\frac{1}{2}$ lb. of coffee cost 5s. $9\frac{3}{4}$d., what is the price of $13\frac{1}{2}$ bags each 3 cwt. 2 qrs. 18 lb.?

64. If $24\frac{15}{16}$ masses of lead, each 16 tons 17 cwt. 3 qrs. 8 lb., be purchased for £83,171 3s. $2\frac{4}{6}\frac{3}{4}$d., what is the cost of 1 cwt.?

65. What income-tax has a gentleman to pay, whose annual income is £[illegible] at [illegible]d. in the pound?

66. Gold to the value of £[illegible] arrived from Melbourne; what is its weight, the price being £3 15s. per ounce?

67. A bankrupt's effects amounted to 2548 guineas, and his debts to £[illegible] 18s.; what will his creditors receive in the pound?

68. Bought [illegible] cwt. [illegible] qrs. [illegible] lb. of sugar at £1 2s. 10d., per cwt., and paid £[illegible] [illegible]s. [illegible]d. charges on it: what does it stand me in per cwt.?

69. How many cwt. of sugar at £1 5s. 6d. per cwt. can I buy for £[illegible]?

70. How many ounces of silver at 5s. per ounce, can I buy for £[illegible] 5s. [illegible]d.?

71. A mass of gold weighed by a false pair of scales of 10½ ounces to the pound weighed 18 lb. 11 oz. 3 dwt. 12 grs.; what is its true weight?

72. If [illegible] barrels of pork weigh each 9 cwt. 2 qrs. 21 lb., but are afterwards found to be short weight, every cwt. making only [illegible] lbs., what is its true weight?

73. What quantity of building ground can be purchased in Dale Street, Liverpool, for £5731 17s. 11¼d., at £2 0s. 6d. the square yard?

74. What is the income of a person who pays £22 7s. 5d. income-tax at the rate of 7d. in the pound?

75. What income does a person possess who pays £229 1s. 4¼d. income-tax at 7d. in the pound?

76. In 1846 the mortality in every 10,000 inhabitants in Liverpool was 384, but by sanitary improvements the mortality in 1857 was only 299. What is the annual saving of life to Liverpool, its population being about 420,000.

77. If a quantity of bread serve 800 men 7 weeks at the rate of 1¼ lb. per day, how many men will the same bread serve for 8 weeks at 1 lb. per day?

78. If 22 persons out of every 1000 die annually, what was the population of England in 1857 when 419,815 died?

COMPOUND PROPORTION.

63. *Compound* proportion combines two or more simple proportions united in one question, and has five or more terms given to find what is required.

For the convenience of arrangement, let each question be divided into two parts: viz., the *supposition or data*, and the *demand*. The former expresses the *conditions*, and consists of *two quantities*; the latter, the base for the thing sought, and consists of *one* quantity only.

Rule.—Write for the third term that part of the supposition or data, whether simple or compound, which embraces the quantity of the same name as the answer; place the demand in the second term, and the other part of the supposition will be the first. Then proceed as in simple proportion, observing that if the third term be a compound, the fourth will be a compound, and must, therefore, be divided by the factor or factors corresponding with those not of the answer's name in the third term.

EXAMPLE (1).

If £3 5s. be paid for the carriage of 13 cwt. 30 miles, what must be paid at the same rate for the carriage of 9 cwt. 65 miles? *

Let x = the answer.

$$\begin{array}{lccccccccccc} & \text{cwt.} & & \text{m.} & & \text{cwt.} & & \text{m.} & & \text{£} & \text{s.} & \\ \text{As} & 13 & \times & 30 & : & 9 & \times & 65 & :: & 3 & 5 & : x \\ & & & & & & & & & 20 & & \\ & & & & & & & & & \text{—} & & \end{array}$$

$$\text{and } \frac{9 \times 65 \times 65}{13 \times 30} = 97\text{s. } 6\text{d.} = \text{£}4\ 17\text{s. } 6\text{d.}$$

Remark.—In considering this question £3 5s. is paid, not for the carriage of 13 cwt., but for the carriage of 13 cwt. *a distance of* 30 *miles*; while the query is not, what must be paid for 9 cwt., but for 9 cwt. carried 65 miles; but 13 cwt. carried 30 miles, and 9 cwt. carried 65 miles, will be the same as 13 cwt. × 30 = 390 cwt.,— and 9 cwt. × 65 = 585 cwt.— carried one mile.

By two applications of simple proportion.

As 13 cwt. : 9 cwt. : : £3 5s. : x = £2 5s. the cost of the carriage of 9 cwt. 30 miles.

As 30 m. : 65 : : £2 5s. : y = £4 17s. 6d., the cost of the carriage of 9 cwt. 65 miles.

EXAMPLE (2).

If 7 men mow 126 acres of meadow in 12 days, how many men, working a like, will mow 72 acres in 3 days?

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* In this question, 13 cwt, carried 30 miles, and £3 5s. represent the two quantities which constitute the supposition or data, while 9 cwt. carried 65 miles imply the demand.

Let x = the number of men.

$$\begin{array}{l} \quad\ \text{ac.} \quad \text{ac.} \quad \text{m.} \quad \text{d.} \quad \text{m.} \quad \text{d.} \\ \text{As } 126 : 72 :: 7 \times 12 : x \times 3 \end{array}$$

$$\frac{72 \times 7 \times 12}{126} = 48.$$

And $x = 48 \div 3 = 16$ men *Ans.*

EXAMPLE (3).

If 240 men in 5 days, of 10 hours each, can dig a trench 210 yards long, 3 broad, and 2 deep; in how many days, of 8 hours long, will 250 men working with the same effect, dig a trench 280 yards long, 5 broad, and 3 deep, of equal hardness and difficulty?

Let x = the number of days.

$$\begin{array}{l} \text{yds.} \qquad\qquad\qquad \text{yds.} \qquad\qquad\qquad \text{m.} \quad \text{d.} \quad \text{h.} \quad \text{m.} \quad \text{d.} \quad \text{h.} \\ \text{As } 210 \times 3 \times 2 : 280 \times 5 \times 3 :: 240 \times 5 \times 10 : 250 \times x \times 8 \end{array}$$

$$\frac{280 \times 5 \times 3 \times 240 \times 5 \times 10}{210 \times 3 \times 2} = 40000$$

And $x = 40000 \div 250 \times 8 = 20$ days *Ans.*

EXERCISES 15.

1. If 20 cwt. of goods be carried 50 miles for £1, what sum must be paid for the carriage of 60 cwt. 75 miles.
2. If in 8 days 24 bushels of oats be eaten by 32 horses, how many bushels will 16 horses eat in 6 days at the same rate?
3. If 14 lb. of soft sugar cost 5s. 3d., what should be the price of 35 lb. of lump, when 4 lb. of the former are equal in value to 3 lb. of the latter?

   As 14 lb. × 3 lb. : 35 lb. × 4 lb. : : 5s. 3d. : x.
4. If a regiment of 960 soldiers consume 320 quarters of wheat in 138 days, how many soldiers will consume 1600 quarters in 46 days?*
5. If 30 masons can build a wall in 10 days of 12 hours working, what time will it take 75 masons working 8 hours a day?†
6. If 7 men can cut 112 acres of grass in 8 days, how many men can mow 84 acres in 3 days?

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* In examples of this kind it is taken for granted that *all* the agents in the same question act *uniformly*.

† Since the work is the same, the labour employed must be equal, hence $30 \times 10 \times 12 = 75 \times x \times 8$, which put into a proportion to realise what is required, will be—As $75 \times 8 : 30 \times 12 :: 10 : x = 6$.

7. If 75 men can dig a trench 440 yards long, 8 feet 4 inches wide, and 8 feet 6 inches deep, in 28 days of 12 hours each; how many men would dig a trench a mile long, 3 feet 8 inches broad, and 2 feet 6 inches deep, in 22 days of 9 hours each?

8. If 40 men can make a drain 800 miles long in 8 days, working 7 hours a day, in how many days will 51 men do 459 yards of similar labour, working 6 hours a day?

9. If 5 men can reap 30 acres of wheat in 8 days, how many acres would 6 men reap in 15 days?

10. If 12 horses in 1 day plough 17 acres, how many acres will 186 horses plough in 9 days?

11. If £100 in one year gain £5, how much will £729 8s. 4d. gain in $1\frac{4}{5}$ years?

12. If 15 bushels of oats serve 12 horses for 80 days, how long will 20 bushels serve 16 horses?

13. If a garrison of 1200 men in 35 days eat a certain quantity of bread, at $1\frac{1}{3}$ lb. a day to each man; how many men in 45 days, receiving $1\frac{1}{3}$ lb. a day, will eat three times the quantity?

14. If 112 men in 5 days, of 10 hours each, dig a trench 70 yards long, 8 ft. wide, and 2 ft. deep; what length of trench 5 ft. wide, 8 ft. deep, and $1\frac{1}{2}$ times as hard, can be dug by 400 men in 9 days, of 12 hours each?

15. If £100 in 365 days gain £4, what ought £6159 7s. 6d. to gain from the 19th of April to the 24th of November, at the same rate?

16. If 4 compositors in 8 days, of 12 hours each, can compose 28 sheets of 16 pages each, each page 40 lines of 42 letters each; in how many days, of 10 hours long, may 6 compositors compose a book of 60 sheets, of 12 pages in a sheet, 48 lines in a page, and 49 letters in a line?

17. If a family of 12 persons spend £36 in butchers' meat in 9 months, when meat is 6d. per lb.; how much money, at the same rate, should a family of 15 persons spend in 8 months, when meat is $7\frac{1}{2}$d. per lb.?

18. If 248 men in $5\frac{1}{2}$ days, of 11 hours each, dig a trench of 7 degrees of hardness, $232\frac{1}{2}$ yards long, $3\frac{2}{3}$ wide, and $2\frac{1}{2}$ deep; in how many days, of 9 hours each, will 24 men dig a trench of 4 degrees of hardness, $337\frac{1}{2}$ yards long, $5\frac{2}{3}$ wide, and $3\frac{1}{2}$ deep?

FRACTIONS.

64. The multiplication of the unit generates all integer numbers, and its division into equal parts produces FRACTIONS.

65. If a unit, therefore, be divided into any number of equal parts, one or more of these equal parts will be a *fraction* of this unit.*

Thus, if one cwt. be divided into 7 equal parts, one, two, or three of such parts will be fractions, and are read: *one-seventh, two-sevenths, three-sevenths of a cwt.*, and expressed by figures in the following way: $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$ cwt.

The lower figure is called the *denominator*, and shews the number of equal parts into which the unit is divided; and the upper figure, the *numerator*, which shows the number of these equal parts taken—

$$\frac{8}{7}\ \begin{matrix}\dots\\\dots\end{matrix}\ \frac{\text{numerator}}{\text{denominator}},\quad \textit{or}\ \frac{\text{dividend}}{\text{divisor}}.$$

66. *A fraction is mentioned with the following distinctions.*

1. A *proper* fraction is one in which the numerator is less than the denominator; as $\frac{3}{7}$.
2. An *improper* fraction is one in which the numerator is not less than the denominator; as $\frac{7}{7}$, $\frac{8}{7}$.
3. A *mixed* fraction is a whole number, with a fraction annexed; as $4\frac{3}{7}$.
4. A *compound* fraction is the fraction of a fraction; as $\frac{2}{3}$ of $\frac{3}{7}$.
5. A *complex* fraction is one which has a fraction, or mixed number, in either the enumerator or denominator, or in both; as

$$\frac{\frac{3}{7}}{4},\ \frac{\frac{3}{7}}{\frac{3}{4}},\ \frac{4\frac{3}{7}}{5},\ \frac{3\frac{3}{7}}{4\frac{1}{2}}.$$

6. The *reciprocal* of a fraction is 1 divided by the fraction; as the reciprocal of $\frac{3}{4}$ is $\frac{1}{\frac{3}{4}}$ or $\frac{4}{3}$, that is, the fraction inverted.

The first great principle in fractions.

67. If the numerator and denominator of a fraction be both mul-

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* A fraction may be conceived to occur, by considering a large unit to be composed of a number of small units;—one or more of such small units would be a fraction of the large unit. Thus a sovereign is realised by a collection of 10 florins; and since each florin is itself a unit, one, two, or three of such florins, would be a fraction of that sovereign.

tiplied, or both divided by the same number, the value of the fraction will not be altered; as $\frac{6}{8} = \frac{18}{24} = \frac{42}{56} = \frac{3}{4}$.[*]

Note.—Hence every fraction is capable of beiny expressed by an infinite variety of numbers without altering its value, since there are an infinite variety of multipliers.

68. *The greatest common measure of two numbers.*

1. If a number divides another without a remainder, it is said to *measure* it.
2. If a number divides two or more numbers, it is said to be a *common measure* of these numbers.
3. The *largest* number, dividing two or more numbers, is said to be the *greatest common measure* of these numbers.

Thus, 4 is a *measure* of 16.
4 is a *common measure* of 16 and 24.
But, 8 is the *greatest common measure* of 16 and 24.

69. *The least common multiple of two or more numbers.*

1. A *multiple* of a number, is that quantity which the number will divide without a remainder.
2. A *common multiple* is a number capable of being divided by two or more numbers without a remainder.
3. The *least common multiple* is the least number that can be so divided.

Thus, 216 is a *multiple* of 27.
216 is a *common multiple* of 27 and 36.
But, 108 is the *least common multiple* of 27 and 36.

70. *To find the greatest common measure of two numbers.*

Rule I.—Divide the greater number by the less, then divide the divisor by the remainder, and thus continue until nothing remains; the last divisor will be the greatest common measure.

EXAMPLE.

Find the greatest common measure of 273 and 351.

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[*] This is only saying what has already been stated in whole numbers, viz., that if the dividend and divisor be both multiplied, or both divided, by the same number, the quotient does not alter; as, $\frac{20}{4} = \frac{60}{12} = \frac{10}{2}$, each yielding 5 in the quotient.

273) 351 (1
273
—
78) 273 (3
234
—
39) 78 (2
78

39 is the greatest common measure.*

To find the greatest common measure of three or more numbers.

Rule II.—Find the greatest common measure of two of the numbers; then find the greatest common measure of the number so found and the third number, and so on; the last found will be the greatest common measure required.

EXAMPLE.

Find the greatest common measure of 273, 351, and 65.

By the preceding rule 39 is the greatest common measure of 273 and 351.

Then of 39 and 65 find the greatest common measure.

39) 65 (1
39
—
26) 39 (1
26
—
13) 26 (2
26

13 is the greatest common measure of 273, 351, and 65.

71. *To find the least common multiple of several numbers.*

Rule.—Arrange the numbers in a line and reject all those that measure any of the others; then divide the remaining numbers by any number that will divide the most of them; write the quotients and the numbers not divided in a line underneath; proceed with this line in a similar way, and thus continue, until a line is obtained in which no two numbers will admit of a divisor. The continued product of the numbers in the last line and the several divisors, will be the least common multiple.

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* This is evident, since $273 = 3 \times 13 \times 7$,
and $351 = 3 \times 13 \times 3 \times 3$.

Now, since 3×13 occurs once as factors in each, hence 3×13, or 39, is a common measure to each, and the *greatest;* because 7 and 3×3, or 9, are prime to one another. That 39 is a common measure may be traced through the process. Thus, since 39 measures 78, it must measure $78 \times 3 + 39$, or 273, which is one of the given numbers; and, 39 measuring 273, and 78, it measures their sum 351,—the other given number.

EXAMPLE.

Find the least common multiple of 3, 8, 12, 14.*

$$\begin{array}{r|rrr} 2 & 8 & 12 & 14 \\ \hline 2 & 4 & 6 & 7 \\ \hline & 2 & 3 & 7 \end{array}$$

$$2 \times 3 \times 7 \times 2 \times 2 = 168 \textit{ Ans.}$$

Note.—That the figure 3 *need not be taken into the computation is evident, since any number which is a multiple of* 12 *will be a multiple of* 3; *therefore* 8, 12, 14, *are alone requisite for the calculation.*

EXERCISES 16.

1. Required the least common multiple of 91 and 117.
2. „ „ „ „ 8, 10, 15.
3. „ „ „ „ 24, 27, 36, 45.
4. „ „ „ „ 2, 3, 4, 5, 6, 7, 8.
5. „ „ „ „ the 9 digits.
6. „ „ „ „ 3, 6, 9, 21, 24.

REDUCTION OF FRACTIONS.

72. **REDUCTION OF FRACTIONS** is the mode of changing the form or expression of any fraction into another without altering its value.

To reduce a fraction to its lowest terms.†

EXAMPLE (1).

Reduce $\frac{84}{119}$ to it lowest terms.

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* Here $\left\{\begin{array}{l} 8 = 4 \times 2 \\ 12 = 4 \times 3 \\ 14 = 2 \times 7 \end{array}\right\}$ The figures 8 and 12 contain the common factors 4; and striking it out of one, we shall have $4 \times 3 \times 2$ for a common multiple of 8 and 12; but 14 has the factor 2, which is also in this common multiple, and by removing it from one, we obtain $4 \times 3 \times 2 \times 7$, or 168 for the least common multiple of the three numbers.

† Fractions are in their *lowest terms* when the numerator and denominator will admit of no common divisor; that is, when they are prime to each other.

85) 119 (1
85
—
84) 85 (2
68
—
17) 34 (2
34
—

17) $\frac{85}{119}$ ($\frac{5}{7}$ *Ans.*

The greatest common measure is 17, and its lowest terms $\frac{5}{7}$.

EXAMPLE (2).

Reduce $\frac{65}{117}$ to its lowest terms.

65) 117 (1
65
—
52) 65 (1
52
—
13) 52 (4
52
—

13) $\frac{65}{117}$ ($\frac{5}{9}$ *Ans.*

The greatest common measure is 13.

EXAMPLE (3).

Reduce $\frac{112}{280}$ to its lowest terms.

112) 280 (2
224
—
56) 112 (2
112

56)$\frac{112}{280}$ ($\frac{2}{5}$ *Ans.*

The greatest common measure is 56.

EXERCISES 16—(*continued*).

7. Find the greatest common measure of 255 and 357.
8. „ „ „ „ 432 and 840.
9. „ „ „ „ 91 and 117.
10. „ „ „ „ 214 and 506.
11. „ „ „ „ 100 and 128.
12. „ „ „ „ 210, 252, and 273.
13. „ „ „ „ 54, 2502, and 2511.
14. „ „ „ „ 561, 1122, and 3399.

15. Find the greatest common measure of $\frac{182}{196}$, and then reduce it to its lowest terms.
16. Find the greatest common measure of $\frac{949}{2920}$, and then reduce it to its lowest terms.
17. Find the greatest common measure of $\frac{6278}{7300}$, and then reduce it to its lowest terms.
18. Find the greatest common measure of $\frac{1176}{1792}$, and then reduce it to its lowest terms.
19. Find the greatest common measure of $\frac{285}{1080}$, and then reduce it to its lowest terms.
20. Find the greatest common measure of $\frac{407297}{8261867}$, and then reduce it to its lowest terms.
21. Find the greatest common measure of $\frac{5582}{8912}$, and then reduce it to its lowest terms.
22. Find the greatest common measure of $\frac{740664}{746496}$, and then reduce it to its lowest terms.
23. Find the greatest common measure of $\frac{2105}{2240}$, and then reduce it to its lowest terms.
24. Find the greatest common measure of $\frac{231}{1568}$, and then reduce it to its lowest terms.
25. Find the greatest common measure of $\frac{4113}{20628}$, and then reduce it to its lowest terms.
26. Find the greatest common measure of $\frac{1413}{7536}$, and then reduce it to its lowest terms.
27. Find the greatest common measure of $\frac{29792}{324128}$, and then reduce it to its lowest terms.
28. Find the greatest common measure of $\frac{2736}{8244}$, and then reduce it to its lowest terms.
29. Find the greatest common measure of $\frac{138428}{208698}$, and then reduce it to its lowest terms.
30. Find the greatest common measure of $\frac{5200}{16315}$, and then reduce it to its lowest terms.
31. Find the greatest common measure of $\frac{2826}{15072}$, and then reduce it to its lowest terms.
32. Find the greatest common measure of $\frac{4113}{20628}$, and then reduce it to its lowest terms.
33. Find the greatest common measure of $\frac{43488}{93312}$, and then reduce it to its lowest terms.
34. Find the greatest common measure of $\frac{32676}{96768}$, and then reduce it to its lowest terms.
35. Find the greatest common measure of $\frac{351936}{627264}$, and then reduce it to its lowest terms.

36. Find the greatest common measure of $\frac{10784}{19488}$, and then reduce it to its lowest terms.

37. Find the greatest common measure of $\frac{343472}{559104}$, and then reduce it to its lowest terms.

38. Find the greatest common measure of $\frac{6356384}{11715952}$, and then reduce it to its lowest terms.

39. Find the greatest common measure of $\frac{169022}{109352}$, and then reduce it to its lowest terms.

40. Find the greatest common measure of $\frac{133261}{549661}$, and then reduce it to its lowest terms.

41. Find the greatest common measure of $\frac{18213808}{28069216}$, and then reduce it to its lowest terms.

73. *To reduce an improper fraction to a whole or mixed number; also, to reduce a mixed number to an improper fraction.*

Rule (1).—Divide the numerator by the denominator, and the quotient will be the integral part; then place the remainder, if any, over the denominator, for the fractional part.

(1).—Multiply the whole number by the denominator of the fraction, and to the product add the numerator; the sum will be the numerator to the given denominator.

EXAMPLE.

Reduce $\frac{23}{5}$ to a mixed number.

$$23 \div 5 = 4\tfrac{3}{5}.$$

EXERCISES 17.

A.

1. Reduce $\frac{68}{9}$ to a mixed number.
2. „ $\frac{38}{7}$ „ „
3. „ $\frac{293}{17}$ „ „
4. „ $\frac{67}{13}$ „ „
5. „ $\frac{68}{11}$ „ „

EXAMPLE.

Reduce $3\frac{3}{4}$ to an improper fraction.

$$3\tfrac{3}{4} = \frac{3 \times 4 + 3}{4} = \tfrac{15}{4} \text{ Ans.}$$

17.

EXERCISES B.

1. Reduce $5\frac{5}{6}$ to an improper fraction.
2. „ $17\frac{5}{8}$ „ „ „
3. „ $9\frac{11}{12}$ „ „ „
4. „ $37\frac{13}{28}$ „ „ „
5. „ $14\frac{7}{9}$ „ „ „
6. „ $15\frac{5}{11}$ „ „ „

74. *To reduce a compound fraction to a simple one.*[*]

Rule.—Multiply all the numerators together for the numerator of the simple fraction, and all the denominators together for its denominator; then reduce the new fraction to its lowest terms.

EXAMPLE.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{10}$ to a simple fraction.

$$\frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{7}{10} = \frac{2 \times 3 \times 7}{3 \times 4 \times 10} = \frac{42}{120} = \frac{7}{20}.$$

17.

EXERCISES C.

1. Reduce $\frac{4}{9}$ of $\frac{7}{8}$ of $\frac{11}{12}$ to a simple fraction.
2. " $\frac{15}{16}$ of $\frac{4}{5}$ of $\frac{9}{10}$ of 15 to a simple fraction.
3. " $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{5}{12}$ of $\frac{7}{8}$ of 24 to a simple fraction.
4. " $\frac{5}{8}$ of $2\frac{2}{5}$ of $1\frac{1}{3}$ of $3\frac{5}{6}$ of $3\frac{1}{3}$ of 5 to a simple fraction.
5. " $\frac{7}{8}$ of $\frac{5}{7}$ of $33\frac{5}{9}$ of $5\frac{1}{2}$ of $\frac{8}{25}$ to a simple fraction.

75. *To reduce a complex fraction to a simple one.*

Rule.—If the numerator and denominator be mixed numbers, reduce them to improper fractions. Then multiply the numerator of the upper fraction by the denominator of the lower, for the numerator of the simple fraction; and the numerator of the lower fraction by the denominator of the upper, for its denominator.[†]

EXAMPLE (1).

Reduce $\dfrac{4\frac{2}{3}}{6\frac{3}{4}}$ to a simple fraction.

$$\frac{4\frac{2}{3}}{6\frac{3}{4}} = \frac{\frac{14}{3}}{\frac{27}{4}} = \frac{14 \times 4}{27 \times 3} = \frac{56}{81} \quad \textit{Ans.}$$

EXAMPLE (2).

Reduce $\dfrac{\frac{3}{7}}{4}$ to a simple fraction.

$$\frac{\frac{3}{7}}{4} = \frac{3 \times 1}{4 \times 7} = \frac{3}{28}.$$

---

[*] A fraction is said to be in its simple form when it is expressed by two numbers, as $\frac{3}{4}$, $\frac{7}{8}$, &c.

[†] This rule merely means that if the numerator and denominator of a complex fraction be multiplied by the same number (viz., one capable of converting both into whole numbers), the resulting fraction is a simple one.

$$\text{Thus, } \frac{4\frac{2}{3}}{6\frac{3}{4}} = \frac{4\frac{3}{4} \times 12}{6\frac{3}{4} \times 12} = \frac{56}{81}.$$

17.

EXERCISES D.

1. Reduce $\dfrac{4\frac{3}{7}}{5}$ to a simple fraction.
2. " $\dfrac{3\frac{3}{7}}{4\frac{1}{2}}$ " "
3. " $\dfrac{2\frac{1}{2}}{3\frac{1}{2}}$ " "
4. " $17\dfrac{2\frac{1}{2}}{3}$ " "
5. " $\dfrac{4\frac{1}{2}}{3\frac{1}{3}}$ " "
6. " $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$ " "

76. *To reduce given fractions to equivalent fractions, having a common denominator.*

Rule (1).—Find the least common multiple of the denominators, which will be the least common denominator. Then divide the common denominator by each denominator, and multiply the quotients by the corresponding numerators; the several products will be the numerators of the required fractions.

Rule (2).—Multiply each numerator into all the denominators, except its own, for a new numerator; and all the denominators together, for the common denominator.*

EXAMPLE (1).

Reduce $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{13}{14}$, to equivalent fractions, having a common denominator.

By Rule (1).

$$\begin{array}{r|rrr} 2 & 8 & 12 & 14 \\ \hline 2 & 4 & 6 & 7 \\ \hline & 2 & 3 & 7 \end{array}$$

$2 \times 3 \times 7 \times 2 \times 2 = 168$, the least common multiple of 8, 12, & 14.

168 least common denominator.

Answers.

$\frac{5}{8}$ . . $5 \times 21 = 105$ numerator, The fractions are $\dfrac{5 \times 21}{8 \times 21} = \dfrac{105}{168}$

$\frac{7}{12}$ . . $7 \times 14 = 98$ " " " $\dfrac{7 \times 14}{12 \times 14} = \dfrac{98}{168}$

$\frac{13}{14}$ . . $13 \times 12 = 156$ " " " $\dfrac{13 \times 12}{14 \times 12} = \dfrac{156}{168}$

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* The principle on which rules (1) and (2) are conducted is, that if the numerator and denominator of a fraction be multiplied by the same number its value is not altered; which multiplication is clearly seen in the process.

By Rule (2).— *Answers.*

$$\tfrac{5}{8} \ldots 5 \times 12 \times 14 = 840, \text{ numerator of } \frac{5 \times 12 \times 14}{8 \times 12 \times 14} = \frac{840}{1344}$$

$$\tfrac{7}{12} \ldots 7 \times 8 \times 14 = 784, \text{ numerator of } \frac{7 \times 8 \times 14}{12 \times 8 \times 14} = \frac{784}{1344}$$

$$\tfrac{13}{14} \ldots 13 \times 8 \times 12 = 1248, \text{ numerator of } \frac{13 \times 8 \times 12}{14 \times 8 \times 12} = \frac{1248}{1344}$$

Common denominator $8 \times 12 \times 14 = 1344$.

EXAMPLE (2).

Compare the values of $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{9}$.

Find the least common multiple of 5, 6, 9.

$$\begin{array}{r|ccc} 3 & 5 & 6 & 9 \\ \hline & 5 & 2 & 3 \end{array}$$

$5 \times 2 \times 3 \times 3 = 90 =$ least common multiple of 5, 6, 9.

90 the least common denominator.

$$\tfrac{4}{5} \ldots 4 \times 18 = 72 \text{ numerator. The fractions are } \frac{4 \times 18}{5 \times 18} = \frac{72}{90}$$

$$\tfrac{5}{6} \ldots 5 \times 15 = 75 \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \frac{5 \times 15}{6 \times 15} = \frac{75}{90}$$

$$\tfrac{7}{9} \ldots 7 \times 10 = 70 \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \frac{7 \times 10}{9 \times 10} = \frac{70}{90}$$

Therefore $\frac{5}{6}$ is the greatest, $\frac{4}{5}$ the next, and $\frac{7}{9}$, the least.

17.

EXERCISES E.

(1). Reduce $\frac{7}{8}$, $\frac{13}{16}$, and $\frac{9}{10}$ to equivalent fractions, having a common denominator.

(2). Reduce $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{2}{5}$ to equivalent fractions, having a common denominator.

77. *To reduce a quantity to a fraction of a higher name, without altering its value—termed* reduction ascending.

Rule.—Reduce the given quantity to the lowest name it contains, for the numerator, and the specified integer to the same name, for a denominator. This fraction, reduced to its lowest terms, will be the one required.

EXAMPLES.

Reduce 6s. 3d. to the fraction of a £.*

6s. 3d. = 75d. for the numerator.

£1 = 240d. for the denominator.

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* Here reduction is conducted as in whole numbers; for if pence are to be converted into pounds, the process is to divide them by 12, and that quotient by 20, or to divide by 240 at once. This is what is termed reduction *ascending*

Hence 75 ÷ 240 or $\frac{75}{240}$ is the required fraction, which, reduced to its lowest terms = $\frac{5}{16}$. *Ans.*

Reduce 13s. 6¾d. to the fraction of a £.

13s. 6¾d. = 651f. for the numerator.
£1 = 960f. for the denominator.

Hence 651 ÷ 960 or $\frac{651}{960}$ is the required fraction, which reduced to its lowest terms = $\frac{217}{320}$. *Ans.*

17.

EXERCISES F.

1. Reduce 3s. 9d. to the fraction of a £.
2. „ 14 lb. to the fraction of 1 cwt.
3. „ 11 cwt. to the fraction of 1 ton.
4. „ 1 rod 25 sq. p. to the fraction of 1 acre.
5. „ 16s. to the fraction of £1.
6. „ 12s. 3d. to the fraction of 1 guinea.
7. „ 3 oz. 4 dwts. to the fraction of 1 lb. troy.
8. „ 9¾d. to the fraction of £1.
9. „ 3 furlongs, 8 poles to the fraction of 1 mile.

78. *To express the value of a fraction in a lower denomination—termed* reduction descending.

Rule.—Reduce the numerator, as in common reduction, to the lower name required; then divide by the denominator, and the quotient will be the value in that denomination.

EXAMPLES.

In £$\frac{4}{5}$, how many shillings?

£4 = 80s. and 80s. ÷ 5 = $\frac{80}{5}$ = 16s. *Ans.*

In £$\frac{3}{8}$, how many shillings?

£3 = 60s. and 60s. ÷ 8 = $\frac{60}{8}$ = 7s. 6d. *Ans.*

17.

EXERCISES G.

1. Find the value of £$\frac{3}{16}$ in a lower denomination.
2. „ „ $\frac{5}{12}$ guinea „ „
3. „ „ $\frac{3}{8}$ acre „ „
4. „ „ $\frac{7}{8}$ ton „ „
5. „ „ $\frac{5}{8}$ lb troy „ „
6. „ „ $\frac{7}{8}$ ton „ „
7. „ „ $\frac{3}{5}$ mile „ „
8. „ „ $\frac{3}{8}$ degree „ „
9. „ „ $\frac{5}{16}$ lb. „ „
10. „ „ $\frac{2}{3}$ of $\frac{3}{4}$ guinea „ „
11. „ „ $\frac{5}{7}$ hhds. „ „
12. „ „ $\frac{2}{3}$ of $\frac{5}{7}$ cwt. „ „

From what has been already said on the nature of a fraction, £$\frac{4}{5}$ is to be viewed as £4 divided by 5; that is, 80 shillings divided by 5, equal 16s. This is simply performing the operation of reduction *descending* on the numerator, with the result divided by the denominator.

ADDITION OF FRACTIONS.

79.

Rule.*—If the fractions have not a common denominator, reduce them to one; then add their numerators together, under which, place the common denominator, for the required sum.

EXAMPLES.

(1.) Add $\frac{26}{13}+\frac{39}{13}+\frac{52}{13}=\frac{26+39+52}{13}=\frac{117}{13}=9$ *Ans.*

(2.) Add $\frac{11}{18}+\frac{13}{18}+\frac{17}{18}+\frac{5}{18}=\frac{11+13+17+5}{18}=\frac{46}{18}=2\frac{5}{9}$ *Ans.*

(3.) Add $\frac{28}{4}+\frac{12}{3}+\frac{18}{6}=\frac{84}{12}+\frac{48}{12}+\frac{36}{12}=$
$\frac{84+48+36}{12}=\frac{168}{12}=14$ *Ans.*

(4.) Add $\frac{75}{120}+\frac{108}{120}+\frac{84}{120}=\frac{75+108+84}{120}=2\frac{9}{40}$ *Ans.*

(5.) Add $\frac{5}{8}+\frac{9}{10}+\frac{7}{12}$

$$\begin{array}{r|rrr} 2 & 8 & 10 & 12 \\ \hline 2 & 4 & 5 & 6 \\ \hline & 2 & 5 & 3 \end{array}$$

$2\times5\times3\times2\times2=120$ the least common multiple.

Hence the least common multiple of 8, 10, and 12 is 120, therefore 120 is the least common denominator.

120 common denominator.

$$\begin{array}{lrcr} \text{Then} & \frac{5}{8}\ldots5\times15 & = & 75 \\ & \frac{9}{10}\ldots9\times12 & = & 108 \\ & \frac{7}{12}\ldots7\times10 & = & 70 \\ \hline \end{array}$$

$$\frac{253}{120}=2\tfrac{13}{120},\ or\ \frac{75}{120}+\frac{108}{120}+\frac{70}{120}=$$

$$\frac{75+108+70}{120}=2\tfrac{13}{120}$$

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* **The fundamental nature of both addition and subtraction demands that the *units* of the numerators should be of the same name, and of the same size, before being either added or subtracted.**

EXERCISES 18.

1. Add $\frac{34}{17} + \frac{51}{17} + \frac{68}{17} + \frac{85}{17}$.
2. " $\frac{42}{14} + \frac{28}{7} + \frac{70}{14} + \frac{21}{7}$.
3. " $\frac{9}{3} + \frac{8}{2} + \frac{20}{10}$.
4. " $6\frac{5}{16} + 3\frac{7}{16} + 4\frac{11}{16} + 5\frac{13}{16}$.
5. " $\frac{2}{3} + \frac{4}{5} + \frac{7}{9}$.
6. " $\frac{7}{8} + \frac{15}{16} + \frac{9}{10}$.
7. " $\frac{3}{5} + \frac{5}{9} + 1\frac{1}{2} + 3\frac{7}{10} + 3\frac{11}{16}$.
8. " $5\frac{7}{7} + 8\frac{7}{12} + 6\frac{3}{8} + 4\frac{5}{5}$.
9. " $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{10}$ to $\frac{3}{4} + \frac{7}{8} + \frac{2}{5}$ *.
10. " $7\frac{1}{20} + 4\frac{61}{80} + 6\frac{127}{160}$.
11. " $6\frac{3}{8} + \frac{17}{48} + 11\frac{15}{32}$.
12. " $7\frac{3}{4} + 5\frac{365}{832}$.
13. " $5\frac{3}{4} + 8\frac{1}{2} + 12\frac{1}{3} + 18\frac{1}{4} + 26\frac{1}{8} + 33\frac{1}{2}$.
14. " $3 + 7\frac{1}{2} + 3\frac{3}{4} + 1\frac{1}{14}$.
15. " $23\frac{3}{8} + 27\frac{3}{4} + 24\frac{2}{3}$.
16. " $14\frac{83}{160} + 2\frac{469}{480} + 5\frac{149}{160} + \frac{13}{40}$.
17. " $2\frac{421}{448} + 9\frac{83}{112}$.
18. " $15\frac{5}{8} + 13\frac{5}{6} + 17\frac{4}{5}$.
19. " $5\frac{5}{8}$ guineas + £$3\frac{3}{16}$.
20. " $11\frac{11}{14} + 9\frac{33}{56} + 18\frac{109}{112} + 9\frac{109}{224}$.
21. " 18s. $11\frac{1}{2}$d. + 9s. $5\frac{3}{4}$d. + $8\frac{1}{8}$d. + $5\frac{75}{128}$d.
22. " $2\frac{2}{3}$ cwt. + $7\frac{5}{8}$ qr. + $18\frac{3}{8}$ lb.
23. " 9 cwt. 1 qr. $25\frac{1}{12}$ lb. + 3 cwt. 2 qr. $5\frac{9}{32}$ lb.
24. " $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{7}{16} + 1\frac{3}{28}$.
25. " $15\frac{5}{8} + 13\frac{5}{6} + 17\frac{4}{5}$.
26. " $89\frac{3}{4} + 69\frac{11}{16} + 43\frac{2}{3}$.
27. " $2\frac{7}{12} + 7\frac{5}{6} + 5\frac{3}{8}$.
28. " $1\frac{7}{12} + 7\frac{13}{16} + 3\frac{5}{9}$.
29. " $1\frac{7}{16} + 2\frac{2}{3} + 4\frac{3}{8}$.
30. " $3\frac{3}{8}$ guineas + $3\frac{5}{16}$ crowns + $17\frac{2}{3}$ half sov. + $4\frac{3}{8}$ crowns.
31. " $4\frac{1}{3} + 5\frac{3}{4} + 6\frac{4}{5}$.
32. " £3 5s. $10\frac{1}{2}$d. + 16s. $5\frac{5}{8}$d. + 4s. $8\frac{13}{28}$d. + 1s. $2\frac{13}{112}$d. + $3\frac{237}{448}$d.
33. " £143 2s. $11\frac{7}{16}$d. + £22 0s. $9\frac{123}{385}$d.
34. " $10\frac{1}{2} + 3\frac{3}{4} + 7\frac{7}{8} + 9\frac{15}{16} + 10\frac{31}{32} + 11\frac{127}{160} + 5\frac{287}{320}$.
35. " $7\frac{1}{2} + 9\frac{3}{4} + 1\frac{18}{20} + \frac{38}{40} + \frac{38}{80} + \frac{39}{160} + \frac{13}{80} + \frac{143}{5120}$.

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* Compound and complex fractions must be reduced to simple ones preparatory to their being operated with, in the rules of addition, subtraction, multiplication and division.

36. Bought $4\frac{3}{5}$ cwt. of tea, and afterwards $24\frac{5}{8}$ lb. and $13\frac{1}{2}$ ounces; how much did I buy in all?

37. What is the amount of $456\frac{7}{8} + 474\frac{13}{16}$?

38. What is the amount of £85 13s. $3\frac{3}{8}$d., and £68 5s. $1\frac{1}{32}$d.?

39. Add $4\,\frac{2\frac{1}{3}}{3} + 1\,\frac{1\frac{1}{2}}{2} + 4\,\frac{\frac{1}{2}}{3} + 3\,\frac{1\frac{1}{3}}{1\frac{1}{2}}$.

40. „ $4\,\frac{1\frac{1}{3}}{2\frac{1}{2}} + 8\,\frac{2\frac{1}{4}}{4} + 9\,\frac{2\frac{1}{2}}{3\frac{1}{3}}$.

41. „ $\frac{3\frac{1}{2} + 1}{3\frac{1}{3}} + \frac{4\frac{1}{2} + 2\frac{1}{4}}{3\frac{1}{2} - 1\frac{4}{3}} + 4$.

42. „ $4\frac{2}{3} + 3\frac{3}{8} + 1\frac{1}{2} + 7\frac{3}{4} + 5\frac{2}{3}$.

43. „ 7s. $10\frac{1}{2}$d. to $9\frac{747}{5083}$d.

44. „ £35 2s. $9\frac{253}{800}$ + £15 17s. $8\frac{18749}{58400}$d.

45. „ $2\frac{2}{3}$ cwt. + $7\frac{5}{8}$ qr. + $18\frac{3}{8}$ lb.

46. „ £$2\frac{7}{8}$ + $1\frac{4}{5}$ guineas + $3\frac{7}{9}$ crowns + $7\frac{9}{16}$ florins + $\frac{13}{16}$ shilling.

47. „ £$7\frac{1}{9}$, $8\frac{5}{8}$ guineas, $3\frac{7}{16}$ crowns, $3\frac{1}{7}$ shillings, and $4\frac{3}{4}$ pence.

48. „ $3\frac{5}{8} + \frac{2\frac{1}{2}}{3} + \frac{3\frac{1}{2}}{4\frac{1}{2}} + \frac{2\frac{1}{3}}{3\frac{1}{4}} + 6\,\frac{3\frac{1}{2}}{3\frac{2}{3}}$.

SUBTRACTION OF FRACTIONS.

80.

Rule.—If the fractions have not a common denominator, reduce them to one; then subtract the numerator of the lower fraction from the upper, and under the difference place the common denominator. But if the numerator below be greater than that above, to the latter add the common denominator, and from the sum subtract the former; underneath the remainder place the common denominator as before, and carry one to the figure below, as in simple subtraction.

EXAMPLES.

(1.) From $\frac{85}{17}$ take $\frac{34}{17}$. Here $\frac{85}{17} - \frac{34}{17} = \frac{85 - 34}{17} = \frac{51}{17} = 3$ *Ans.*

(2.) From $\frac{42}{6}$ take $\frac{8}{4}$. Here $\frac{42}{6} - \frac{8}{4} = \frac{84}{12} - \frac{24}{12} = \frac{84 - 24}{12} = \frac{60}{12} =$ 5 *Ans.*

(3.) From 8 take $3\frac{2}{9}$. Since $8 = 7\frac{9}{9}$, then $7\frac{9}{9} - 3\frac{2}{9} = 4\,\frac{9 - 2}{9} =$ $4\frac{7}{9}$ *Ans.*

(4.) From $\frac{33}{37}$ take $\frac{18}{37}$. Here $\frac{33}{37} - \frac{18}{37} = \frac{33 - 18}{37} = \frac{15}{37}$ *Ans.*

(5.) From $\frac{4}{5}$ take $\frac{5}{8}$. Here $\frac{4}{5} - \frac{5}{8} = \frac{32}{40} - \frac{25}{40} = \frac{32 - 25}{40} = \frac{7}{40}$ *Ans.*

EXERCISES 19.

1. From 3 take $1\frac{3}{8}$.
2. „ $3\frac{13}{16}$ take $1\frac{5}{16}$.
3. „ $33\frac{17}{35}$ take $18\frac{29}{35}$.*
4. „ $13\frac{1}{2}$ take $10\frac{5}{16}$.
5. „ $13\frac{3}{8}$ take $10\frac{2}{3}$.
6. „ $14\frac{421}{448}$ take $9\frac{93}{112}$.
7. „ $4\frac{1}{12}$ take $\frac{3}{8}$.
8. „ $37\frac{3}{16}$ take $11\frac{51}{416}$.
9. „ $13\frac{59}{100}$ take $7\frac{5431}{36500}$.
10. „ $24\frac{1}{2}$ lb. take $8\frac{13}{16}$ lb.
11. „ 12 take $3\frac{3}{14} + 3\frac{45}{56}$.
12. „ $10\frac{31}{40}$ take $5\frac{127}{140}$.
13. „ 15s. $5\frac{43}{112}$d. take 4s. $0\frac{3}{4}$d.
14. „ $7\frac{1}{2}$ take $2\frac{39}{80}$.
15. „ £51 15s. $2\frac{421}{448}$d. take £47 8s. $9\frac{93}{112}$d.
16. „ $8\frac{2}{3} + 10\frac{3}{4}$ take $3\frac{1}{2}$.
17. „ $18\frac{1469}{1740}$ take $6\frac{4889}{5040}$.
18. „ $(\frac{1}{3} + \frac{1}{2})$ take $(\frac{4}{21} + \frac{7}{11})$.
19. „ $(18\frac{5}{6} + 17\frac{3}{8})$ take $6\frac{1}{3}$.
20. „ 13s. $7\frac{109}{320}$d. take 6s. $3\frac{9}{16}$d.
21. „ $18\frac{2}{3} + 10\frac{1}{4}$ take $1\frac{1}{2}$.
22. „ $8\frac{2}{3} + 10\frac{3}{4}$ take $3\frac{1}{2}$.
23. „ $19\frac{2}{3}$ take $10\frac{2}{5}$.
24. „ $89\frac{7}{8} + 17\frac{2}{3} + 13\frac{5}{6}$ take $29\frac{1}{2} + 15\frac{5}{9} + 17\frac{7}{12}$.
25. „ £371 13s. $2\frac{49}{1120}$d. take £344 2s. $6\frac{25}{32}$d.
26. „ $8\frac{7}{8}$ take $5\frac{3}{4}$.
27. „ $108\frac{13}{16}$ take $17\frac{17}{18}$.
28. „ $15\frac{13}{24}$ take $1\frac{2}{3}$.
29. „ $12\frac{137}{144}$ take $2\frac{5}{8}$.
30. „ $8\frac{23}{48}$ take $1\frac{5}{12}$.
31. „ £86 6s. $6\frac{3}{4}$d. take £80 17s. $8\frac{53}{56}$d.
32. „ £74 15s. $5\frac{43}{112}$ take £8 4s. $0\frac{3}{4}$d.

MULTIPLICATION OF FRACTIONS.

81.

Rule.—If the quantities to be multiplied are not in the form of simple fractions, reduce them to that form. Then multiply all the numerators together for the numerator of the product, and all the denominators together for its denominator.

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* From $33\frac{17}{35} = 32\frac{35}{35} + \frac{17}{35} = 32\frac{52}{35}$
Take $18\frac{23}{35} =$ $18\frac{29}{35}$
$14\frac{23}{35}$ *Ans.*

EXAMPLES.

(1). Multiply $\frac{15}{5}$ by $\frac{6}{2} = \frac{15}{5} \times \frac{6}{2} = \frac{90}{10} = 9$ *Ans.*

(2). * „ $\frac{3}{7}$ by $6 = \frac{3}{7} \times \frac{6}{1} = \frac{18}{7} = 2\frac{4}{7}$ *Ans.*

(3). „ $\frac{3}{7}$ by $\frac{6}{11} = \frac{3}{7} \times \frac{6}{11} = \frac{18}{77}$ *Ans.*

(4). „ $\frac{4}{5}$ by $\frac{5}{6} = \frac{4}{5} \times \frac{5}{6} = \frac{20}{30} = \frac{2}{3}$ *Ans.*

(5). „ $3\frac{4}{5}$ by $\frac{2}{3} = \frac{19}{5} \times \frac{2}{3} = \frac{38}{15} = 2\frac{8}{15}$ *Ans.*

(6). „ $3\frac{3}{4}$ by $2\frac{2}{3} = \frac{15}{4} \times \frac{8}{3} = \frac{120}{12} = 10$ *Ans.*

(7.) „ $\dfrac{3\frac{3}{7}}{4\frac{1}{2}}$ by $\dfrac{\frac{3}{7}}{\frac{3}{4}}$ since $\dfrac{3\frac{3}{7}}{4\frac{1}{2}} = \dfrac{16}{21}$, and $\dfrac{\frac{3}{7}}{\frac{3}{4}} = \dfrac{4}{7}$, therefore $\dfrac{3\frac{3}{7}}{4\frac{1}{2}} \times \dfrac{\frac{3}{7}}{\frac{3}{4}} = \dfrac{16}{21} \times \dfrac{4}{7} = \dfrac{64}{147}$ *Ans.*

EXERCISES 20.

1. $\frac{15}{5} \times \frac{8}{4}$.
2. $\frac{21}{7} \times \frac{6}{2}$.
3. $\frac{7}{2} \times \frac{12}{4}$.
4. $\frac{10}{4} \times \frac{12}{2}$.
5. $3\frac{1}{2} \times \frac{8}{2}$.
6. $3\frac{1}{2} \times 3\frac{1}{2}$.
7. $2\frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2}$.
8. $1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2}$.
9. $21\frac{1}{2} \times 21\frac{3}{5}$.
10. $3\frac{1}{8} \times \frac{6}{8}$.
11. $\frac{5}{6} \times 13\frac{3}{5}$.
12. $3 \times 1\frac{2}{5}$.
13. $2\frac{1}{2} \times 17\frac{1}{3}$.
14. $2\frac{1}{3} \times 17\frac{1}{2}$.
15. $\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6}$.
16. $\frac{2}{3} \times \frac{5}{9} \times 1\frac{2}{5} \times 13\frac{5}{8} \times 14\frac{2}{3} \times \frac{8}{9}$.
17. $(17\frac{3}{8} + 19\frac{7}{12}) \times 10\frac{1}{4} \times 225$.
18. $(3\frac{1}{2} + 5\frac{3}{5}) \times 2\frac{2}{3}$.
19. $22\frac{3}{4} \times 20\frac{3}{4} \times 18\frac{1}{2}$.
20. $21\frac{2}{3} \times 21\frac{3}{4} \times 17\frac{1}{2}$.

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* In multiplying $\frac{3}{7}$ by 6, if 3 alone be multiplied by 6, the result (18) is seven times too large, because the factor 3 is seven times greater than it should be; hence 18 must be divided by 7, making $\frac{18}{7}$ the product. Again, to multiply $\frac{3}{7}$ by $\frac{6}{11}$, if $\frac{3}{7}$ be multiplied by 6, the product is $\frac{18}{7}$, but the factor 6 is eleven times greater than it ought to be; therefore $\frac{18}{7}$ must be divided by 11, or 18 divided by 7 times 11, making the real product $\frac{18}{77}$; hence the rule.

Note.—Multiplying $\frac{3}{7}$ by 6, means $\frac{3}{7}$ added to itself six times, which makes $\frac{18}{7}$.

21. Multiply $21\frac{2}{3}$ by $24\frac{5}{8}$.

22. „ $14\frac{1}{8} \times 7\frac{1}{16}$.

23. „ $10\frac{47}{144} \times 7\frac{1}{16}$.

24. „ $468\frac{3}{4} \times 66\frac{2}{3}$.

25. „ $22209288\frac{3881}{4032}$ by 3, 4, 6, 7, 8, successively.

26. „ $5985666396\frac{1025}{1152}$ by 2, 3, 4, 6, 8, successively.

27. „ £3 19s. $1\frac{28483}{45360}$d. by 3, 4, 5, 6, 7, 9, successively.

28. „ £6 2s. $3\frac{10063}{11340}$d. by 2, 3, 5, 6, 7, 9, successively.

29. „ 10 tons 19 cwt. 1 qr. 8 lb. 6 oz. $2\frac{569}{720}$ drs. by 2, 3, 4, 5, 6, successively.

30. „ $119\frac{11}{16}$ by $12\frac{4}{9}$.

31. The divisor is $14\frac{1}{4}$, and the quotient $3\frac{1}{3}$; what is the dividend?

32. Multiply $5\frac{1}{2} \times 5\frac{1}{2} \times 5\frac{1}{2}$.

33. „ 7 yrs. 304 days 0 h. 24 m. $8\frac{722}{1008}$ sec. by 2, 3, 4, 6, 7.

34. „ 6 lb. 3 oz. 3 dwt. $14\frac{59}{1260}$ grs. by 2, 3, 5, 6, 7.

35. „ £2 11s. $3\frac{22231}{30240}$d. by 2, 5, 7, 3, 8, 9.

36. „ £2 5s. $5\frac{23737}{24192}$d. by 5, 7, 9, 3, 4, 8.

37. „ $37\frac{5}{16} \times 16\frac{17}{18} \times 158\frac{3}{4}$.

38. „ $5\frac{151}{375} \times 27\frac{19}{24}$.

39. „ $(8\frac{1}{3} + 3\frac{3}{4} + 6\frac{1}{9}) \times 4\frac{2}{3}$.

40. „ $(8\frac{2}{3} + 3\frac{3}{4}) \times 8\frac{3}{5}$.

41. „ $14\frac{107}{240} \times 28\frac{3}{8} \times 43\frac{1}{4}$.

42. How many yards in $8\frac{2}{3}$ pieces of cloth, each $(16\frac{7}{8} + 16\frac{5}{6})$ yds.?

43. What is the weight of $45\frac{1}{2}$ pieces of plate, each $(14\frac{1}{2} + 16\frac{2}{3} + 15\frac{5}{9})$ oz.?

44. What is the weight of $7\frac{3}{4}$ flitches of bacon, each $(15\frac{2}{3} + 18\frac{5}{8})$ lb.?

45. Multiply 4 cwt. 2 qr. $26\frac{13}{24}$ lb. by $2\frac{3}{4}$.

46. How much are 4 times $(15\frac{5}{8} + 13\frac{5}{6} + 17\frac{4}{5})$?

47. How much are $3\frac{1}{2}$ times $(19\frac{2}{3} - 10\frac{1}{2})$?

48. How much are $8\frac{2}{3}$ times $(18\frac{7}{8} + 16\frac{5}{6})$ yds.?

49. What is the amount of $21\frac{1}{2} \times 21\frac{1}{4} + 13\frac{3}{8} \times 35\frac{1}{2}$?

50. What is the cost of $456\frac{7}{8}$ yards at 3s. 9d. per yard?

51. What is the cost of $35\frac{1}{2}$ times $13\frac{3}{8}$ yards at 2s. $10\frac{1}{2}$d. per yard?

52. What is the amount of 3 times $(4\frac{2}{3} + 3\frac{3}{8} + 1\frac{1}{2})$ added to 35 times $(7\frac{3}{4} + 5\frac{2}{3})$?

53. What is the price of $5\frac{2}{3}$ pieces of cloth, each $37\frac{3}{8}$ yards, at 7s. $10\frac{1}{2}$d. per yard?

54. Multiply $(8\frac{2}{3} + 10\frac{3}{4} - 3\frac{1}{2}) \times 5\frac{5}{8}$.

55. What is the difference between the products of $3\frac{2}{3}$ and $2\frac{1}{2}$, and $3\frac{1}{2}$ and $2\frac{2}{3}$?

56. What is the cost of $16\frac{3}{5}$ packs of cloth, each containing $23\frac{2}{3}$ pieces of $49\frac{1}{2}$ yards, at 7s. $3\frac{1}{2}$d. per yard?

57. Find the amount of $(8\frac{2}{3} + 10\frac{1}{4} - 1\frac{1}{2}) \times 4\frac{2}{3}$.

58. What is the cost of $13\frac{2}{5}$ casks of butter, each weighing $2\frac{7}{8} + 1\frac{5}{8}$ cwt., at $8\frac{3}{4}$d. per lb.?

DIVISION OF FRACTIONS.

82.

Rule.—Reduce the quantities, if not already so, to the form of simple fractions, then invert the terms of the divisor, and proceed as in multiplication. The resulting fraction is the quotient required.

EXAMPLES.

(1). Divide $\frac{48}{8}$ by 3 that is $\frac{48}{8} \div \frac{3}{1} = \frac{48}{8} \times \frac{1}{3} = \frac{48}{24} = 2$ *Ans.*

(2). „ $\frac{42}{7}$ by $\frac{12}{6}$ that is $\frac{42}{7} \div \frac{12}{6} = \frac{42}{7} \times \frac{6}{12} = \frac{252}{84} = 3$ *Ans.*

(3). „ $\frac{7}{8}$ by 3, that is $\frac{7}{8} \div \frac{3}{1} = \frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$ *Ans.*

(4). „ $\frac{252}{9}$ by $\frac{42}{6}$ that is $\frac{252}{9} \div \frac{42}{6} = \frac{252}{9} \times \frac{6}{42} = \frac{1512}{378} = 4$ *Ans.*

(5). „ * $\frac{4}{7}$ by $\frac{3}{5}$ that is $\frac{4}{7} \div \frac{3}{5} = \frac{4}{7} \times \frac{5}{3} = \frac{20}{21}$ *Ans.*

(6). „ $2\frac{5}{9}$ by $3\frac{3}{4}$ that is $\frac{23}{9} \div \frac{15}{4} = \frac{23}{9} \times \frac{4}{15} = \frac{92}{135}$ *Ans.*

(7). „ $\dfrac{3\frac{3}{7}}{4\frac{1}{2}}$ by $\dfrac{\frac{3}{7}}{4}$ that is $\dfrac{16}{21} \div \dfrac{3}{28} = \dfrac{16}{21} \times \dfrac{28}{3} = \dfrac{64}{9} = 7\frac{1}{9}$ *Ans.*

(8). „ $\frac{4}{5}$ by $\frac{5}{8}$ that is $\frac{4}{5} \div \frac{5}{8} = \frac{4}{5} \div \frac{8}{5} = \frac{32}{25} = 1\frac{7}{25}$ *Ans.*

EXAMPLE.

Divide 57890532879 by 2, 3, 4, 5, 6, 7, 9, and by 3, 4, 5, 6, 7, 9, 2.

2) 57890532879	3) 57890532879
3) $28945266439\frac{1}{2}$	4) 19296844293
4) $9648422146\frac{1}{2}$	5) $4824211073\frac{1}{4}$
5) $2412105136\frac{5}{8}$	6) $964842214\frac{13}{20}$
6) $482421107\frac{13}{40}$	7) $160807035\frac{31}{40}$
7) $80403517\frac{71}{80}$	9) $22972433\frac{191}{280}$
9) $11486216\frac{471}{560}$	2) $2552492\frac{1591}{2520}$
Ans. $1276246\frac{1591}{5040}$	*Ans.* $1276246\frac{1591}{5040}$

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* "*Divide* $\frac{4}{7}$ *by* $\frac{3}{5}$." If $\frac{4}{7}$ be divided by 3, it is the same as 4 divided by 7 and that quotient again divided by 3, which is equivalent to 4 divided by the product of 7 times 3, or $\frac{4}{21}$; but, in dividing $\frac{4}{7}$ by 3, instead of by $\frac{3}{5}$, a divisor

$$\begin{array}{rcr}
1276246\frac{1591}{5040} & \qquad\qquad & 1276246\frac{1591}{5040} \\
2 & & 3 \\ \cline{1-1}\cline{3-3}
2552492\frac{1591}{2520} & & 3828738\frac{1591}{1680} \\
3 & & 4 \\ \cline{1-1}\cline{3-3}
7657477\frac{751}{840} & & 15314955\frac{331}{420} \\
4 & & 5 \\ \cline{1-1}\cline{3-3}
30629911\frac{121}{210} & & 76574778\frac{79}{84} \\
5 & & 6 \\ \cline{1-1}\cline{3-3}
153149557\frac{37}{42} & & 459448673\frac{9}{14} \\
6 & & 7 \\ \cline{1-1}\cline{3-3}
918897347\frac{2}{7} & & 3216140715\frac{1}{2} \\
7 & & 9 \\ \cline{1-1}\cline{3-3}
6432281431 & & 28945266439\frac{1}{2} \\
9 & & 2 \\ \cline{1-1}\cline{3-3}
57890532879 \ \textit{Proof.} & & 57890532879
\end{array}$$

EXERCISES 21.

1. $\frac{84}{7} \div \frac{16}{5}$.
2. $\frac{12}{3} \div \frac{8}{4}$.
3. $\frac{144}{9} \div \frac{32}{8}$.
4. $\frac{126}{7} \div \frac{42}{15}$.
5. $7\frac{1}{5} \div 3$.
6. $28\frac{15}{24} \div 3$.
7. $469\frac{7}{12} \div 35$.
8. $40\frac{5}{8} \div 2\frac{1}{3}$.
9. $769\frac{1}{6} \div 21\frac{2}{3}$.
10. $211\frac{13}{24} \div 5\frac{2}{3}$.
11. $381\frac{13}{16} \div 10\frac{1}{4}$.
12. $4\frac{1}{2} \div 1\frac{4}{5}$.
13. $13\frac{1}{2} \div 4\frac{1}{2}$.
14. $\frac{117}{9} \div \frac{12}{4}$.

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5 times too great has been used; hence $\frac{4}{21}$ is 5 times too little, and must be multiplied by 5, producing $\frac{20}{21}$ the correct quotient, which shows the application of the rule. Or, again, if $\frac{4}{7}$ and $\frac{3}{5}$ be reduced to fractions having a common denominator, the question becomes $\frac{20}{35}$ divided by $\frac{21}{35}$; now, if the dividend and divisor be multiplied by the same number (35), the quotient remains the same; hence the result will be 20 divided by 21, or $\frac{20}{21}$ the quotient; which also shows the rule.

15. Divide 89547853097 by 3, 4, 6, 7, 8, and by 4, 6, 7, 8, 3.

16.* „ 38789047 by 2, 3, 4, 6, 7, 9, 5, and by 3, 4, 6, 7, 9, 5, 2.

17. „ 508953467 by 2, 3, 4, 5, 6, 7, and by 3, 4, 5, 6, 7, 2.

18. „ 790456789 by 2, 3, 5, 7, 9, 8, 4, and by 5, 7, 9, 8, 4, 3, 2.

19. „ £13875 13s. $5\frac{1}{2}$d. by 2, 3, 4, 5, 6, 7, 9, and by 3, 4, 5, 6, 7, 9, 2.

20. By what must I multiply 35 that the product may be 570?

21. By what must I multiply $1\frac{3}{5}$ that the product may be $\frac{14}{15}$?

22. Divide $(8\frac{1}{3} + 17\frac{5}{9} + 11\frac{5}{18})$ by $(1\frac{3}{4} + 1\frac{5}{7} + 2\frac{1}{2})$.

23. „ 765347 by 3, 4, 5, 7, 9, successively.

24. Find the value of $(17\frac{2}{3} + 19\frac{7}{12}) \times 10\frac{1}{4} \times 225 \div 4\frac{1}{2}$.

25. If $868\frac{1}{2}$ lb. of tea cost £229 15s. $9\frac{3}{4}$d., what is it a lb.?

26. If $141\frac{3}{4}$ ounces of silver cost £36 9s. $5\frac{1}{16}$d., what is it an ounce?

27. Divide £79 10s. $10\frac{5}{8}$d. by $881\frac{13}{16}$.

28. „ £374 19s. $4\frac{1}{2}$d by $769\frac{1}{8}$.

29. „ $16\frac{53}{60}$ by $3\frac{1}{5}$.

30. „ 6895487689287 by 2, 3, 4, 6, and 8, successively.

31. „ £89735 13s. $5\frac{1}{2}$d. by 3, 4, 5, 6, 7, and 9, „

32. „ £69357 13s. 7d. by 2, 3, 5, 6, 7, and 9, „

33. „ 7895 tons 13 cwt. 3 qrs. 17 lb. 13 oz. 9 drs. by 2, 3, 4, 5, and 6, successively.

34. „ $21\frac{101}{105}$ by $15\frac{7}{12}$.

35. The product of two numbers is $42\frac{2}{3}$, and one of them is $\frac{3}{4}$, what is the other?

36. By what number is $19\frac{3}{8}$ to be divided, that the quotient may be $6\frac{2}{3}$?

37. Of what number is $6\frac{3}{4}$ the $\frac{5}{6}$th?

38. By what is $3\frac{3}{4}$ to be multiplied, that the product may be $23\frac{1}{16}$?

39. Divide 7895 years 213 days 21 h. 39 m. 43 sec. by 2, 3, 4, 6, 7, and 9, successively.

40. „ 7893 lb. 9 oz. 17 dwts. 21 grs. by 2, 3, 5, 6, and 7, successively.

41. „ 89547853097 by 3, 4, 6, 7, and 8, successively.

42. „ £38765 13s. $9\frac{1}{2}$d. by 2, 5, 7, 3, 8, and 9, successively.

43. By what number must the sum of $4\frac{2}{3} + 3\frac{2}{5}$ be multiplied to give the product $240\frac{118}{120}$?

44 Divide £68793 17s. $9\frac{3}{4}$d. by 5, 7, 9, 3, 4, and 8, successively.

45. „ $\left(3\frac{5}{8} + \dfrac{3\frac{1}{2}}{4\frac{1}{2}} + \dfrac{2\frac{1}{2}}{3} + \dfrac{2\frac{1}{3}}{3\frac{1}{4}} + 6\dfrac{3\frac{1}{2}}{3\frac{2}{3}}\right)$ by 2.

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* This and similar questions may be made to embrace two, three, or more exercises in division, and their proof by multiplication would be good exercise for the learner.

46. What is the quotient, if $(8\frac{2}{3} + 3\frac{3}{4}) \times 8\frac{3}{5} \times 18\frac{7}{8}$ be divided by $3\frac{1}{3}$?

47. If $931\frac{11}{16}$ lbs. of tea cost £158 18s. $4\frac{13}{32}$, what is it per lb.?

48. If $211\frac{19}{24}$ gallons of rum cost £91 9s. $3\frac{9}{16}$d., what is it a gallon?

49 Divide 3515 cubic yards 20 c. feet $1199\frac{13}{16}$ c. inches, by $47\frac{11}{16}$.

50. If $37\frac{1}{2}$ times $33\frac{3}{8}$ lbs. of tea cost £232 1s. $2\frac{17}{32}$d., what is it a lb.?

51. If $21\frac{2}{3}$ pieces of cloth, each containing $35\frac{3}{5}$ yards, cost £171 18s. $10\frac{1}{3}$d., what is it a yard?

52. If 3 times $47\frac{1}{4}$ yards + 6 times $46\frac{1}{2}$ yards + 18 times $48\frac{1}{4}$ yds. cost £339 9s. $11\frac{13}{16}$d., what is it a yard?

53. If $16\frac{3}{5}$ packs, each containing $23\frac{2}{3}$ pieces of $49\frac{1}{2}$ yards, cost £7090 0s. $3\frac{3}{4}$d., what is it a yard?

54. Find the value of $17\dfrac{2\frac{1}{2}}{3} \times \dfrac{3\frac{1}{2} + 1}{3\frac{1}{3}} + \dfrac{4\frac{1}{2} + 2\frac{1}{3}}{3\frac{1}{2} - 1\frac{2}{3}} + 2\frac{1}{2} \times 1\frac{3}{5}$.

55. Divide $(18\frac{2}{3} + 17\frac{3}{4} + 5) - (2\frac{2}{3} \times 5\frac{3}{4})$ by $1\frac{2}{5}$ of $4\frac{2}{3}$.

56. „ £78954 13s. $10\frac{1}{2}$d. by 25920.

Simplify the expression $\dfrac{3\frac{1}{2} + 1}{3\frac{1}{3}} + \dfrac{4\frac{1}{2} + 2\frac{1}{3}}{3\frac{1}{2} - 1\frac{2}{3}} + 2\frac{1}{2} \times 1\frac{3}{5}$.

$$\frac{4\frac{1}{2}}{3\frac{1}{3}} = 1\tfrac{7}{20}$$

$$\frac{4\frac{1}{2} + 2\frac{1}{3}}{3\frac{1}{2} - 1\frac{2}{3}} \text{ or } \frac{6\frac{5}{6}}{1\frac{5}{6}} = 3\tfrac{8}{11}$$

$$2\tfrac{1}{2} \times 1\tfrac{3}{5} = \underline{4}$$

Ans. $9\frac{17}{220}$.

Simplify the expression $\left(\dfrac{2\frac{1}{2}}{3\frac{1}{2}} + \dfrac{3}{2\frac{1}{3}} + 1\frac{3}{8} \text{ of } \frac{3}{5}\right) \times 1\frac{2}{3} \times 3\frac{1}{3}$,

which is, $(\frac{5}{7} + \frac{9}{7} + \frac{33}{40}) \times 1\frac{2}{3} \times 3\frac{1}{3}$.

Or, $2\frac{33}{40} \times 1\frac{2}{3} \times 3\frac{1}{3} = 15\frac{25}{36}$ *Ans.*

Simplify the expression $\dfrac{3\frac{1}{2} \text{ of } \frac{5}{9}}{2\frac{2}{3} \text{ of } 2\frac{1}{2}} \times \dfrac{2\frac{2}{9} \text{ of } \frac{7}{10}}{2\frac{2}{3} \text{ of } 3\frac{1}{2}} \div \dfrac{\frac{3}{4}}{\frac{5}{6}}$

$$\frac{3\frac{1}{2} \text{ of } \frac{5}{9}}{2\frac{2}{3} \text{ of } 2\frac{1}{2}} = \frac{1\frac{17}{18}}{6\frac{2}{3}} = \tfrac{7}{24}$$

$$\frac{2\frac{2}{9} \text{ of } \frac{7}{10}}{2\frac{2}{3} \text{ of } 3\frac{1}{2}} = \frac{1\frac{5}{9}}{9\frac{1}{3}} = \tfrac{1}{6}$$

$$\frac{\frac{3}{4}}{\frac{5}{6}} = \tfrac{9}{10}$$

And the expression becomes $(\frac{7}{24} \times \frac{1}{6}) \div \frac{9}{10} = \frac{35}{648}$ *Ans.*

57. Simplify $\dfrac{\frac{2}{3} + \frac{3}{4} + 1\frac{1}{2} \text{ of } 1\frac{2}{5}}{1\frac{2}{3} + \dfrac{1}{2\frac{1}{2}} + \dfrac{2}{3\frac{1}{2}}} \div \dfrac{4\frac{1}{2} - 1\frac{1}{3}}{3\frac{1}{2} - \dfrac{2\frac{1}{2}}{1\frac{1}{3}}}$

58. Reduce $\frac{5}{8}$ of $2\frac{3}{5}$ of $1\frac{1}{3}$ of $3\frac{5}{6}$ of $\frac{6}{7}$ of 7 of $3\frac{1}{3}$, to a simple fraction.

59. Divide $\frac{7}{8}$ of $\frac{6}{7}$ of $33\frac{5}{8}$ of $5\frac{1}{2}$ by $\frac{3}{5}$ of $\frac{5}{6}$ of $1\frac{2}{3}$ of $3\frac{3}{4}$.

60. Simplify $\left(\frac{1\frac{1}{3}+\frac{7}{8}}{1\frac{1}{2}-\frac{1}{12}}+\frac{7}{8}\text{ of }\frac{3\frac{2}{3}\times 3}{1\frac{1}{2}\text{ of }3\frac{2}{3}}-\frac{2\frac{1}{2}-\frac{2}{3}}{1\frac{1}{3}\times\frac{1}{4}}\right)$.

PROPORTION AND OTHER EXERCISES IN FRACTIONS.

83.

Rule.—Arrange the terms as directed in the rule of proportion for integers (Art. 61); then when the prescribed law of reducing the first and second terms to the same name is fulfilled, multiply the second and third terms together, and divide by the first, according to the methods for multiplying and dividing fractions.

EXAMPLE.

What is the price of $2\frac{5}{12}$ cwt. of tobacco, if $1\frac{2}{3}$ lb. cost $5\frac{5}{8}$s.?

As $1\frac{2}{3}$ lb. : $2\frac{5}{12}$ cwt. : : $5\frac{5}{8}$s.

$$\frac{5}{3} \qquad \frac{29}{12}-\frac{112}{1} \qquad \frac{45}{8}$$

$$\frac{3}{5}-\frac{3248}{12}-\frac{45}{8}=\frac{29232}{32}\text{s.}=£45\ 13\text{s. }6\text{d.}$$

If $2\frac{5}{8}$ acres of land be rented for £13 18s. 9d., what will be the rent of 79 a. 3 r. 35 p.?

As $2\frac{5}{8}$ a. : 79 a. 3 r. 35 p. : : £13 18s. 9d.

$$\frac{21}{8}-\frac{160}{1} \qquad 79\times 4=319,\ 319\times 40=12795 \qquad 13\times 20=278,\ 278\times 12=3345$$

$$\frac{3360}{8} \qquad \frac{12795}{1} \qquad \frac{3345}{1}$$

$$\frac{8}{3360}-\frac{42799275}{1}=\frac{342394200}{3360}\text{d.}=£424\ 11\text{s. }11\tfrac{1}{28}\text{d.}$$

EXERCISES 22.

1. What is the rent of $73\frac{3}{5}$ acres of land at £$\frac{5}{8}$ the rood?
2. Bought $45\frac{1}{2}$ pieces of cloth, each containing $46\frac{13}{15}$ yards, at 17s. 10d. for $3\frac{2}{3}$ yards; find the entire value of the cloth.
3. What is the price of $533\frac{13}{24}$ yards of cloth, if $12\frac{3}{5}$ yards cost £3 15s. 7d.?

4. If $9\frac{3}{5}$s. buy 2 yds. 3 qrs. of cloth, how much will £13 8s. 10d. buy?

5. What is the cost of $460\frac{1}{10}$ lbs. of tobacco, if $2\frac{1}{12}$ lbs. cost $11\frac{7}{48}$s.?

6. If $2\frac{1}{3}$ yards of cloth cost 18s. $5\frac{1}{2}$d., what is the price of $309\frac{17}{36}$ yards?

7. What is the cost of $469\frac{7}{12}$ ounces of silver, if $28\frac{5}{8}$ ounces cost £9 1s. $10\frac{1}{2}$d.?

8. Sold $3018\frac{4}{5}$ gallons of rum, at the rate of $6\frac{2}{3}$ gallons for $63\frac{3}{5}$s.; what is the entire selling price?

9. If 18s. 9d. buy $4\frac{1}{2}$ yards of cloth, how many yards will £79 10s. $10\frac{3}{8}$d. buy?

10. If $1\frac{3}{5}$s. buy $\frac{3}{8}$ lb of tea, how much will £$2\frac{5}{18}$ purchase?

11. What cost $29\frac{3}{5}$ cwt. of tobacco, if $1\frac{7}{8}$ lb. cost $7\frac{3}{16}$s.?

12. If $\frac{5}{18}$ of a ship be bought for £227 12s. 1d., what share of her will £273 2s. 6d. purchase?

13. What cost $5\frac{7}{9}$ cwt. of tea, if $(\frac{3}{4} + \frac{5}{8})$ lb. cost $8\frac{4}{15}$ shillings?

14. If $\frac{7}{16}$ lb. of coffee cost $\frac{13}{14}$ of a shilling, what will $1\frac{1}{24}$ cwt. cost?

15. What is the cost of $8\frac{3}{4}$ pieces of cloth, each containing $(9\frac{5}{8} + 7\frac{5}{6})$ yards, if $4\frac{2}{3}$ yards cost £$2\frac{7}{8}$?

16. Divide the sum of $4\frac{1}{3} + 5\frac{3}{4} + 6\frac{4}{5}$ by the difference between $8\frac{7}{8}$ and $5\frac{3}{4}$, and multiply the quotient by $(16\frac{1}{6} + 11\frac{5}{8})$.

17. Out of a purse containing $93\frac{7}{10}$ guineas, £$19\frac{6}{7}$, $7\frac{7}{18}$ crowns, $3\frac{3}{5}$ shillings, and $5\frac{5}{12}$ pence, there are paid $23\frac{5}{16}$ guineas, £$16\frac{7}{9}$, $17\frac{13}{18}$ crowns, $7\frac{4}{5}$ shillings, and $5\frac{3}{4}$ pence; what sum is left?

18. What is the cost of $18\frac{5}{9}$ casks of butter, each weighing $1\frac{4}{5}$ cwt., at $7\frac{5}{8}$s. for $8\frac{5}{9}$ lbs.?

19. Sold $29\frac{7}{8}$ pieces of cloth, each containing $(3\frac{2}{5} + 4\frac{2}{3})$ yards, at $8\frac{5}{8}$s. for $2\frac{1}{3}$ yds.; required the selling price?

20. The owner of $\frac{4}{17}$ths of a ship sold his share for £$912\frac{4}{33}$; what would $\frac{2\frac{1}{2}}{4\frac{1}{4}}$ of $\frac{2}{5}$ths of the vessel cost at the same rate?

21. Bought $14\frac{3}{8}$ yards of cloth at 2s. $5\frac{3}{4}$d. the yard, and $17\frac{11}{16}$ yards at 3s. $9\frac{1}{2}$d. the yard; what is the average price of 1 yard?

22. What is the cost of $37\frac{5}{18}$ pieces of cloth, each containing $(4\frac{2\frac{1}{2}}{3} + 5\frac{2}{4\frac{1}{2}} + 6\frac{2\frac{1}{2}}{3\frac{3}{4}})$ yards, at the rate of 13s. $2\frac{3}{4}$d. for every 3 yards $3\frac{3}{5}$ quarters?

23. Divide the sum of $4\frac{7}{12} + 5\frac{1}{2} + 6\frac{4}{5}$, by the difference of $9\frac{7}{8}$ and $6\frac{3}{4}$, and multiply the quotient by the sum of $10\frac{1}{6} + 11\frac{3}{8} + 6\frac{1}{4}$.

24. Bought $133\frac{7}{8}$ yards of cloth at 13s. 8d. the yard, and $137\frac{5}{8}$ yds. at 15s. 3d. the yard; what is the average-price of 1 yard?

Find the result of the expression $(2\frac{7}{12} + 7\frac{5}{6} + 5\frac{3}{8} - 1\frac{2}{3}) \times (1\frac{7}{16} + 2\frac{2}{3} + 4\frac{3}{8} - 1\frac{5}{12})$.

What is the result of $(4\frac{3}{8} + 1\frac{7}{16} + 2\frac{2}{3} - 1\frac{5}{12}) \times (1\frac{7}{12} + 7\frac{13}{16} + 3\frac{5}{9} - 2\frac{5}{8})$?

What is the quotient of $199\frac{33}{64} + 145\frac{221}{1152}$ divided by 9 times $\frac{7}{8}$?

Bought $4\frac{2}{3}$ pieces of cloth, each containing $8\frac{1}{3} + 3\frac{3}{4} + 6\frac{1}{9}$ yards, at $19\frac{7}{8}$s. for every $8\frac{2}{3}$ yards; what is the cost?

What is the cost of $3\frac{5}{8} + \frac{2\frac{1}{2}}{3} + \frac{3\frac{1}{2}}{4\frac{1}{2}} + \frac{2\frac{1}{3}}{3\frac{1}{4}} + 6\,\frac{3\frac{1}{2}}{3\frac{2}{3}}$ yards of cloth, if 3 times $\frac{7}{8}$ yards cost $15\frac{2}{3}$ shillings?

How many cwt. of butter can I buy for £$83\frac{5}{8}$ + $73\frac{13}{16}$ guineas, at £3 14s. 9d. the cwt.?

If $529\frac{3}{4}$ yards of cloth are sold for £205 5s. $8\frac{3}{8}$d, how much is it a yard?

Bought $23\frac{3}{8}$ yards of silk at 4s. $7\frac{1}{2}$d., $27\frac{3}{4}$ yards at 4s. $9\frac{3}{4}$d., and $24\frac{2}{3}$ yards at 3s. $10\frac{3}{4}$d. per yard; what is the average price of 1 yard?

What cost $\frac{1}{2}$ of $\frac{3}{4}$ths of a ship, if $\frac{4}{9}$ths of her cost £1875 10s. $3\frac{1}{2}$d.?

Find the cost of $4\frac{2\frac{1}{3}}{3} + 1\frac{1\frac{1}{2}}{2}$ cwt. of pork, if $4\frac{\frac{1}{2}}{3} + 3\frac{1\frac{1}{3}}{1\frac{1}{2}}$ lb. cost $\frac{1\frac{1}{4}}{2\frac{1}{3}} + \frac{2\frac{1}{2}}{3\frac{1}{2}}$ guineas.

If $\frac{11}{12}$ths of a ship cost £68249 18s. $5\frac{3}{4}$d., what must $\frac{3}{4}$ths of her be sold for to gain £116 14s. 6d.?

Sold $7\frac{1}{2}$ barrels of beef, each weighing $1\frac{2}{3} + 1\frac{3}{4} + 2\frac{3}{8}$ cwt., at 2s. $7\frac{1}{2}$d. for every $3\frac{3}{4}$ lb.; what was the amount received?

In the making of bread 1 lb. of dough requires $\frac{2}{3}\frac{2}{5}$ lb. of flour with $\frac{1}{3}\frac{3}{5}$ lb. of water; how much flour, and how much water will it take to make 513 lbs. of bread, knowing that $1\frac{1}{6}$ lb. of dough will be reduced to 1 lb. of bread in baking?

Find the amount of $\left(\frac{1\frac{3}{4} + 2\frac{5}{6}}{5\frac{1}{2} + 4\frac{1}{5}} \times \frac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}} \times 6\frac{2}{5} \div \frac{2\frac{1}{2}}{1\frac{1}{3}}\right)$ yards, and their cost at $15\frac{5}{6}$s. for 3 times $1\frac{7}{8}$ yards.

Of a field, $1\frac{1}{5}$ is meadow, $\frac{3}{8}$ arable, and the remainder (1 acre 3 roods 26 perches), pasture land; find the quantity of meadow and arable land, and the rent of the entire field, at £3 17 $9\frac{1}{2}$ per acre.

Bought $21\frac{1}{2}$ packs of cloth, each containing $21\frac{2}{3}$ pieces of $13\frac{3}{4}$ yards each, at 3 times $4\frac{2}{5}$s. for $3\frac{1}{3}$ yards; what is the entire price?

41. Find the price of $8\frac{2}{3}$ pieces of cloth, each containing $18\frac{7}{8} + 16\frac{5}{8}$ yards, at 13s. $5\frac{1}{2}$d. for $2\frac{1}{2}$ yards.

42. What is the cost of $15\frac{1}{3}$ pieces of plate, each weighing $5\frac{3}{8} + 9\frac{5}{6}$ ounces, when $1\frac{2}{3}$ times $1\frac{3}{4}$ oz. cost $1\frac{1}{8}$ times £1 5s. 6d.?

43. Find the price of 4 times $(15\frac{5}{8} + 13\frac{5}{8} + 17\frac{4}{5})$ yards, when $8\frac{1}{2}$ times $(19\frac{2}{3} - 10\frac{1}{2})$ yards cost £3 16s. $10\frac{1}{2}$d.

44. Bought $21\frac{2}{3}$ packages of cloth, each containing $21\frac{3}{4}$ pieces of $17\frac{1}{2}$ yards each, at 3 times $1\frac{2}{3}$ yards for $2\frac{1}{3}$ times $17\frac{1}{3}$s.; what is the cost?

45. Sold $22\frac{3}{4}$ packs of cloth, each containing $20\frac{3}{4}$ pieces of $18\frac{1}{2}$ yards each, at the rate of 3 times $2\frac{2}{5}$ yards for $2\frac{1}{3}$ times $17\frac{1}{2}$s.; what is the amount?

46. If £339 17s. 11d. gain £35 1s. $5\frac{1}{2}$d., what would £100 gain at the same rate?

47. If $7\frac{1}{16}$ guineas be gained on $5\frac{2}{3}$ times $37\frac{3}{8}$ gallons of rum, what is the gain on 1 gallon?

48. Bought $5\frac{5}{6}$ times $(8\frac{2}{3} + 10\frac{3}{4} - 3\frac{1}{2})$ lb. of coffee, at the rate of $2\frac{2}{3}$ times $(5\frac{3}{5} + 3\frac{1}{2})$s. for $(19\frac{2}{3} - 10\frac{2}{5})$ lb.; what is the cost?

49. Sold $16\frac{3}{5}$ packs of cloth, each containing $23\frac{2}{3}$ pieces of $49\frac{1}{2}$ yards each, at £2 3s. 9d. for every $5\frac{2}{3}$ yards; what is the entire selling price?

50. How many pieces of cloth, of $37\frac{3}{8}$ yards each, can be purchased for £83 7s. $10\frac{5}{16}$d., at 7s. $10\frac{1}{2}$d. per yard?

51. How many bushels of wheat can be purchased for £149 3s. $7\frac{38}{229}$d, if 3 times $(4\frac{2}{3} + 3\frac{3}{8} + 1\frac{1}{2})$ bushels cost £$3\frac{3}{16}$ + $5\frac{5}{8}$ guineas?

52. Find the value of $(\frac{1}{3} + \frac{1}{2} - \frac{191}{231}) + (\frac{2\frac{1}{2} - \frac{1}{3}}{2\frac{1}{3} - 1\frac{1}{4}}) + (\frac{38}{39} \times 2\frac{1}{7} \times \frac{2\frac{1}{4}}{4\frac{3}{4}} \div \frac{5}{14})$ when divided by $3\frac{3}{4}$.

53. If 3 times $(6\frac{3}{4} + 1\frac{2}{5})$ ounces of silver cost $3\frac{2}{7}$ times $1\frac{5}{8}$ guineas, what is the cost of $17\frac{5}{8}$ times $(\frac{4\frac{1}{2}}{3\frac{1}{3}} + \frac{4\frac{1}{2} + 2\frac{1}{3}}{3\frac{1}{2} - 1\frac{2}{3}} + 2\frac{1}{2} \times 1\frac{3}{5})$ ounces?

54. What is the value of $(\frac{2}{3} + \frac{3}{4}) \times \frac{8}{9} \div \frac{3}{10}$, and this quotient $\times \frac{4}{5\frac{2}{3}} \times \frac{3}{20} \times \frac{2\frac{1}{4}}{1}$?

55. What cost $(\frac{7}{8} + \frac{3}{4} + \frac{4}{7})$ lbs. of coffee, if $(\frac{5}{4} + \frac{5}{3} - \frac{1\frac{1}{3}}{8})$ lbs. cost $\frac{5}{8} + \frac{7}{9} + \frac{2\frac{1}{3}}{4}$s.?

56. Bought $\left\{\begin{array}{lllll} \text{7869} & \text{bushels of wheat} & & \text{at 8s. } 10\frac{1}{2}\text{d.} & \text{per bushel,} \\ \text{4863} & \text{do.} & \text{do.} & \text{at 8s. } 7\frac{1}{2}\text{d.} & \text{do.,} \end{array}\right\}$ find the average price of a bushel.

. Sold $21\frac{2}{3}$ packs of cloth, each containing $18\frac{1}{2}$ pieces of $21\frac{3}{4}$ yards at 3 times $5\frac{2}{3}$s. for $1\frac{2}{3}$ times 4 yards; what sum was realised?

. Find the value of $(2\frac{2}{3} + 3\frac{3}{4}) \times 3\frac{1}{1\frac{1}{2}} + (18\frac{5}{12} - 4\frac{3}{5}) \div 4\frac{1\frac{1}{3}}{2}$ multiplied by $\frac{3}{4}$ of $2\frac{2}{5}$.

. To the 4th proportional of $\frac{2}{3}$, $3\frac{3}{8}$, and 3 times $3\frac{2}{2\frac{1}{3}}$, add the 4th proportional of $1\frac{2}{3}$, $3\frac{5}{8}$, and 3 times $3\frac{1}{1\frac{1}{2}}$.

. What is the cost of $7\frac{3}{4}$ flitches of bacon, each weighing $18\frac{3}{8} + 15\frac{2}{3}$ lb., at £21 17s. 6d. for $4\frac{1}{2}$ cwt.?

. Bought $45\frac{3}{16}$ gallons of rum at 16s. 9d., $75\frac{3}{16}$ galls. at 18s. 5d., and $67\frac{5}{81}$ gallons at 17s. 10d. per gallon; find the entire cost and price of 1 gallon.

. Bought $21\frac{2}{3}$ pieces of cloth, each containing 27 yards, at 8s. $10\frac{1}{2}$d. per yard, and $27\frac{3}{4}$ pieces each 28 yards, at 8s. $7\frac{1}{2}$d. per yard; what is the entire cost and average price of 1 yard?

Find the entire cost and average price of 1 yard, when 3 pieces, each $47\frac{1}{2}$ yards, cost 5s. $1\frac{3}{4}$d. per yard; 6 pieces each $46\frac{1}{2}$ yards, cost 5s. 3d. per yard; and 18 pieces, each $48\frac{1}{4}$ yards, cost 5s. $1\frac{1}{2}$d. per yard.

What cost $4\frac{2}{3}$ nuggets of gold, each weighing $(\frac{1}{3} - \frac{4}{21} + \frac{1}{2} - \frac{7}{11})$ lb., if $3\frac{1}{5}$ times $\frac{4}{5}$ lb. be bought for $53\frac{1}{2}$ guineas?

What cost 4 times $(15\frac{5}{8} + 13\frac{5}{6} + 17\frac{4}{5})$ yards, when $3\frac{1}{2}$ times $(19\frac{2}{3} - 10\frac{1}{2})$ yards cost £3 16s. $10\frac{1}{2}$d.?

Find the cost of $17\frac{5}{8}$ chests of tea, each weighing $2\frac{2}{3}$ cwt., at 3 times $1\frac{4}{5}$s. for $(\frac{2}{3} + \frac{3}{4})$ lb.

DECIMAL FRACTIONS.

4. The division of unity by any number produces, as before ted (Art. 64), a *fraction*; but when its division is by 10 or any ver of 10, such a fraction is termed *a decimal*.

A decimal fraction is therefore a fraction whose denominator is or some power of 10, as 100, 1000, &c.

5. (1). Decimal fractions are expressed by writing the numerator h a point (·) before it; thus, $\frac{7}{10}$, $\frac{45}{100}$, $\frac{875}{1000}$, are decimal fractions, l are written, ·7, ·45, ·875.*

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* The system of decimals is only an extension of the ordinary notation of le numbers, where the local values of figures increase in a tenfold ratio for a advance from right to left, or decrease in a tenfold ratio by each removal

(2). But when the numerator does not consist of as many figures as there are ciphers in the denominator, the deficiency must be supplied by prefixing ciphers to it; thus, $\frac{7}{100}$, $\frac{45}{1000}$, $\frac{875}{100000}$, are respectively written ·07, ·045, ·00875.

86. The denominator of a decimal fraction, although not expressed, can always be ascertained from the numerator. It consists of 1, with as many ciphers annexed, as the numerator has places; thus, ·07, ·045, ·00375, signify respectively, $\frac{7}{100}$, $\frac{45}{1000}$, $\frac{375}{100000}$.

EXERCISES 23.

1. Express 3·65 as a fraction.
2. " ·0375 " "
3. " 56.78125 " "
4. " 493·0035 " "
5. " $1\frac{35}{100}$ as a decimal.
6. " $11\frac{5}{1000}$ " "
7. " $25\frac{375}{10000}$ " "
8. " $\frac{7456985}{100000}$ " "
9. " $\frac{28170355}{10000}$ " "
10. " $\frac{4}{10}+\frac{7}{1000}+\frac{8}{100000}$ as a decimal.

---

from left to right. Continue this tenfold scale to the right of the unit's place, and it is obvious that the local value of the first figure on the right of units will be *tenths*, the second *hundredths*, the third *thousandths*, &c.;—diminishing in a tenfold ratio from left to right, and thus rendering the value of any figure of a decimal dependant upon its distance from the unit's place. For 365·875 means 365 as a whole number, with $\frac{8}{10}+\frac{7}{100}+\frac{5}{1000}=\frac{875}{1000}=$ ·875 as a fraction.

The decomposition and tenfold ratio is shown thus:—

$\frac{2222}{1}=2222$ written 2222.

$\frac{2222}{10}=222+\frac{2}{10}$ " 222·2.

$\frac{2222}{100}=22\frac{2}{10}+\frac{2}{100}$ " 22·22.

$\frac{2222}{1000}=2+\frac{2}{10}+\frac{2}{100}+\frac{2}{1000}$ " 2·222.

$\frac{2222}{10000}=\frac{2}{10}+\frac{2}{100}+\frac{2}{1000}+\frac{2}{10000}$ " ·2222.

REDUCTION OF VULGAR FRACTIONS TO DECIMALS.

87. *To transform a vulgar fraction into an equivalent decimal.*

Rule.—Annex ciphers to the numerator, then divide by the denominator. If there be not so many figures in the quotient as there were ciphers annexed, supply the deficiency by prefixing ciphers to it, which will be the decimal required.*

Transform $\frac{3}{8}$ to a decimal.†
8) 3000

·375 *Ans.*

Transform $\frac{1}{160}$ to a decimal.
160) 100000

·00625 *Ans.*

Transform $\frac{3}{512}$ to a decimal.

512) 3000 (·00585375 *Ans.*
2560

4400
4096

3040
2560

4800
4608

1920
1536

3840
3584

2560
2560

---

* The process enjoined by this rule merely implies that principle so often inculcated in the course of this work, viz., that if both the terms of a fraction be multiplied by any number, and divided by any other number, its value remains unaltered; thus $\frac{3}{8} = \frac{3000}{8000} = \frac{375}{1000}$ where the process consists in multiplying the terms of the fraction $\frac{3}{8}$ by 1000, and then dividing them by 8.

† To transform $\frac{3}{8}$ into a decimal fraction, merely requires its terms to be multiplied by such a number as will make the denominator of the required fraction 10, 100, 1000, &c.; this 125 will do in the foregoing. Therefore, if $\frac{3}{8}$ be multiplied above and below by 125, there results the decimal fraction $\frac{375}{1000}$, written ·375.

Transform $\frac{2}{3}$ to a decimal.

```
3 ) 2000
    ————
    ·666 Ans.
```

Transform $\frac{5}{6}$ to a decimal.

```
6 ) 5000
    ————
    ·833
```

Transform $\frac{3}{11}$ to a decimal.

```
11 ) 30000
     —————
     ·2727
```

Transform $\frac{15}{22}$ to a decimal.

```
22 ) 150 ( ·68181 Ans.
     132
     ———
      180
      176
      ———
       40
       22
       ———
       180
       176
       ———
         40
         22
         ——
         18
```

88. Decimals are of two sorts, finite or infinite.

(1). A *finite* decimal is a decimal in which the division terminates, as $\frac{3}{8}$ = ·375.

(2). An *infinite* decimal is a decimal in which the division does not terminate, and is called :—

* A *pure repeater*, when the same figure is repeated from the decimal point ; as $\frac{2}{3}$ = ·666 &c. = $\cdot\dot{6}$.

A *mixed repeater*, when the repeater is removed one or more figures from the decimal point, as $\frac{5}{6}$ = ·833 &c. = $\cdot 8\dot{3}$.

A *circulating decimal*, when two or more figures repeat from the decimal point, as $\frac{3}{11}$ = ·2727, &c. = $\cdot\dot{2}\dot{7}$.

A *mixed circulating decimal*, when the circulating decimal is removed one or more figures from the decimal point, as $\frac{15}{22}$ = ·68181 &c. = $\cdot 6\dot{8}\dot{1}$.

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* A *pure repeater* is indicated by a dot over the figure repeated, and a *circulate* by dots placed over the first and last of the recurring figures.

EXERCISES 24.

1. * Transform $\frac{1}{2}$ into a decimal.
2. " $\frac{3}{4}$ " "
3. " $\frac{3}{40}$ " "
4. " $\frac{5}{8}$ " "
5. " $\frac{1}{16}$ " "
6. " $\frac{1}{100}$ " "
7. " $\frac{7}{32}$ " "
8. " $\frac{7}{320}$ " "
9. " $\frac{12}{25}$ " "
10. " $\frac{12}{2500}$ " "
11. " $\frac{3}{50}$ " "
12. " $\frac{3}{500}$ " "
13. " $\frac{3}{256}$ " "
14. " $\frac{3403}{80000}$ " "
15. " $\frac{7}{256}$ " "
16. " $\frac{12}{625}$ " "
17. " $\frac{13}{128}$ " "
18. " $\frac{18}{25}$ " "
19. " $\frac{7}{640}$ " "
20. " $\frac{241}{320}$ " "
21. " $\frac{4}{625}$ " "
22. " $\frac{6428}{1280}$ " "
23. " $\frac{79}{160}$ " "
24. " $\frac{331}{640}$ " "
25. " $\frac{241}{15625}$ " "
26. " $\frac{63}{64}$ " "
27. " $\frac{113}{128}$ " "
28. " $\frac{6849}{10240}$ " "

Infinite Decimals.

29. Transform $\frac{4}{9}$ into an infinite decimal
30. " $\frac{1}{6}$ " " "
31. " $\frac{6}{11}$ " " "
32. " $\frac{4}{27}$ " " "
33. " $\frac{7}{36}$ " " "
34. " $\frac{107}{111}$ " " "
35. " $\frac{3}{7}$ " " "
36. " $\frac{17}{22}$ " " "

---

* The answers to these exercises, are the converse exercises next following, nd *vice versa.*

37. Transform $\frac{26}{365}$ into an infinite decimal.
38. ” $\frac{51231}{73000}$ ” ”
39. ” $\frac{290}{365}$ ” ”
40. ” $\frac{64537}{77440}$ ” ”
41. ” $\frac{893}{968}$ ” ”
42. ” $\frac{137}{174}$ ” ”
43. ” $\frac{3}{238}$ ” ”
44. ” $\frac{13}{45}$ ” ”
45. ” $\frac{34080}{999}$ ” ”
46. ” $\frac{1100}{21}$ ” ”
47. ” $\frac{360}{899}$ ” ”

89. *To convert terminating decimals into vulgar fractions.*

Rule.—Divide the numerator and denominator of the decimal fraction by a common divisor, and the result will be a vulgar fraction.—If the greatest common divisor be used, the vulgar fraction will be in its lowest terms, as in (Art. 72).

EXERCISES 25.

1. Convert ·5 into a vulgar fraction.
2. ” ·75 ” ”
3. ” ·075 ” ”
4. ” ·625 ” ”
5. ” ·0625 ” ”
6. ” ·00625 ” ”
7. ” ·21875 ” ”
8. ” ·021875 ” ”
9. ” ·48 ” ”
10. ” ·0048 ” ”
11. ” ·06 ” ”
12. ” ·006 ” ”
13. ” ·01171875 ” ”
14. ” ·0437875 ” ”
15. ” ·02734375 ” ”
16. ” ·0192 ” ”
17. ” ·1015625 ” ”
18. ” ·72 ” ”
19. ” ·0109375 ” ”
20. ” ·753125 ” ”
21. ” ·0064 ” ”
22. ” ·502265625 ” ”
23. ” ·49375 ” ”
24. ” ·5484375 ” ”

25. Convert ·015625 into a vulgar fraction.
26. " ·984375 " "
27. " ·9296875 " "
28. " ·66884765625 " "

90. A vulgar fraction is converted into an infinite series after the lowing manner:—Thus $\frac{2}{3} = \frac{6}{9} = \frac{6}{10-1}$

$$\begin{array}{llll}
\text{Dividing } 10-1\Big) & 6 & & \Big(\frac{6}{10}+\frac{6}{100}+\frac{6}{1000}+\frac{6}{10000} \\
 & 6-\frac{6}{10} & & \\
\hline
\Big) & \frac{6}{10} & \Big(\frac{6}{100} & \\
 & \frac{6}{10}-\frac{6}{100} & & \\
\hline
\Big) & \frac{6}{100} & \Big(\frac{6}{1000} & \\
 & \frac{6}{100}-\frac{6}{1000} & & \\
\hline
\Big) & \frac{6}{1000} & \Big(\frac{6}{10000} & \\
 & \frac{6}{1000}-\frac{6}{10000} & & \\
\hline
\end{array}$$

Hence $\frac{2}{3}$ or $\frac{6}{9} = \frac{6}{10}+\frac{6}{100}+\frac{6}{1000}+\frac{6}{10000}$, &c. $= \cdot\dot{6}$.

EXAMPLE (2).

$$\frac{3}{11}=\frac{27}{99}=\frac{27}{100-1}$$

$$\begin{array}{llll}
\text{and } 100-1\Big) & 27 & & \Big(\frac{27}{100}+\frac{27}{10000}+\frac{27}{1000000} \\
 & 27-\frac{27}{100} & & \\
\hline
\Big) & \frac{27}{100} & \Big(& \\
 & \frac{27}{100}-\frac{27}{10000} & & \\
\hline
\Big) & \frac{27}{10000} & \Big(& \\
 & \frac{27}{10000}-\frac{27}{1000000} & & \\
\hline
\end{array}$$

Hence $\frac{3}{11}$ or $\frac{27}{99} = \frac{27}{100}+\frac{27}{10000}+\frac{27}{1000000}$, &c. $= \cdot\dot{2}\dot{7}$.

EXAMPLE (3).

In a similar way

$$\frac{5}{27} \text{ or } \frac{185}{999} = \frac{185}{1000-1} = \frac{185}{1000} + \frac{185}{1000000} + \frac{158}{1000000000} \text{ \&c.} = \cdot\dot{1}8\dot{5}.$$

EXAMPLE (4).

Also $\frac{5}{6} = \frac{75}{90}$, which multiplied by 10 gives $\frac{750}{90} = 8\frac{3}{9} = 8\frac{3}{10-1}$ $= 8 + \frac{3}{10} + \frac{3}{100}$, &c. $= 8\cdot\dot{3}$, and this divided back by 10 will give $\cdot 8\dot{3}$.

Wherefore

$\frac{2}{3}$ or $\frac{6}{9} = \frac{6}{10} + \frac{6}{100} + \frac{6}{1000} +$ &c. continued to infinity.
$\frac{3}{11}$ or $\frac{27}{99} = \frac{27}{100} + \frac{27}{10000} + \frac{27}{1000000} +$ &c., continued to infinity.
$\frac{5}{27}$ or $\frac{185}{999} = \frac{185}{1000} + \frac{185}{1000000} +$ &c., continued to infinity.
and $\frac{5}{6} \times 10 = 8 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$, &c., continued to infinity.

And the converse

$\frac{6}{10} + \frac{6}{100} + \frac{6}{1000} + \frac{6}{10000} +$ &c., *or* $\cdot\dot{6} = \frac{6}{9}$
$\frac{27}{100} + \frac{27}{10000} + \frac{27}{1000000} +$ &c., *or* $\cdot\dot{2}\dot{7} = \frac{27}{99}$
$\frac{185}{1000} + \frac{185}{1000000} +$ &c., *or* $\cdot\dot{1}8\dot{5} = \frac{185}{999}$
$8 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} +$ &c., *or* $8\cdot\dot{3} = 8\frac{3}{9}$ which if divided by 10 gives $\frac{8}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000}$, &c. *or* $\cdot 8\dot{3} = \frac{75}{90} = \frac{5}{6}$.

But $\frac{6}{10} + \frac{6}{100} + \frac{6}{1000} +$ &c., is a geometrical progression* continued to infinity, whose ratio is $\frac{1}{10}$.

Also $\frac{27}{100} + \frac{27}{10000} +$ &c., is a geometrical progression continued to infinity, whose ratio is $\frac{1}{100}$.

In a similar way $\frac{185}{1000} + \frac{185}{1000000} +$ &c., is a geometrical progression continued to infinity, whose ratio is $\frac{1}{1000}$.

In the expression $8 + \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} +$ &c. the terms $\frac{3}{10} + \frac{3}{100} +$ &c., form a progression whose ratio is $\frac{1}{10}$.

But the sum of a geometrical progression continued to infinity $= \frac{\text{1st term}}{1 - \text{ratio}}$.

$$\text{Wherefore } \frac{6}{10} + \frac{6}{100} + \text{\&c.} = \frac{\frac{6}{10}}{1-\frac{1}{10}} = \frac{6}{9} \text{ or } \frac{2}{3}$$

$$\frac{27}{100} + \frac{27}{10000} + \text{\&c.} = \frac{\frac{27}{100}}{1-\frac{1}{100}} = \frac{27}{99} \text{ or } \frac{3}{10}$$

$$\frac{185}{1000} + \frac{158}{1000000} + \text{\&c.} = \frac{\frac{185}{1000}}{1-\frac{1}{1000}} = \frac{185}{999} \text{ or } \frac{5}{27}$$

---

* A geometrical progression is a series of numbers increasing or decreasing by a common ratio.

$8 + \frac{3}{10} + \frac{3}{100} + \&c. = 8 + \frac{\frac{3}{10}}{1 - \frac{1}{10}} = 8 + \frac{3}{9}$, and $8\frac{3}{9}$ divided by 10 will give $\frac{75}{90}$ or $\frac{5}{6}$.

91. Whence by induction,—*To convert a repeating or circulating decimal to its equivalent vulgar fraction.*

Rule.—Make the repeater, or circulate, the numerator of the fraction, and the denominator will be as many nines, as there are figures in this numerator. This fraction reduced to its lowest terms will be the fraction required.

EXAMPLES.

Reduce the following, $\cdot\dot{5}$, $\cdot\dot{4}\dot{5}$, $\cdot\dot{4}7\dot{2}$, $\cdot\dot{7}1428\dot{5}$ to vulgar fractions.

(1.) $\cdot\dot{5} = \frac{5}{9}$

(2.) $\cdot\dot{4}\dot{5} = \frac{45}{99} = \frac{5}{11}$

(3.) $\cdot\dot{4}7\dot{2} = \frac{472}{999}$

(4.) $\cdot\dot{7}1428\dot{5} = \frac{714285}{999999} = \frac{142857 \times 5}{142857 \times 7} = \frac{5}{7}$

92. *To convert a mixed repeater, or a mixed circulate, to its equivalent vulgar fraction.*

Rule.—Multiply the decimal by such a number (10, 100, &c.) as will render the non-circulating part a whole number; then convert the circulating part into a fraction, to which unite the whole number already formed. This mixed fraction, if divided by the first multiplier, gives the equivalent fraction; which, reduced to its lowest terms, will be the fraction required.

EXAMPLES.

Reduce the following, $\cdot 8\dot{3}$, $\cdot 6\dot{8}\dot{1}$, $\cdot 70\dot{1}7945205\dot{4}$, to vulgar fractions.

(1). $\cdot 8\dot{3}$ if multiplied by 10 gives $8\cdot 3 = 8\frac{3}{9}$, which divided by 10 gives $\frac{8\frac{3}{9}}{10} = \frac{5}{6}$.

(2). $\cdot 6\dot{8}\dot{1}$ if multiplied by 10, gives $6\cdot 81 = 6\frac{81}{99}$, and this divided back by $10 = \frac{6\frac{81}{99}}{10} = \frac{15}{22}$.

(3). $\cdot 70\dot{1}7945205\dot{4}$ if multiplied by 1000 gives $701\cdot\dot{7}945205\dot{4} = 701\frac{79452054}{99999999}$, and this divided back by 1000 gives $\frac{701\frac{79452054}{99999999}}{1000} = \frac{70179451353}{99999999000} = \frac{1369863 \times 51231}{1369863 \times 73000} = \frac{51231}{73000}$.

EXERCISES 25 (*continued*).

Convert the followng infinite decimals to vulgar fractions.

29. $\cdot\dot{4}$.
30. $\cdot 1\dot{6}$.
31. $\cdot\dot{5}\dot{4}$.
32. $\cdot\dot{1}4\dot{8}$.
33. $\cdot 19\dot{4}$.
34. $\cdot\dot{9}6\dot{3}$.
35. $\cdot\dot{2}8571\dot{4}$.
36. $\cdot 7\dot{7}\dot{3}$.
37. $\cdot\dot{0}684931\dot{5}$.
38. $\cdot 701\dot{7}945205\dot{4}$.
39. $\cdot\dot{7}945205\dot{4}$.
40. $\cdot 7042484\dot{5}04132231404958677685\dot{9}$. Remainder, 39040.
41. $\cdot 922\dot{5}20661157024793388429\dot{7}$.
42. $\cdot 78\dot{7}35632183908045977011494252\dot{8}$.
43. $\cdot 0\dot{1}2605042016806722689075630252100840336134453781\dot{5}$. Remainder, 30.
44. $\cdot 2\dot{8}$.
45. $3\dot{4}\cdot 1\dot{2}$, $3\dot{4}\cdot 1\dot{2} = 3\cdot\dot{4}1\dot{2} \times 10 = 3\frac{412}{999} \times 10 = \frac{34090}{999}$.
46. $\dot{5}2\cdot 380\dot{9}$, $\dot{5}2\cdot 380\dot{9} = \cdot\dot{5}2380\dot{9} \times 100 = \frac{523809}{999999} \times 100 = \frac{1100}{21}$
47. $\dot{2}\cdot\dot{6}$.

REDUCTION.

93. *To transform money, weights, and measures of a lower name to equivalent decimals of a higher, or* reduction ascending.

Rule.—Reduce the given quantity to a vulgar fraction (Art. 77), and that fraction to a decimal.

EXAMPLE.

Transform 13s. 6¾d. into the decimal of a £.*

---

* Reduction in this case, as already stated in reference to vulgar fractions, is conducted on the same principle as reduction in whole numbers; for if farthings are to be converted into pounds, the farthings must be divided by 4, that quotient by 12, and the next by 20, or the whole by 960 at once.

Now 13s. $6\frac{3}{4}$d. = 651 f., and 960f. = £1.

Then 651 ÷ 960 *or* £$\frac{651}{960}$ = £·678125. *Ans.*

or thus,

4) 651 farthings.

12) 162·75 pence.

20) 13·5625 shillings.

£·678125 *Ans.*

or thus,

12) 6·75d.

·5625s. = $6\frac{3}{4}$d.

23) 13·5625

£·678125.

Transform 1 qr. 21 lb. into the decimal of a cwt.

Now 1 qr. 21 lb. = 49 lb., and 112 lb. = 1 cwt.

Then 49 ÷ 112 *or* $\frac{49}{112}$ cwt. = ·4375 cwt. *Ans.*

or thus,

28) 49

4) 1·75

·4375 cwt.

or thus,

28) 21 lb.

·75 quarter.

4) 1·75 quarter.

·4375 cwt.

EXERCISES 26.

A.

1. Reduce 6s. 3d. to the decimal of a £.
2. „ 3s. 9d. to the decimal of a £.
3. „ 14 lb. to the decimal of a cwt.
4. „ 11 cwt. to the decimal of a ton.
5. „ 3 nails to the decimal of a yard.
6. „ 1 rood 25 perches to the decimal of an acre.
7. „ 12s. $6\frac{3}{4}$d. to the decimal of a £.
8. „ 15s. $9\frac{3}{4}$d. to the decimal of a £.
9. „ $4\frac{1}{2}$d. to the decimal of a £.
10. „ $3\frac{1}{2}$d. to the decimal of a £.
11. „ 3s. 8d. to the decimal of a £.
12. „ 2 qrs. 21 lb. to the decimal of a cwt.
13. „ $10\frac{1}{2}$ lb. to the decimal of a cwt.
14. „ 6 fur. 27 p. 5 yd. 2 ft. 9 in. to the decimal of a mile.

94. *To express the value of a decimal quantity in numbers of an inferior denomination, or* reduction descending.

Rule.—Multiply the given decimal by the number which will reduce it to the next lower denomination, and point off from the right of the product as many decimals as there are figures in the given decimal. Again multiply this decimal so as to reduce it to the next lower denomination, and point off as many decimals as were then multiplied; proceed thus to the lowest denomination. The several figures to the left of the points will be the required value of the given decimal.

EXAMPLE (1).

What is the value of £·675?

£·675
20
———
13·50
12
———
Ans. 13s. 6d. 6·0

EXAMPLE (2).

What is the value of ·34625 acres?

·34625 acres.
4
———
1·38500
40
———
15·400
Ans. 1 rood 15·4 perches.

EXERCISES.

B.

1. Find the value of £·7.
2. ,, ,, ·7 crown.
3. ,, ,, £·26875.
4. ,, ,, £·375.
5. ,, ,, ·375 of 5s. 6d.
6. ,, ,, ·475 ton.
7. ,, ,, £·93125.
8. ,, ,, £·075.
9. ,, ,, ·325 mile.
10. ,, ,, ·28 degree.
11. ,, ,, ·875 gallon.
12. ,, ,, ·5625 day.
13. ,, ,, ·625 lb. troy.
14. ,, ,, ·25 of ·25 yard.
15. ,, ,, ·075 acre.
16. ,, ,, ·725 crown.

17. Find the value of ·075 cubic foot.
18. „ „ ·25 of 365¼ days.
19. „ „ * $\cdot\dot{6}\dot{3}$ mile = $\frac{7}{11}$ mile.
20. „ „ $2\cdot 5\dot{7}$ acres.
21. „ „ £$\cdot 58\dot{8}$.
22. „ „ $\cdot\dot{5}3\dot{7}$ mile.
23. „ „ $\cdot 541\dot{6}$ hour.

ADDITION† OF DECIMALS.

Rule.—Place the given numbers so that the decimal points shall be directly under one another; then add as in whole numbers, and from the sum point off from the right hand as many figures for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

Note.—Or in the sum, place the point directly under the other points.

EXAMPLES.

(1.) Add 18·3 + 45·935 + ·0185 + 10·00456.

Proof by vulgar fractions.

$$\begin{aligned}
18\cdot 3 &= 18\tfrac{3}{10} &&= 18\tfrac{300000}{1000000}\\
45\cdot 935 &= 45\tfrac{935}{1000} &&= 45\tfrac{935000}{1000000}\\
\cdot 0185 &= \tfrac{185}{10000} &&= \tfrac{18500}{1000000}\\
10\cdot 00456 &= 10\tfrac{456}{1000000} &&= 10\tfrac{456}{1000000}\\
\hline
\textit{Ans. } 74\cdot 25806 &= &&\ \ 74\tfrac{258060}{1000000} = 74\cdot 25806
\end{aligned}$$

(2.)

89·507
478·63
5444·798
7·00125
6·7509

Ans. 6026·68715

(3.)

£35	3	6·6025359375
33	10	0·57384375
31	18	1·689375
30	7	9·0375
28	18	9·75
27	11	3
26	5	0
£213	14	6·6532546875 *Ans.*

---

* The value of an infinite decimal is best found by expressing it as a vulgar fraction, and then reducing it.

† Decimals are added, subtracted, multiplied, and divided, in every respect as in whole numbers; the only difficulty consists in fixing the decimal point.

EXERCISES 27.

1. Add 0·4 + ·614 + 4·21 + 78·6 + ·00047.
2. „ 4·0625 + 45·625 + 4·5 + 1·1875 + 6·4875 + 5·75.
3. „ 6·6025359375 + ·57384375 + 1·689375 + 9·0375 + 9·75 + 8.
4. „ 30·3765 + 5·4285714 + 23·705 + 20·60714285 + 14·8125 + 7·235.
5. „ 7·5489 + 7·525 + 89·502.
6. „ ·654 + 73·6 + 4·21 + ·00047.
7. „ £23 7s. 11·321088d. + £22 9s. 11·3472d. + £21 12s. 7·68d.
8. „ 97·375 + 1·611875 + ·4296875 quarters.
9. „ 8·175 + 2·1875 + 1·632 cwt.
10. „ 14·378 + 10·6858 + ·0632 cwt.
11. „ 1·785 + 8·87 + 2·8958 + 0·7852 yards.
12. „ 4·625 + 8·4575 + 2·875 + 1·075 lbs.
13. „ £195 18s. 8·376d. + £8 12s. 8·616d. + £29 0s. 6·952d. + £94 11s. 7·25d.
14. „ 68 cwt. 1 qr. 2·24 lbs. + 3 cwt. 2 16·8 23 cwt. + 3 10·5 + 14 cwt. 3 18·875.
15. „ £6 11s. 0·51d. + £66 18s. 9·84d. + £71 14s. 1·815d.
16. „ 8·745 cwt. + 4 cwt. 2·65 qrs. + 3 cwt. 3 qrs. 18·48 lb. + 2 cwt. 1 qr. 14·672 lb.
17. „ 8s. 6d. + 13s. 9d. + ·875s. + 10s. 6d. + ·564d. and reduce the sum to the decimal of a pound.
18. „ ·203125 quarters + ·7635 bushels of wheat.
19. „ 97·375 qrs. + 12·895 bushels + 73·75 pecks of wheat.
20. „ 3·35 cwt. + 3 cwt. 2·75 qrs. + 8 cwt. 3 qrs. 17 lb.
21. „ 37 cwt. 3 qrs. 14 lb. + 7·2735 cwt. + $6\frac{13}{16}$ cwt. + 47·32 cwt. + $32\frac{5}{8}$ cwt. + $27\frac{19}{32}$ cwt.
22. „ $68\frac{7}{8}$ + 13·375 + $68\frac{11}{16}$ + 487·4325.
23. „ ·3211 guinea + $\frac{3}{8}$ crown + ·6925 shilling.
24. Find the amount of ·375 guinea + $\frac{3}{16}$ crown + (·3 of 7s. 6d.) — ($\frac{3}{8}$ of 2d).

SUBTRACTION OF DECIMALS.

96.

Rule.—Write the less number under the greater, so that the decimal points shall be directly under each other; then subtract as in whole numbers, and in the remainder place the decimal point, as directed in addition of decimals.

EXAMPLE.

From 17·38 take 45·756.

By vulgar fractions.

$$\begin{array}{rcrcr} 87{\cdot}38^{*} & = & 87\frac{38}{100} & = & 87\frac{380}{1000} \\ 45{\cdot}756 & = & 45\frac{756}{1000} & = & 45\frac{756}{1000} \\ \hline 41{\cdot}624 & & & & 41\frac{624}{1000} = 41{\cdot}624. \end{array}$$

EXERCISES 28.

1. From 9·8355 take 1·885.
2. „ 14·215 take 025.
3. „ 15·3875 take 2·6475.
4. „ 6·9945 cwt. take 2·12625 cwt.
5. „ 8·375 + 4·4845 + 17·05 take 8·8395.
6. „ £64 2s. 6·96871d. take £60 17s. 4·0985d.
7. „ £58 8 9·75 take £51 19 9·6225.
8. „ 8·05 + 9·885 + 5·4375 take 8·875 + 2·1685.
9. „ 18·375 + 16·4875 + 82·825 take 13·56735.
10. „ 17·457 + 18·6 + 19·4375 take 6·86735.
11. „ 8·375 + 6·4875 + 2·005 take 8·5 + 2·05 + 1·005.
12. „ £64 2s. 6·96371d. take £60 17s. 4·0935d.

MULTIPLICATION OF DECIMALS.

97.

Rule.—Multiply as in whole numbers, and from the right of the product point off as many decimals as there are decimal places in the factors; if there are fewer figures in the product than there are decimals in the factors, supply the defect by prefixing ciphers.

EXAMPLES.

(1.) Multiply 7·43 by 2·5. *Proof by vulgar fractions.*

$$\begin{array}{rcrcr} 7{\cdot}43 & = & 7\frac{43}{100} & = & \frac{743}{100} \\ 2{\cdot}5 & = & 2\frac{5}{10} & = & \frac{25}{10} \\ \hline \end{array}$$

And $\frac{743}{100} \times \frac{25}{10} = \frac{18575}{1000} = 18{\cdot}575.$

$$\begin{array}{r} 3715 \\ 1486 \\ \hline 18{\cdot}575 \end{array}$$ *Ans.*

---

* In subtracting 6, cipher or 0 is supposed to be above it, as ·380. For affixing a cipher or ciphers will not alter the value of the decimal, as $\frac{38}{100} = \frac{380}{1000} = \frac{3800}{10000}$, that is ·38 = ·380 = ·3800.

(2.) What is the product of ·94 × ·5 × ·12?

$$\begin{array}{r} \cdot 94 \\ \cdot 5 \\ \hline \cdot 470 \\ \cdot 12 \\ \hline \cdot 05640 \end{array}$$ *Ans.*

Note.—To secure the number of decimals required by the rul
(0) is here prefixed.

Proof by vulgar fractions.

$$\tfrac{94}{100} \times \tfrac{5}{10} \times \tfrac{12}{100} = \tfrac{5640}{100000} = \cdot 05640.$$

EXERCISES 29.

1. Find the products of 118·25 × 9·125.
2. „ „ ·040625 × 24.
3. „ „ 7·9375 × 24.
4. „ „ 1·08125 × 24.
5. „ „ 163 4375 × 223·5.
6. „ „ ·1035 × ·198.
7. „ „ 89.964285 × 112.
8. „ „ 17·625 × ·0023.
9. „ „ ·123 × ·0203 × ·02.
10. „ „ 84·125 × 15·5 × ·5.
11. „ „ ·2 × ·2 × ·2 × ·2 × ·2
12. „ „ ·05 × ·05 × 05 × ·05.
13. „ „ ·003 × 1728.
14. „ „ $\cdot 12^2$.
15. „ „ $6 \cdot 879^2$.
16. „ „ $\cdot 12^3$.
17. „ „ 27·045 × 18·325 × 3·05·
18. „ „ $1 \cdot 04^4$.
19. „ „ $\cdot 13^3$.
20. „ „ 21·5 × 21·35 × 21·6.
21. „ „ 18 3225 × 2 8375.
22. „ „ 25·0765 × 43·625.
23. „ „ 122523·779 × 3·75.
24. „ „ 1845·4 × 14·19.
25. „ „ 16·325 × 37·65 × 28.
26. „ „ 4·38 × 32·75 × 112.
27. Bought 13·625 cwt. of coffee at £·065625 the lb.; find the co
28. Multiply (19 cwt. 2 qr. 22·792 lb.) by 37·675.
29. „ (19 cwt. 2 qr. 26·345 lb.) by 31·875.
30. „ 2210·345 by 277·875 and by 31·875.

31. Find the products of 475·45 × 12·74 × 34·35.
32. " " 9·12075 × 34 375 × 245·675.
33. " " 6·435 × 5·75 × 1·645.
34. " " 18·3225 × 112 × 6·25.
35. Bought 174·9375 yards of silk at £·28125 the yard; required the cost.
36. Multiply (8·35 + 3.6875 + 8·9017) by 4·25.
37. " (8·375 cwt. + 2·1685 cwt. — 6375 cwt.) by 9 5.
38. " £3 18s. 6d. by 51·9448.
39. How much is 1·5 times (·85 of 2·25).
40. Multiply 8s. 7½d. by 9914·94.
41. " (37·875 + 7·2735 + 6·8125 + 47·32 + 32·625 + 27·59375) cwt. by 2 0625.
42. Find the price of 7 hogsheads 29·75 gallons of gin, at £·440625 per gallon.
43. Multiply (18·375 + 16·4375 + 32 825 — 13 56735) by 25·25.
44. Bought 337·35 cwt. of pork at ·739s. per lb.; what did the whole cost?
45. What is the price of 3·6985 cwt. of coffee, at 1·6875s. per lb?
46. Sold 23·4375 cwt. of sugar at £ 0203125 per lb.; what was the entire selling price?
47. (8·45 + 7·045 + 6·0045) × 3·5.
48. What is the weight of 9·75 loads of lime, each weighing 13 cwt. 1 qr. 19·25 lb.?
49. What is the amount of (4.0625 + 45·625 +4 5 + 1·1875 + 6·4375 + 5·75) × 81·5 × 8?
50. What is the value of (3·20076 + 27·0058 + 7238·5 + 12·75 + ·54344) × 5·375 × 63·75?
51. (478·63 + 89·005 + 00125 + 5437·25) × 10·5 × 46 5.
52. What is the price of (30·3765 + 5·4285714 + 23·705 + 20·607142857 + 14·8125 + 7·235) sacks of flour, at £2 93125 per sack?

CONTRACTED MULTIPLICATION.

98. *To contract the operation of multiplication, so as to retain only as many decimals in the product, as may be thought necessary.*

Rule.—Place the unit's figure of the multiplier under that figure of the multiplicand having the same decimal position as that intended for the product, (annexing ciphers if the decimals are too few). Then write all the other figures of the multiplier so that the entire row may be reversed.

Then in multiplying by this inverted multiplier, reject all the figures of the multiplicand which lie to the right of the multiplying figure, carrying from these rejected figures what would have been carried had they been retained, and set down the products so that their right-hand figures may be in the same vertical line. To compensate for the rejected figures of the multiplicand, 1 is carried for any number rejected between 5 and 14, 2 for any between 15 and 24, 3 for any between 25 and 34, &c.; the sum will be an approximate product to the true product.

EXAMPLE.

Multiply 13·57493, by 92·41035 so as to retain 4 places of decimals.

Contracted method.	*Common method.*
13·57493	13·57493
5301429	92·41035
12217437	6787465
271499	4072479
54299	13574930
1357	54299720
41	27149860
7	122174370
1254·4640	1254·4640325255

53. Multiply 46·163112 by 46·163112, retaining 4 decimal places in the product.

54. Multiply 2131·0330 by 46·163112, retaining 4 decimal places in the product.

Division of Decimals.

99.

Rule.—Expunge the decimal point of the divisor, and remove that of the dividend as many places to the right as there are decimal places in the divisor, using ciphers if necessary.* Then proceed as in common division, recording one decimal place in the quotient for every decimal place in the changed dividend which is used.

EXAMPLES.

Divide 18·575 by 2·5.

25) 185·75 (7·43. *Ans.*
175
107
100
75
75

Proof by vulgar fractions.

$$18{\cdot}575 \div 2{\cdot}5 = 18\tfrac{575}{1000} \div 2\tfrac{5}{10} = \tfrac{18575}{1000} \div \tfrac{25}{10} = \tfrac{18575}{1000} \times \tfrac{10}{25} = \tfrac{743}{100} = 7\tfrac{43}{100} = 7{\cdot}43.$$

---

* The signification of this rule simply requires that the dividend and divisor should be multiplied by such a power of 10, as will render the latter a whole number.

2. Divide 4475·99992 by 89·964285.

```
89964285 ) 4475999920 ( 112 Ans.
           89964285
           ---------
            47957142
            89964285
            ---------
             79928570
             79928570
```

Remark.—Since the divisor has 6 and the dividend but 5 decimals, a cipher is annexed to the latter.

3. Divide 18·43 by ·768.

```
768 ) 18430 ( 23·99739583̇.     768 ) 3040 ( 395833
      1536                            2304
      ----                            ----
       3070                            7360
       2304                            6912
       ----                            ----
        7660                            4480
        6912                            3840
        ----                            ----
         7480                            6400
         6912                            6144
         ----                            ----
          5680                            2560
          5376                            2304
          ----                            ----
           304                             2560
                                           2304
                                           ----
                                            256
```

4. Divide 3·2572 by ·958

```
958 ) 3257·2 (3 4
      2874
      ----
       3832
       3832
```

EXERCISES 30.

1. Find the quotient of 2495222·8125 ÷ 5·375 × 63·75.

2. „ 754·7694 ÷ ·7854.

3. „ 14656·6875 ÷ 9·75.

4. Divide 1430·9 by 40.
5. „ 437·619 by 34·35.
6. „ 208065·95355 by 475·45.
7. „ 159293·316 by 3·25.
8. „ 77025·44630859375 by 245·675.
9. „ 1211463·61875 by 313·52578125.
10. „ 1639·57386 by 1·6836.
11. „ 60·86705625 by 6·435 × 5·75.
12. „ 12825 75 by 6·25 × 112.
13. „ 2052·12 × 6·25 by 112.
14. „ 7335·4510640625 by 99 4165625.
15. „ £49 4s. 2·8125d. by 174·9375.
16. „ £117 7s. 11 3136d by 51·9448.
17. „ 742 cwt. 1 qr. 8·8886 lb. by 87·675.
18. „ £1797 1s. 7·89d. by 9914·94.
19. „ 629 cwt. 0 qr. 6·746875 lb. by 81·875.
20. „ £328 19s. 4·37625d. by 159·49975.
21. If 1365·2712875 yards cost £2568 15·158548625s., what cost 1 yard?
22. If £1·6836 purchase 1 cwt. of sugar, how many cwt. can be bought for £1639 11s. 5·7264d.?
23. If 18 cwt. 3 qrs. 24·5 lb. of beef cost £32 0s. 2·34375d., what is it a cwt.?
24. If 37 acres 2 roods 37½ perches of land be rented for £148 11s. 6·984375d., what is it an acre?
25. If 85 acres 2 roods 37 perches of land be rented for £316 2s. 8·15625d., what is it an acre?
26. How many barrels of flour, at £1·6875 per barrel, can be bought for £151 12s. 5·25d.?
27. What must be the weight of a nugget of gold, sold at £3·8625 per ounce, which brought £447 1s. 8·25d.?
28. How many gallons of brandy, at £1·08125, can be had for £116 14s. 8¼d.?
29. How many cwt. of iron at 18·625s. per cwt., can be had for £76 2s. 0·140625d.?
30. How many bushels of wheat at 9·125s. per bushel can be had for £25 16s. 8·4375d.?
31. What quantity of land can be had for £167 3s. 4¼·25 farthings, at £16·875 per acre?
32. How many cwt. of lead, at 15·875s., per cwt. can be bought for £45 18s. 9d. 0·75 f.?

33. What quantity of gold dust, at £1·65 per ounce, can be bought for £207 2s. 8·85d.?

34. Divide 650729·55 by 365.

35. „ 926·895525 „ 238.

36. How many yards of cloth, at 16s. 3d. per yard, can I have for £988 8s. 10·95d.?

37. How many bushels of wheat, at 7·7s. per bushel, can be had for £92 5s. 7·125d.

38. Divide 10948·5 by 87.

39. If 39·4375 barrels of flour cost £56 8s. 10¾·125f. what is it a barrel?

40. How many times does £128 0s. 11¾·625f. contain 405f.?

41. Divide 1814671·155 by 365.

42. If (13·15 + 18·2 + 58·5) tons of coal cost £75 16s. 2·625d., what is it a ton?

43. Divide 301470·3 by 8·4.

CONTRACTED DIVISION.*

100. Contracted division is a method of finding the quotient to as many figures, as the divisor deprived each time of the right hand figure will admit.

Rule.—Find the first figure of the quotient as in the uncontracted method, and thence the remainder. Divide this remainder as a new dividend, by the divisor with its right hand figure cut off for the next quotient figure. Divide this second remainder by the divisor (with two figures cut off) for the next quotient figure, and so on till all the figures of the divisor are exhausted.

In multiplying the quotient figure by the curtailed divisor observe to carry what should be carried from the figure cut off in each step. The number of figures in the quotient is equal to the number in the divisor, consequently the number of figures in the quotient may be increased or diminished, by increasing or diminishing those of the divisor. At a single glance it can be seen how many places of whole numbers the quotient ought to contain.

EXAMPLE 1.

Divide 756·145 by 35·451675.

---

* Contracted division is invaluable in abridging the decimal process in extracting the cube root by the plan derived from Horner's method of solving numerical equations.

Contracted Method.

8,5·,4,5,1,6,7,5) 756·14500 (21·328893
70903350
4711150
8545168
1165982
1063550
102432
70903
31529
28361
3168
2836
332
319
13
11
2

Common Method.

85451675) 756145000 (21·328893
709·03350
47·111500
85451675
116598250
106355025
102432250
70903350
325289000
283613400
316756000
283613400
331426000
319065075
123609250
106355025
17254225

44. Divide 85·51788 by 5·569906, having six figures in the quotient.

5·,5,6,9,9,0,6) 85·51788 (6·37675
8341944
209844
167097
42747
38989
3758
3341
417
389
28
28

45. Divide 19·895 by 6·3926, having four figures in the quotient.

46. Divide 87·076326 by 9·365407, so as to have eight figures in the quotient.

PROPORTION AND MISCELLANEOUS EXERCISES IN DECIMALS.

EXAMPLE.

Find the cost price of 102·5464 cwt. of coffee, when 9·46 lb. cost 13s. 7½d.

lb. cwt. s. d.
As 9·46 : 102·5464 : : 13 7·5 : x = £827 1 10·08
112 12
11485·1968 163·5
163·5
12
946) 187782967·68 (198502·08
946
2,0) 1654,1 10·08
9318
8514
£827 1 10·08 *Ans.*
8042
7568
4749
4730
1967
1892
7568
7568

EXERCISES 31.

1. What weight of tobacco can be bought for £230 2s. 7·2d., if 8·8 lb. cost 13s. 2·4d.?
2. How many acres of land can be had for £155 10s. 6·09375d., at £7·9375 per acre?
3. Find the rent of 83 a. 3 r. 33 p. of land, at £3 13s. 9d. per acre.
4. What is the price of 9 lb. 8 oz. 7·8824 dwts. of silver, at £·39 for 1·5 ounce?
5. Bought 7 hhds. 29·75 gals. of Cape wine, at £·440625 the gallon; what is the cost?
6. How many cwt. can be bought for £106 12s. 9·75d., at £·040625 the lb.?
7. What is the result of (4·87 + 3·435 + 6·895 + ·1875 — 2·6475) × 34·35 × 112 × 3·25?
8. If ·375 + ·0625 + ·04375 lb. of coffee cost £·0475 + ·875s. what is the cost of 1·075 ton?
9. If (1·785 + 3·87 + 2·8953 + ·7852 — 1·385) lb. of tea cost (7·35 + 2·365 + ·0375 + 4·4625 — ·025)s., what will (4·625 + 3·4375 + 2·875 + 1·075) cwt. come to?
10. Bought 43·625 hhds. of sugar, each weighing (14·378 + 10·6353 + ·0632) cwt. at £1 12s. 9d. the cwt., and sold it at 3·75d. the lb.; what is the gain?
11. Bought 68·27 cwt. of pork at £2·87 the cwt.; 3·65 cwt. at £2 7·32s. the cwt.; 23 cwt. 3·375 qr. at 8·5d. the lb.; and 14 cwt. 2 qrs. 18·875 lb. at £1 18s. 11d. the cwt.; what is the average price of 1 cwt.?
12. Bought 33·75 pieces of cloth, each measuring (3·375 + 4·4845 + 17·05 + 3·8395) yards, at 3·635 guineas for (2·235 + 3·63 + 5·14735 — 3·05) French ells; what is the cost?
13. What is the amount of (3·175 + 2·1875 + 1·632 + 2·12625) × 34·375 × 245·675?
14. What is the price of 973·85 cwt. of lead, at 8s. 6d. + 13s. 9d. + ·875s. + 10s. 6d. + ·564d. the cwt.?
15. Find the rent of 837 a. 3 r. 37 p. of land, at £2 15s. 7d. per acre.
16. What quantity of pork, at 6¼d. the lb., can I buy for £53 8s. 9·75d.?
17. What cost ·203125 quarters, + ·73635 bushels of barley, at £1·365 + ·475s. the quarter?
18. Bought 97·375 quarters, 12·895 bushels, 73·75 pecks of wheat, at £3·555 + 2·685s. per quarter; what is the cost?

19. What is the cost of 9·5 times (3·375 + 2·1685 — ·6375) cwt. of tea, if (3·05 + ·335 + 5·4375) lb. cost 3·5 times £1·25 + $3\frac{3}{8}$s.?

20. Sold 14 cwt. 3 qrs. 7 lb. + 7·235 cwt. + $20\frac{1}{2}\frac{7}{8}$ cwt. + 30·3765 cwt. + 5 cwt. 1 qr. 20 lb. + 23·705 cwt., at £1 13s. $7\frac{1}{2}$d. per cwt.; what is the selling price?

21. Purchased 25·25 pieces of cloth, each containing (18·375 + 16·4375 + 32·825 — 13·56735) yards, at 37·68s. for 7·635 yards; what is the cost?

22. Bought 89·375 cwt. of pork, at £2 15s. $7\frac{1}{2}$d. per cwt.; and 68·485 cwt. at 7·875d. per lb.; what must 1 lb. be sold for so as to gain 13 guineas on the whole?

23. Bought 5·75 hhds. of sugar, each weighing 6·435 cwt., at £1·645 per cwt., and sold 3·4375 cwt. at $3\frac{1}{2}$d. the lb., and the remainder at £1·4375 the cwt.; what is the gain?

24. How many yards of cloth at £·6825 per yard, can be bought for £249 11s. $7\frac{1}{2}$d. + ·45f.?

25. If 159 lb. 9·25 ounces of tea cost £32 17s. $10\frac{1}{2}$d. + ·75f., what is it a lb.?

26. Bought 3 hhds. of tobacco, each 4 cwt. 2 qr. 7·4 lb. at 16s.. $4\frac{1}{2}$d. for 8·4 lb.; what was paid for the whole?

27. Hired a servant man on the 14th of May 1859, at £48 3s. 2·415d. per annum, and he left on the 18th of October following; what wages ought he to receive?

28. Find the price of 17 cwt. 2 qr. $24\frac{1}{2}$ lb. of tea at £358 13s. 9d. per ton.

29. If 14·85 oz. ÷ 24·75 of gold be bought for £2 13s. $7\frac{1}{2}$d., what is the price of 173·5425 oz. × 2·25 ÷ 3·75.?

30. What is the cost of (80·4632 + 48·488 + 9·65) yards ÷ ·12, if ·68 of 3·5 yards cost ·06 of £13·375?

31. Bought 30·3765 cwt. + 5 cwt. 1 qr. 14 lb. + 23·705 cwt. + $20\frac{1}{2}\frac{7}{8}$ cwt. + 14 cwt. 3 qrs. 7 lb. + 7·235 cwt. at £2 18·625s. per cwt.; what was the cost?

32. What is the income tax at 5d. in the pound, on an income of £3887·448617288725584?

33. Bought 3 pieces of cloth for £68 16s. 7·03125d., which was at 6s. $9\frac{1}{2}$d. the yard; how many yards were there in each piece?

34. How many quarters of wheat at £2 16s. per quarter can be bought for £62 0s. 7·86d.?

35. What is the cost of 9 lb. 8 oz. 7·8824 dwt. of silver at 7·8s. for $1\frac{1}{2}$ oz., and 10 lb. 5 oz. 10·75 dwt. at 8·25s. per ounce?

36. What is the price of 5·375 pieces of silver plate, each weighing 7282 ounces, at 5s. $3\frac{3}{4}$d. per ounce?

37. What income tax must be paid on the rent of 89 a. 3 r. 37½ p., valued at £6 17s. 10½d. per acre, at 16d. in the £?

38. What is the price of 10·5 packages of tea, each weighing (478·63 + 89·005 + ·00125 + 5437·25) lbs., at 13s. 6¾d. for every 3·5 lb.?

39. How many pieces of cloth, each 7282 yards, can be had for £15514 1s. 5·625d., if 1 yard be bought for 6s. 3·75d.?

40. How many cwt. of tobacco, at 13s. 2·4d. for 8·8 lb., can be bought for £230 2s. 7·2d.?

41. Sold (47·388 + 81·4632 + 9·75) yards ÷ ·04, at ·45 of 8 yards for ·088 of £3·625; what is the selling price?

Simplify the expression $(\frac{1\cdot5}{\cdot025}) + \frac{2\cdot75}{1\frac{1}{4}} \times \frac{5\cdot385}{1\cdot077} \div \frac{3\cdot75}{5}$

$$\begin{array}{ll}
\frac{1\cdot5}{\cdot025} = 60 & \\
\frac{2\cdot75}{1\frac{1}{4}} = \underline{\quad 2\cdot2 \quad} & \\
\qquad\qquad 62\cdot2 \text{ sum.} & \\
\frac{5\cdot385}{1\cdot077} = \underline{\quad 5 \quad} & \\
\qquad\qquad 811 \text{ product.} & \\
\frac{3\cdot75}{5} = \frac{\qquad}{\cdot75} = 414\cdot6 \textit{ Ans.} &
\end{array}$$

PRACTICE.

101. **Practice** is the name given to certain operations, for finding, by means of *aliquot parts*, the price of any number of articles or part of an article, when the price of one is given.

Note.—A less number is an *aliquot* part of a greater of the same denomination when the less measures the greater exactly; as 3 is *an aliquot part* of 12; 5s. is an *aliquot* part of 20s. or £1.

102. *The process of operation.*—Find the price of the number of articles given at a unit of money, say £1, 1s., or 1d.; multiply that price by the number of units of the chosen denomination contained in the given price, and take aliquot parts for the next lower denominations, if any.

EXAMPLE.*

Find the price of 67 cwt. of pork, at £2 16s. 6d. per cwt.

---

[*] It is evident that the price of 67 cwt. at £2 16s. 6d. the cwt., is the same as 67 cwt. at £2 per cwt., then at 10s., then at 5s., then at 1s., and then at 6d. per cwt.—all added. This process is only another way of performing compound multiplication.

TABLE OF ALIQUOT PARTS.

Of a Pound.		Of a Shilling	Of a Cwt.	Of a lb. Troy.
10s. = $\frac{1}{2}$	1s. = $\frac{1}{20}$	6d. = $\frac{1}{2}$	2 qr. = $\frac{1}{2}$	6 oz. = $\frac{1}{2}$
5s. = $\frac{1}{4}$	8d. = $\frac{1}{30}$	4d. = $\frac{1}{3}$	1 qr. = $\frac{1}{4}$	4 oz. = $\frac{1}{3}$
4s. = $\frac{1}{5}$	6d. = $\frac{1}{40}$	3d. = $\frac{1}{4}$	16 lb. = $\frac{1}{7}$	3 oz. = $\frac{1}{4}$
6s. 8d. = $\frac{1}{3}$	4d. = $\frac{1}{60}$	2d. = $\frac{1}{6}$	14 lb. = $\frac{1}{8}$	Of an Ounce.
3s. 4d. = $\frac{1}{6}$	3d. = $\frac{1}{80}$	1½d. = $\frac{1}{8}$	Of a qr.	10 dwt. = $\frac{1}{2}$
2s. 6d. = $\frac{1}{8}$	Of a Guinea.	Of a Penny.	7 lb. = $\frac{1}{4}$	4 dwt. = $\frac{1}{5}$
2s. = $\frac{1}{10}$	10s. 6d = $\frac{1}{2}$	½d. = $\frac{1}{2}$	4 lb. = $\frac{1}{7}$	5 dwt. = $\frac{1}{4}$
1s. 8d. = $\frac{1}{12}$	7s. = $\frac{1}{3}$	¼d. = $\frac{1}{4}$	3½ lb. = $\frac{1}{8}$	2 dwt. = $\frac{1}{10}$

67 cwt. at £1 per cwt. } = £67
2 = the units of £s in the price.

	134	= price of 67 cwt.	at £2 0 0 per cwt.
10s. = $\frac{1}{2}$ of £1	33 10	„ „	at 0 10 0 „
5s. = $\frac{1}{2}$ of 10s.	16 15	„ „	at 0 5 0 „
1s. = $\frac{1}{5}$ of 5s.	3 7	„ „	at 0 1 0 „
6d. = $\frac{1}{2}$ of 1s.	1 13 6	„ „	at 0 0 6 „
	£189 5 6		£2 16 6

Otherwise,

67 cwt. at 1s. per cwt. = 67s.
56 = the units of shillings in the price.

402
335

3752 = price of 67 cwt. at 56s. 0d.
6d. is $\frac{1}{2}$ of 1s. - 33 6 „ „ at 0 6d.

2,0) 378,5 6 56s. 6d.

£189 5 6

Otherwise,

67 cwt. at 1d. per cwt. = 67d.
678 = the units of pence in the price.

536
469
402

12) 45426 = what 67 cwt. would cost at 678d. per cwt.

2,0) 378,5 6

£189 5 6

EXAMPLE (2.)

Find the price of 873 cwt. of cheese, at £1 6s. 8d. per cwt.

873 cwt. at £1 per cwt. = £873
1 = the unit of £ in the price.

	873	= price of 873 cwt. at	£1 0 0
6s. 8d. = ⅓ of £1 -	291	„ „ at	0 6 8
	£1164		£1 6 8

Otherwise,

873 cwt. at 1s. per cwt. = 873s.
26 = the units of shillings in the price.

5238
1746

	22698 0	= price of 873 cwt. at	26s. 0d.
6d. = ½ of 1s. - -	436 8	„ „ at	0 6
2d. = ⅓ of 6d. - -	145 6	„ „ at	0 2
	2,0) 2328,0 0		26 8
	£1164 0 0		

Otherwise,

873 cwt. at 1d. per cwt. = 873d.
320 = the units of pence in the price.

17460
26190

12) 279360, price of 873 cwt., at 320d. per cwt.

2,0) 2328,0

£1164 0

EXAMPLE (3.)

Bought 467 yards of cloth at 6s. 3¾d. per yard, what is the cost?

67 yards at
£1 per yard = £467

5s. = ¼ of £1 -	116 15 0	= price of 467 yards at	5s. 0d.
1s. = ⅕ of 5s. -	23 7 0	„ „ at	1 0
3d. = ¼ of 1s. -	5 16 9	„ „ at	0 3
¾d. = ¼ of 3d. -	1 9 2¼	„ „ at	0 0¾
	£147 7 11¼	at	6 3¾

Otherwise,

7 yards at 1s. per yard = 467s.

	6 = the units of shillings in the price.		
	2802 = price of 467 yards	at	6s. 0d.
l. = ¼ of 1s. - -	116 9 „ „	at	0 3
l. = ¼ of 3d. - -	29 2¼ „ „	at	0 0¾
	2,0) 294,7 11¼ = - -	at	6 3¾
	£147 7 11¼		

Otherwise,

7 yards at 1d. per yard = 467d.

	75 = the units of pence in the price.	
	2335	
	3269	
	35025 = price of 467 yards	at 75d.
½d. is ½ of 1d.	233½ „ „	at ½d.
¼d. is ½ of ½d.	116¾ „ „	at ¼d.
	12) 35375¼	at 75¾d.
	2,0) 294,7 11¼	
	£147 7 11¼	

Otherwise,

7 yards, at 1 farthing per yard = 467 farthings.

303 = the units of farthings in the price.

1401
14010

4) 141501 = the cost of 467 yards at 303 farthings the yd.

12) 35375¼

2,0) 294,7 11¼

£147 7 11¼

EXAMPLE (4.)

Bought 7805 yards of tape, at ½d. per yard.

7805 yards at ½d. per yard = 7805 halfpence.

1 = the unit of halfpence in the price.

2) 7805 = the cost of 7805 yards, at ½d. per yard.

12) 3902½

2,0) 32,5 2½

£16 5 2½

Otherwise,

7805 yards, at 1d. per yard = 7805d.

½d. = ½ of 1d. 3902½ the price of 7805 yards, at ½d. per yard.

2,0) 32,5 2½

£16 5 2½

Otherwise,

7805 yards, at 1s. per yard = 7805 shillings.

½d. = $\frac{1}{24}$ of 1s. 32,5 2½ = the price of 7805 yards, at ½d. per yard.

£16 5 2½

EXAMPLE (5.)

Find the price of 6903 lb. of oaten-meal, at ¾d. per lb.

6903 lb., at 1 farthing per lb. = 6903 farthings.

3 = the units of farthings in the price.

4) 20709 = the price of 6903 lb., at ¾d. per lb.

12) 5177¼

2,0) 43,1 5¼

£21 11 5¼

Otherwise.

903 lb. at 1d. per lb. = 6903d.

$\frac{1}{2}$d. = $\frac{1}{2}$ of 1d. - 3451$\frac{1}{2}$ = price of 6903 lb., at 0$\frac{1}{2}$d.
$\frac{1}{4}$d. = $\frac{1}{2}$ of $\frac{1}{2}$d. - 1725$\frac{3}{4}$ „ „ at 0$\frac{1}{4}$d.

12) 5177$\frac{1}{4}$ at 0$\frac{3}{4}$

2,0) 43,1 5$\frac{1}{4}$

£21 11 5$\frac{1}{4}$

Otherwise,

903 lb. at 1s. per lb. = 6903s.

1. is $\frac{1}{16}$ of 1s. - - 43,1 5$\frac{1}{4}$ the cost of 6903 lb. at $\frac{3}{4}$d. per lb.

£21 11 5$\frac{1}{4}$

EXERCISES 32.

1. Find the price of 632 yards at 4s. per yard.
2. „ „ 397 „ 5s. „
3. „ „ 1603 „ 2s. „
4. „ „ 873 „ 3s. „
5. „ „ 1746 „ 6s. „
6. „ „ 428 „ 6s. 8d. „
7. „ „ 739 „ 3s. 4d. „
8. „ „ 564 „ 9s. „
9. „ „ 763 „ 10s. „
10. „ „ 385 „ 2s. 6d. „
11. „ „ 639 „ 1s. 8d. „
12. „ „ 856 „ 1s. 4d. „
13. „ „ 489 „ 1s. 3d. „
14. „ „ 1357 „ 6d. „
15. „ „ 675 „ 4d. „
16. „ „ 876 „ 8d. „
17. „ „ 945 „ 2d. „
18. „ „ 836 „ 1d. „
19. „ „ 478 „ $\frac{3}{4}$d. „
20. „ „ 1683 „ $\frac{1}{2}$d. „
21. „ „ 987 „ $\frac{1}{4}$d. „
22. „ „ 836 „ 1$\frac{1}{4}$d. „

23. Find the price of 673 yards at 1½d. per yard.

24. " " 685 " 1¾d. "

25. " " 885 " 2¼d. "

26. " " 675 " 2½d. "

27. " " 836 " 2¾d. "

28. " " 459 " 2s. 4d. "

29. " " 653 " 1s. 5d. "

30. " " 835 " 1s. 1¼d. "

31. " " 673 " 1s. 1½d. "

32. " " 954 " 1s. 1¾d. "

33. " " 555 " 1s. 2½d. "

34. " " 693 " 1s. 2¾d. "

35. " " 876 " 2s. 5½d. "

36. " " 638 " 2s. 10½d. "

37. " " 876 " 1s. 11¾d. "

38. " " 869 " 5s. 10d. "

39. " " 437 " 4s. 6¾d. "

40. " " 685 " 12s. 6d. "

41. " " 985 " 13s. 4d. "

42. " " 865 " 17s. 6d. "

43. " " 639 " 18s. 4d. "

44. " " 183 " 18s. 9d. "

45. " " 1142 " £1 11s. 8d. "

46. " " 1653 " £1 12s. 6d. "

47. " " 473 " £2 8s. 8d. "

48. " " 319 " £2 17s. 4d. "

49. " " 2345 " £2 6s. 8d. "

50. " " 685 " £2 5s. 6d. "

51. " " 943 " £2 13s. 4d. "

52. " " 653 " £3 7s. 6d. "

53. " " 856 " £4 6s. 6d. "

54. " " 658 " £5 2s. 6d. "

EXAMPLE.

Find the price of 84¾ lb. of tea at 3s. 10d. per lb.

3s. 10d.
12

2 6 0
7

16 2 0 = cost of 84 lb.
2 10½ = cost of ¾ lb.

£16 4 10½ = cost of 84¾ lb.

3s. 10d.
3

4) 11 6

2 10½.

Otherwise,

84¾ lb. at £1 per lb. = £84 15s.

	£	s.	d.			
3s. 4d. = ⅙ of £1	14	2	6	= price of 84¾ lb.	at 3s. 4d.	
5d. = ⅛ of 3s. 4d.	1	15	3¾	,, ,, ,,	at 0s. 5d.	
1d. = ⅕ of 5d.	0	7	0¾	,, ,, ,,	at 0s. 1d.	
	£16	4	10½		at 3s. 10d.	

Otherwise,

84¾ lb. at 1s. per lb = 84s. 9d.
3 = the units of shillings in the price.

	s.	d.		
	254	3	= price of 84¾ lb.	at 3s. 0d.
6d. = ½ of 1s. -	42	4½	,, ,, ,,	at 0s. 6d.
4d. = ⅓ of 1s. -	28	3	,, ,, ,,	at 0s. 4d.
20)	32,4	10½		at 3s. 10d.
	£16	4 10½		

Otherwise,

84¾ lb. at 1d. per lb. = 84¾d.
46 = the units of pence in the price.

504
336
34½

3
46
4) 138
34½

12) 3898½ = the cost of 84¾ lb. at 46d. per lb.

2,0) 324 10½

£16 4 10½

Otherwise,

3s. 10d.
17

3 5 2 = cost of 17 lb.
5

£16 5 10 = the cost of 85 lb.
¼ lb. cost 0 11½ - - 0¼ lb.

£16 4 10½ the cost of 84¾ lb.

EXERCISES.

Find the price of		867½ cwt.	at	£1	8	4	per cwt.
"	"	185¾	"	2	12	6	"
"	"	87 23/32	"	2	19	4	"
"	"	85 15/16	"	14	18	8	"
"	"	67 19/30	"	2	16	6	"
"	"	327 31/32	"	3	17	10½	"
"	"	75 15/64	"	2	16	10⅓	"

EXAMPLE.

Find the price of 215cwt. 3qr. 21lb. of soap, at £2 7s. 10d. per cw

	£2	7	10	× 5
			10	
	23	18	4	= cost of 10 cwt.
			7	
	167	8	4	= cost of 70 cwt.
			3	
	502	5	0	= cost of 210 cwt.
	11	19	2	= cost of 5 cwt.
	514	4	2	= cost of 215 cwt.
2 qr. = ½ of 1 cwt.	1	8	11	= cost of 2 qr.
1 qr. = ½ of 2 qr.	0	11	11½	= cost of 1 qr.
14 lb. = ½ of 1 qr.	0	5	11¾	= cost of 14 lb.
7 lb. = ½ of 14 lb.	0	2	11⅞	= cost of 7 lb.
	£516	9	0⅛	= cost of 215 cwt. 3 qr. 21 lb.

Otherwise,

	*£2	7	10	
			6	
	14	7	0	= cost of 6 cwt.
			6	
	86	2	0	= cost of 36 cwt.
			6	
	516	12	0	= cost of 216 cwt.
7 lb. = 1/16 of 112 lb.		2	11⅞	= cost of 7 lb.
	£516	9	0⅛	= cost of 215 cwt. 3 qr. 21 lb.

---

* 215 cwt. 3 qr. 21 lb. want but 7 lb. of being 216 cwt.—If the price of 216 cw be found, and from it be taken the price of 7 lb., the price of 215 cwt. 3qr. 21 l is obtained.

Otherwise,

* 215 cwt. 3 qr. 21 lb., at
£1 per cwt. = £215 18 9
2 = the units of pounds in the price.

					What 215 cwt. 3 qr. 21 lb. would come to.
	431	17	6	$\overbrace{\quad}^{16}$ - -	at £2 0 0
5s. = $\frac{1}{4}$ of £1	53	19	$8\frac{1}{4}$	$= \frac{4}{16}$ - -	at 0 5 0
2s. 6d. = $\frac{1}{2}$ of 5s.	26	19	$10\frac{1}{8}$	$= \frac{2}{16}$ - -	at 0 2 6
3d. = $\frac{1}{10}$ of 2s. 6d.	2	13	$11\frac{13}{16}$	$= \frac{13}{16}$ - -	at 0 0 3
1d. = $\frac{1}{3}$ of 3d.		17	$11\frac{15}{16}$	$= \frac{15}{16}$ - -	at 0 0 1
	£516	9	$0\frac{1}{8}$	$= \frac{34}{16} = 2\frac{1}{8}$ -	at 2 7 10

Otherwise,

† 215 cwt. 3 qr. 21 lb., at
1s. per cwt. = 215s. $11\frac{1}{4}$d.
47 = the units of shillings in the price.

	10149	$0\frac{3}{4}$	= price of	215 cwt.	3 qr	21 lb.		at 47s. 0d.
6d. = $\frac{1}{2}$ of 1s.	107	$11\frac{5}{8}$	,,	,,	,,	,,	,,	at 0s. 6d.
4d. = $\frac{1}{3}$ of 1s.	71	$11\frac{3}{4}$	,,	,,	,,	,,	,,	at 0s. 4d.
2,0)	1032,9	$0\frac{1}{8}$	,,	,,	,,	,,	,,	at 47s. 10d.
	£516 9	$0\frac{1}{8}$						

EXERCISES 33.

1. Find the price of 431 cwt. 3 qr. 14 lb. at 11s. $11\frac{1}{2}$d. per cwt.
2. ,, ,, 14 cwt. 2 qr. 25 lb. at £9 6s. 8d. per cwt.
3. ,, ,, 73 cwt. 2 qr. 23 lb. at £18 13s. 4d. per cwt.
4. ,, ,, 87 cwt. 2 qr. $24\frac{1}{2}$ lb. at £2 19s. 4d. per cwt.
5. ,, ,, 12 cwt. 2 qr. 3 lb. at £7 9s. 4d. per cwt.

---

* At £1 per cwt., the price of 1 qr. is 5s.; of 1 lb. $2\frac{1}{7}$d.; of 1 cwt. 1 qr. 1 lb., £1 5 $2\frac{1}{7}$; and of 215 cwt. 3 qrs. 21 lb., $\left\{\begin{array}{lll} \text{cwt.} & \text{qr.} & \text{lb.} \\ 215 & 3 & 21 \\ \text{£1} & \text{5s.} & 2\frac{1}{7}\text{d.} \end{array}\right\}$

It will be £215 18 9

† At 1s. per cwt., the price of 1 qr. will be 3d.; of 1 lb. $\frac{3}{28}$d., and the price of 215 cwt. 3 qr. 21 lb., $\left\{\begin{array}{lll} \text{cwt.} & \text{qr.} & \text{lb.} \\ 215 & 3 & 21 \\ \text{1s.} & \text{3d.} & \frac{3}{28} \end{array}\right\}$

would be 215s. $11\frac{1}{4}$d.

EXAMPLE.

Find the rent of 3 roods 35 perches of land at £5 13s. 4d. per acre.

		3 r. 35 p.							
		5s.	1½d.						
The rent of 3 roods 35 perches at £1 per acre	=		19	4½					
				5 = the units of pounds in price.					
		£4	16	10½	= rent of 3a. 35p.		at £5	0	0
10s. = ½ of £1 -		0	9	8¼	,,	,,	at 0	10	0
3s. 4d. = ⅓ of 10s. -		0	3	2¾	,,	,,	at 0	3	4
		£5	9	9½			at £5	13	4

Otherwise,

£5 13 4

2 r. = ½ of 1 acre -	£2	16	8	rent of	2 roods.
1 r. = ½ of 2 r. -	1	8	4	,,	1 rood.
20 p. = ½ of 1 r. -	0	14	2	,,	0 20 perches.
10 p. = ½ of 20 p. -	0	7	1	,,	0 10 ,,
5 p. = ½ of 10 p. -	0	3	6½	,,	0 5 ,,
	£5	9	9½		3 35

Otherwise,

	£5	13	4	rent of	1 acre.
5 p. = $\frac{1}{32}$ of 1 acre	0	3	6½	,,	0 5 perches.
	£5	9	9½		3 35 perches.

* *Otherwise, by Proportion.*

acre.		r.	p.		£	s.	d.			£	s.	d.
As 1	:	3	35	::	5	13	4	:	x =	5	9	2½

EXAMPLE.

Find the price of 16 cwt. 3 qrs. 21 lb. of butter at £93 6s. 8d. per ton.

---

* All practice sums can be solved by proportion; but the converse does not hold, because all questions resolvable by practice must have the price of *one* given.

	£ s. d.		
	£93 6 8		
10 cwt. = ½ of 1 ton	46 13 4	= the cost of 10 cwt.	
5 cwt. = ½ of 10 cwt.	23 6 8	„ „	5 cwt.
1 cwt. = ⅕ of 5 cwt.	4 13 4	„ „	1 cwt.
2 qrs. = ½ of 1 cwt.	2 6 8	„ „	0 2 qrs.
1 qr. = ½ of 2 qrs.	1 3 4	„ „	0 1 qr.
14 lb. = ½ of 1 qr.	0 11 8	„ „	0 0 14 lb.
7 lb. = ½ of 14 lb.	0 5 10	„ „	0 0 7 lb.
	£79 0 10		16 3 21

Otherwise,

16 cwt. 3 qrs 21 lb.
at £1 per ton = 16s. 11¼d.

	93	= the units of pounds in the price.	
	1575 2¼	= price of 16 cwt. 3 qr. 21 lb. at	£93 0 0
6s. 8d. = ⅓ of £1	5 7¾	„ „ „ at	0 6 8
	2,0) 158,0 10 0	„ „ „	£93 6 8
	£79 0 10		

6. Find the rent of 255 a. 0 r. 32 sq. p. at £1 14s. 0¾d. per acre.
7. Find the price of $5\frac{1}{16}$ puncheons of rum, each 82 gals. 3 qts., at 15s. 3d. per gallon.
8. Find the rent of 97 a. 3 r. 25 sq. p. at £2 5s. 6d. per acre.
9. Find the price of $79\frac{17}{196}$ barrels of flour at £1 12s. 8d. per barrel.
10. Find the price of 53 cwt. 2 qrs. 25 lb. of tobacco at £5 14s. 4d. per cwt.

EXERCISES 34.

1. What is the cost of 169 cwt. 2 qr. 21 lb. of sugar, at £3 5s. 4d. per cwt.?
2. Bought 79 cwt. 3 qr. 21 lb. of butter, at £2 18s. 4d. per cwt., what is the cost?
3. What is the cost of 247 cwt. 3 qr. 14 lb. of rice, at £1 14s. 2d. per cwt.?
4. Rented 138 acres, 3 roods, 35 perches of land, at £3 13s. 4d. per acre, what is the rent?
5. Bought 2 qr. 23 lb. of tea, at £29 17s. 4d. per cwt., what is the cost?
6. What is the cost of 18 cwt. 2 qr. 14 lb. of coffee, at £149 6s. 8d. per ton?

7. Bought 3 tons 16 cwt. 2 qr. 23 lb. of sugar, at £74 13s. 4d. per ton, what is the cost?
8. Gave a £300 note to pay for 17 cwt. 2 qr. $24\frac{1}{2}$ lb. of tea, at £298 13s. 4d. per ton, what change do I get back?
9. What is the cost of 1 ton 18 cwt. 2 qr. 16 lb. of tea, at £107 6s. 8d. per ton?
10. Find the price of 148 cwt. 0 qr. 7 lb. of iron, at 17s. 4d. per cwt.
11. Find the cost of 23 hhds. 19 gals. 2 qts. of gin, at 7s. 2d. per gal.
12. Bought 18 cwt. 3 qr. $10\frac{1}{2}$ lb. of tea, at £515 13s. 4d. per ton, what is the cost?
13. Find the price of 13 cwt. 2 qr. 23 lb. of coffee, at £128 6s. 8d. per ton.
14. Find the price of $47\frac{5}{8}$ bags of coffee, each weighing 1 cwt. 2 qr. 16 lb., at £3 19s. 4d. per cwt.
15. Gave a £50 note to pay for 13 cwt. 2 qr. $24\frac{1}{2}$ lb. of coffee, at £54 13s. 4d. per ton, what change do I get back?
16. Find the price of $22\frac{3}{16}$ pieces of cloth, each 7 yards 3 qr. 3 n., at 10s. 8d per yard.
17. What is the cost of 18 cwt. 3 qr. $12\frac{1}{4}$ lb. of butter, at £5 18s. 8d. per cwt.?
18. What is the cost of 10 tons 13 cwt. 2 qr. 14 lb. of sugar, at £16 6s. 8d per ton.
19. Bought 46 tons 17 cwt. 3 qr. 14 lb. of lead, at £19 13s. 4d. per ton; what is the cost?
20. Find the price of 387 yards 3 qr. 3 nails of cloth, at 14s. 8d. per yard.
21. Gave a £50 note to pay for 16 cwt. 3 qr. $24\frac{1}{2}$ lb. of pork, at £54 13s. 4d. per ton, what change am I to get back?
22. Gave a £50 note to pay for 17 cheeses, each weighing 3qr. 25 lb.. at £2 13s. 8d. per cwt., what change is returned?
23. Gave a £300 note to pay for 18 cwt. 2 qr. $27\frac{1}{2}$ lb. of tea, at £298 13s. 4d., what change is returned?
24. Bought $47\frac{5}{8}$ chests of tea, each weighing 1 cwt. 2 qr. 16 lb., at £14 4s. 8d. per cwt.; find the cost.
25. Bought $51\frac{1}{4}$ chests of tea, each weighing 12 cwt. 3 qr. 21 lb. 13oz., at £14 18s. 8d. per cwt.; find the cost.
26. What is the price of $22\frac{3}{16}$ pieces of cloth, each containing 17 yds. 3 qr. 3n., at 5s. 4d per yard?
27. Purchased $36\frac{7}{16}$ plots of land, each containing 45 ac. 3 r. 26 p., at £20 3s. 4d. the acre; what is the cost?
28. Find the price of $21\frac{9}{32}$ bags of wheat, each containing 27 quarters 4 bushels, at £3 10s. per quarter.

29. Find the cost of $26\frac{1}{3}\frac{2}{3}$ pieces of cloth, each containing 84 yds. 3 qr. 3 n., at 6s. 8d. per yard ?

30. Find the price of $17\frac{1}{3}\frac{7}{8}$ hhds. of sugar, each weighing 23 cwt. 2 qr. 13 lb., at £1 17s. 4d. the cwt.

31. Find the rent of 139 ac. 3 r. 19 p. of land, at £2 7s. 10d. the acre.

32. What is the cost of 449 cwt. 3 qr. 25 lb. of rice, at £1 5s. 11d the cwt. ?

33. Find the price of 324 cwt. 3 qr. 23 lb of tallow, at £2 17s. 10½d. the cwt.

34. Find the rent of 179 ac. 3 r. 33 p. of land, at £3 17s. 7d. the acre.

35. Find the rent of 29 $\frac{2}{3}\frac{1}{2}$ plots of land, each plot containing 29 ac. 2 r. 17 p., at £2 13s. 4d. the acre.

36. What is the price of 79 cwt. 3 qrs. 27¾ lb. of tallow at £2 17s. 11¾d. per cwt.

37. What is the expense of excavating $35\frac{4}{6}\frac{2}{4}$ pieces of railway land, each 73 cubic yards 19 cubic feet 999 cubic inches, at £16 4s. the cubic yard ?

38. What is the cost of purchasing in Castle Street, Liverpool, 26½ pieces of ground, each containing 29 square yards 7 square feet 89 square inches, at £18 4s. 6d. the square yard ?

39. What is the cost of excavating $47\frac{1}{1}\frac{1}{6}$ pieces of railway land, each piece containing 73 cubic yards 19 cubic feet 999 cubic inches, at £2 0s. 6d. the cubic yard ?

40. Find the rent of 37047780 square inches at £2 19s. 3½d. per acre.

41. What is the rent of 13½ plots of land, each plot containing 7 acres 3 roods 23½ perches at £2 5 florins 7 cents 9 mills per acre ?

42. What is the cost of digging 23¾ plots of railway land, each containing 29 cubic yards 23 cubic feet 793 cubic inches, at £25 19s. 9d. per cubic yard ?

43. What is the rent of 37 acres 2 roods 37½ perches of land, at £3 18s. 9d. per acre ?

44. Find the rent of 85 acres 2 roods 37 perches of land, at £3 13s. 9d. per acre.

45. Bought 5⅜ brewings of ale, each 6 butts 2 barrels 33 gallons 3 quarts, at £1 11s. 6d. per barrel ; what was the cost ?

46. Find the price of 6⅞ nuggets of gold, each weighing 4 lb. 9 oz. 17 dwts. 20 grs., at £3 17s. 10½d. per ounce.

47. Bought 7⅝ pieces of gold ore, each weighing 5 lb. 10 oz. 12 dwts., at £41 12s. 6d. per lb ; what was paid for it ?

48. Find the sale of $37\frac{11}{16}$ packages of Epsom salts, each weighing 5 lb. 7 oz. 5 dr. 2 sc. 17 grs. at 1s. $1\frac{1}{2}$d. per lb.

49. Find the price of $26\frac{13}{16}$ packages of soda, each weighing 3 lb. 5oz. 7 dr. 2 sc. 17 gr., at 1s. $4\frac{1}{2}$d. per lb.

50. What is the cost of 87 cwt. 2 qrs. $8\frac{19}{32}$ lb. of beef at £2 16s. 6d. per cwt?

51. Find the total cost of 6973 lb. of cotton at $6\frac{5}{8}$d. per lb., and 4576 lb. at $7\frac{7}{8}$d. per lb.

52. Find the cost of $18\frac{13}{16}$ loads of wheat, each 15 qrs. 3 bushels 2 pecks, at £2 18s. 6d. per quarter.

53. Find the cost of $3\frac{3}{4}$ packages of chlorate of potassa, each weighing 9 lb. 4 oz. 3 dr. 2 sc. 12 grs., at 3s. 4d. per lb.

54. Find the price of 449 cwt. 3 qrs. 25 lb. of cheese at £1 5s. 11d. per cwt.

55. Find the price of $4\frac{2}{3}$ parcels of nitrate of potassa, or salt petre, each weighing 85 lb. 7 oz. 5 dr. 2 sc. 18 grs., at 1s. 3d. per lb.

56. What is the purchase money of $48\frac{11}{16}$ pieces of land, each containing 37 square yards 7 square feet 96 square inches, at £1 7s. per square yard?

57. What sum can I earn in $5\frac{2}{3}$ periods of time, each 2 years 300 days 17 hours 35 minutes 48 seconds, at £150 per annum of 365 days to the year?

TARE AND DRAFT.

103. In the purchase of goods certain deductions are made from the entire gross weight, called *draft* and *tare*, before calculating their price.

104. *Draft* is an allowance made to cover loss in weight arising from want of exactitude in weighing or from any other cause.

105. *Tare* is a deduction made for the cask, package, &c., containing the goods.

The deduction for draft is made first, and for tare afterwards—(*palm-oil* excepted)—when all the necessary deductions are made, the remainder is called the *net* weight.

Commercial allowances are regulated by the usage of the market where the goods are sold, and are chiefly confined to tare and draft; the terms *tret* and *cloff* met with in books of Arithmetic, are nearly, if not altogether, obsolete as deductions in commerce.

ALLOWANCES.

Scale regulating the deductions for draft.

Weight under ½ cwt. no draft.

Weight	½	„	and under	1 cwt.	1 lb. draft.
„	1	„	„	2 „	2 „
„	2	„	„	3 „	3 „
„	3	„	„	10 „	4 „
„	10	„	„	18 „	7 „
„	18	„	and upwards		9 „

COTTON.

American { *Draft* 1 lb. per bale.
Tare 4 lbs. per cwt.

Surat { *Draft* 1 lb. per bale.
Tare 15 lbs. per bale and 10 lbs. per ½ bale.

PALM OIL.

Tare on butts of 12 cwt. and upwards = 16 lbs.
„ on puncheons of 3½ cwt. to 12 cwt. = 18 lbs.
Draft as per scale.

POTASHES.

Draft, as per scale.
Tare, actual.

WOOL.

Draft, 1 lb. per cwt. on foreign wool.
2 lb. per pack of 240 lb. on fleece wool.
Tare, actual.

TOBACCO.

Draft, 2 lbs. per box.
Tare, 31 lb. per box.

LARD.

Draft as per scale.
Tare, actual.

BEEF, BUTTER, AND PORK.

Tare, according to average weight of tierce, firkin, or barrel.
No draft.

BACON.

Draft, 2 lbs. per bale.
Tare, average weight of bale.

FLOUR.

Tare, 20 lb. per barrel.
No draft.

TEA.

Draft, as per scale.
Tare, actual.

SUGAR.

Draft, as per scale.

		cwt.	qr.	lbs.
Tare on	8 and under 9 cwt.	1	0	7
	12 ,, 13 ,,	1	1	14
	16 ,, 17 ,,	1	2	14
	21 ,, 22 ,,	2	0	0

TALLOW.

Draft on all tallow 2 lbs. per cask.

St. Petersburg	—*Tare,*	12	lb.	per cwt.	
Odessa	,,	10½	,,	,,	
S. America	,,	16	,,	,,	on pipes.
,,	,,	20	,,	,,	half pipes.

LINSEED, SALT-PETRE AND NITRATE OF SODA.

Tare, actual.
No draft.

Find the price of 14 packs of wool, each weighing 2 cwt. 2 qr. 19 lb; *draft* 2 lb. per pack; *tare* 8 lb. per cwt.; at £3 19s. 4d. per cwt. net.

	cwt.	qr.	lb.			£	s.	d.
	2	2	19			3	19	4
			2	draft				30
	2	2	17			119	0	0
			14			15	17	4
	37	0	14		The price of 34 cwt.	134	17	4
8 lb. - $\frac{1}{14}$ of 1 cwt.	2	2	17	tare	1 qr. - $\frac{1}{4}$ -	0	19	10
					14 lb - $\frac{1}{2}$ -	0	9	11
	34	1	25	net weight.	7 lb. - $\frac{1}{2}$ -	0	4	11½
					2 lb. - $\frac{1}{7}$ -	0	1	5
					2 lb. - $\frac{1}{7}$ -	0	1	5
					Ans.	£136	14	10½

Find the price of 15½ large chests of tea, each weighing* 2 cwt. 2qr. 21 lb.; draft 3 lb. per chest; tare 12 lb. per cwt.; at £13 17s. 8d. per cwt.

---

* The questions throughout this work are intended to exercise the pupil, and are therefore proposed irrespective of the real weight of the article named; for instance, a chest of tea weighs from 70 lb. to 80 lb., but here it is given as weighing 2 cwt. 2 qr. 21 lb

	cwt.	qr.	lb.		£	s.	d.	
	2	2	21		13	17	8	
			3 draft				4	
	2	2	18		55	10	8	
			$15\frac{1}{2}$				9	
	39	3	18		499	16	0	
	1	1	9	2qr. - $\frac{1}{2}$ -	6	18	10	$\underbrace{16}$
				1qr. - $\frac{1}{2}$ -	3	9	5	
	41	0	27	7lb. - $\frac{1}{4}$ -	0	17	$4\frac{1}{4}$	$=\frac{4}{16}$
				1lb. - $\frac{1}{7}$ -	0	2	$5\frac{3}{4}$	$=\frac{12}{16}$
8 lb. - $\frac{1}{14}$ of 1 cwt.	2	3	$21\frac{13}{14}$	$\frac{3}{28}$ lb. -	0	0	$3\frac{3}{16}$	$=\frac{3}{16}$
4 lb. - $\frac{1}{2}$ of 1 cwt.	1	1	$24\frac{27}{28}$					
				Ans.	£511	4	$4\frac{3}{16}$	$=\frac{19}{16}=1\frac{3}{16}$
	4	1	$18\frac{25}{28}$ tare					
	36	3	$8\frac{3}{28}$ net weight.					

* Find the cost of $17\frac{1}{2}$ hhds. of sugar, each weighing 5 cwt. 2 qr. 26 lb.; draft 16 lb. per cwt.; tare 14 lb. per cwt.; at £2 16s. $10\frac{1}{2}$d. per cwt., net.

	cwt.	qr.	lb.		£	s.	d.	
	5	2	26		2	16	$10\frac{1}{4}$	
			$17\frac{1}{2}$				9	
	100	1	7		25	11	$10\frac{1}{2}$	
16 lb. - $\frac{1}{7}$	14	1	9 draft				8	
	85	3	26		204	15	0	
14 lb. - $\frac{1}{8}$	10	2	$27\frac{3}{4}$ tare		8	10	$7\frac{1}{2}$	$\underbrace{128}$
	75	0	$26\frac{1}{4}$ net		213	5	$7\frac{1}{2}$	$=\frac{64}{128}$
				16 lb. - $\frac{1}{7}$ -	0	8	$1\frac{1}{2}$	$=\frac{64}{128}$
				8 lb. - $\frac{1}{2}$ -	0	4	$0\frac{3}{4}$	$=\frac{96}{128}$
				2 lb. - $\frac{1}{4}$ -	0	1	$0\frac{3}{16}$	$=\frac{24}{128}$
				$\frac{1}{4}$ lb. - $\frac{1}{8}$ -	0	0	$1\frac{67}{128}$	$=\frac{67}{128}$
				Ans.	£213	18	$11\frac{59}{128}$	$=\frac{315}{128}=2\frac{59}{128}$.

---

* The examples and exercises introduced, are not given with the view of illustrating the prevalent deductions in articles of commerce, but rather of exercising the arithmetical abilities of the pupil.

EXERCISES 35.

1. Find the cost of 3 hhds. of sugar, weighing 10 cwt. 2 qr. 18 lb., 13 cwt. 1 qr. 14 lb., and 9 cwt. 3 qr. 10 lb.; tare 1 qr. 10 lb. per hhd.; at £3 14s. 8d. per cwt. net.
2. Bought 20 hhds. of tobacco, each weighing 14 cwt. 2 qrs. 16 lb.; tare 14 lb. per cwt.; at £19 1s. 9d. per cwt. net; what is the cost?
3. Bought 42 casks of madder, each weighing 9 cwt. 1 qr. 22 lb. gross; tare 12 lb per cwt.; at £1 19s. 8d. per cwt. net; what is the cost?
4. Bought 10½ casks of raisins, each weighing 8 cwt. 3 qr. 20 lb. gross; tare 16 lb. per cwt.; at £5 7s. 4d. per cwt. net; what is the cost?
5. Bought 16 casks of currants, each containing 13 cwt. 1 qr. 10 lb. gross; tare 17½ lb. per cwt.; at £6 7s. 2d. per cwt. net; what is the cost?
6. Find the cost of 36 hhds. of sugar, each weighing 13 cwt. 3 qr. 20 lb. gross; tare 21 lb. per cwt; at £2 7s. 10d. per cwt. net.
7. Purchased 72 hhds. of sugar, each weighing 7 cwt. 3 qr. 9 lb. gross; tare 9 lb. per cwt.; at £2 17s. 9d. the cwt. net; find the cost.
8. Bought 35 hhds. of tobacco, each weighing 7 cwt. 1 qr. 8 lb.; tare 7 lb. per cwt.; at £6 7s. 3d. the cwt. net; find the cost.
9. Bought 5 packs of wool, weighing as follows:—5 cwt. 2 qr. 13 lb.; 6 cwt. 3 qr. 19 lb.; 5 cwt. 1 qr. 23 lb.; 7 cwt. 2 qr. 15 lb.; and 6 cwt. 3 qr. 19 lb.; draft 12 lb. per cwt.; tare 4 lb. per 104 lb.; at 11 guineas per cwt. net, and paid in cash 79¾ guineas; how much is still due?
10. Bought 5 casks of madder, weighing 8·675 cwt., 6·4835 cwt., 7·6945 cwt., 8·305 cwt., and 6·05 cwt.; draft 12 lb. per cwt.; tare 4 lb. per 104 lb.; at £3 13s. 6d. per cwt. net; what is the cost?
11. Find the price of 6 bales of pimento, weighing 2 cwt. 3 qr. 7 lb. 3 cwt. 1 qr. 19 lb., 2 cwt. 2qr. 18 lb., 3 cwt. 3 qr. 17 lb., 2 cwt. 1 qr. 24 lb., and 3 cwt. 2 qr. 15 lb. gross; tare 17½ lb. per cwt.; tret 4 lb. per 104 lb.; at £11 13s. 4d. per cwt. net.
12. Required the value of 3 casks of oil weighing 11 cwt. 1 qr. 7 lb., 10 cwt. 2 qr. 8 lb. and 10 cwt. 3 qr. 21 lb., tare, 18 lb. per cwt., at £50 8s. per tun of 252 gallons—a gallon weighing 9 lb.

13. Find the cost of 18½ hhds. of sugar, each weighing 15½ cwt. 18 lb. gross; tare, 18 lb. per cwt.; at £5 15s. 6d. per cwt. net.

14. Required the net weight and cost of 224 cwt. 3 qrs. 20 lb. of tobacco; tare, 25 cwt. 3 qr.; tret, 4 lb. on every 104 lb.; draft, 2 lb. on every 3 cwt.; at £3 5s. 4d. per cwt. net.

15. Find the net weight and cost of 10 bales of cotton, each weighing 2 cwt. 3 qr. 15 lb.; draft, 2¾ lb. per bale; tret, 4 lb. on every 104 lb., at £3 18s. 6d. per cwt. net.

16. Find the price of 11 bales of cotton, each weighing 3 cwt 2 qrs. 18 lb.; draft, 14 lb. per cwt.; tare, 4 lb. per 104 lb.; at £2 7s. 9d. per cwt. net.

A clerk having lost the weighing notes of 11 bales of cotton, recollected that the draft was 14 lb. per cwt.; the tare 4 lb. per 104 lb.: and the net weight 33 cwt. 3 qr. $14\frac{49}{104}$ lb. What was the average gross weight of each bale?

		cwt.	qr.	lb.	
		83	3	$14\frac{49}{104}$	net weight.
$\frac{1}{25}$	- -	1	1	$11\frac{81}{104}$	tare.
		85	0	26¼	the weight ofter the draft is deducted.
$\frac{1}{7}$	- -	5	0	3¾	draft.
	11)	40	1	2	= gross weight.
	Ans.	3	2	18	= the average weight of each bale.

17. The net weight of 19 boxes of tobacco was 48 cwt. 0 qr. 10½ lb., and the tare was 14 lb. per cwt.; what was the gross weight of each box?

INTEREST.

106. INTEREST is the sum paid for the use or loan of money, and is reckoned at a certain rate* on £100 for one year.

---

* The rate of interest varies in amount, according to the nature of the security given for the loan, the state of the money-market, and the pressing necessity to contract the loan. The Bank of England rates were as follows:—

In	1845	-	2½ per cent.
"	1847	-	5 to 7 per cent.
"	1851	-	3 per cent.
Oct. 15,	1857	-	7 "
" 19,	"	-	8 "
Nov. 5,	"	-	9 "
" 9,	"	-	10 "

(1). The sum lent is called the *principal.*

(2). The consideration given for the use of a £100 for one year is called the *rate per cent.*

(3). The principal increased by the interest is termed the *amount.*

All questions connected with interest are solved by proportion: when the time is one year, by simple proportion; when more or less than one year, by compound proportion.

EXAMPLES.

(1.) Find the interest of £450 for one year, at 6 per cent. per annum,

As £100 : £450 : : £6 : x = £27 interest.

(2). Find the interest of £750 for 3 years, at 5 per cent. per annum.

```
As £100 : £750 : : £5 : x = £37 10s. the interest for 1 year.
                               3
                           --------
                  Ans. £112 10 the interest for 3 years.
```

(3.). Find the interest of £6159 7s. 6d. for 5½ years, at 4 per cent. per annum.

```
       £   s. d.              £   s.  d.
As 100 : 6159 7 6 : : 4 : x = 246  7  6 the interest for one year.
                                      5
                            -------------
                             1231 17  6 interest for 5 years.
                              123  3  9   ,,   for half a year.
                            -------------
                  Ans. £1355  1  3   ,,   for 5½ years.
```

(4). What is the interest on £1231 17s. 6d. for 3 years and 9 months, at 5 per cent. per annum.

```
  £      £   s.  d.              £  s.  d.
As 100 : 1231 17  6 : : 5 : x = 61 11 10½ = interest for 1 year
                  5                    3
     ---------------            -----------
     £61,59  7   6  m.          £184 15  7½ = interest for 3 years
        20          6 - ½ yr.    30 15 11¼ =    ,,    for 6 mo
     -------        3 - ½ 6m.    15  7 11⅝ =    ,,    for 3 mo.
      11,87                    ------------
        12                      £230 19  6⅜ =    ,,    for 3 years
     -------                                        9 months.
      10,50
         2
     -------
       1,00
```

When the Bank of England, on the 9th of November 1857, made their rate of discount 10 per cent., such was the panic, that the most ordinary operations of trade were becoming stagnant, which induced the Premier, Lord Palmerston on the 12th to write to the Governor of the Bank of England, authorising an issue of notes, if necessary, beyond the limits of circulation prescribed by the Act of 1844.

(5). Find the interest of £837 10s. for 2 years and 4 months, at $5\frac{3}{4}$d. per cent. per annum.

$$\begin{array}{lrrlrrrl}
 & £ & \text{s.} & & £ & \text{s.} & \text{d.} & \\
\text{As } 1,00 : & 837 & 10 :: 5\frac{3}{4} : x = & & 48 & 3 & 1\frac{1}{2} & = \text{the interest for 1 year.} \\
 & & 5\frac{3}{4} & & & & 2 & \\
\hline
 & 4187 & 10 & & 96 & 6 & 3 & = \text{the interest for 2 years.} \\
 & 628 & 2\ \ 6 & 4 \text{-} \frac{1}{3} \text{ 1 yr.} & 16 & 1 & 0\frac{1}{2} & = \text{ ,, for 4 mths.} \\
\hline
1,00) & 48,15 & 12\ \ 6 & & £112 & 7 & 3\frac{1}{2} & \textit{Ans.} \\
\hline
 & £48 & 3\ \ 1\frac{1}{2} & & & & &
\end{array}$$

(6). Find the interest of £6159 7s. 6d., from the 19th of April to the 24th of November, at 4 per cent.

$$\begin{array}{lrrrl}
 & £ & \text{s.} & \text{d.} & £ \\
\text{As } £1,00 : & 6159 & 7 & 6 :: 4 : x = & £246\ 7\text{s. } 6\text{d interest for 1 year.} \\
 & & & 4 & \\
\hline
 & 246,37 & 10 & 0 & \\
 & 20 & & & \\
\hline
 & 7,50 & & & \\
 & 12 & & & \\
\hline
 & 6,00 & & &
\end{array}$$

April 11 days,
May 31 ,,
June 30 ,,
July 31 ,,
Aug. 31 ,,
Sept. 30 ,,
Oct. 31 ,,
Nov. 24 ,,
219 ,,

As $\underset{5}{\cancel{365}}$ d. : $\underset{3}{\cancel{219}}$ d. : : £246 7s. 6d. : x = £147 16s. 6d. *Ans.*

$$\begin{array}{rrrr}
 & & & 3 \\
\hline
5) & 739 & 2 & 6 \\
\hline
 & £147 & 16 & 6
\end{array}
\qquad
\begin{array}{rrrrl}
 & £ & \text{s.} & \text{d.} & \\
365) & 246 & 7 & 6 & \\
\hline
 & & 13 & 6 & \text{the interest of 1 day.} \\
 & & 219 & & \\
\hline
 & £147 & 16 & 6 & \text{the interest for 219 days.}
\end{array}$$

Otherwise, by compound Proportion,

As £100 × 365 ; 6159 7 6 × 219 : : 4 : x = £147 16s. 6d. *Ans.* (£ s. d.)

$\frac{6159\ 7\ 6 \times 219 \times 4}{365 \times 100}$. If the numerator and denominator be multiplied by 2, it becomes* $\frac{6159\ 7\ 6 \times 219 \times 8}{73000} = x =$ £147 16s. 6d.

---

* That is, $\frac{\text{Principal,} \times \text{twice rate,} \times \text{No. of days.}}{73000}$ = Interest.

£ s. d.
That is, 6159 7 6
8

49275 0 0
219

73000) 10791225 0 0 (147
73000

349122
292000

571225
511000

60225
20

73000) 1204500 (16
73000

474500
438000

36500
12

73000) 438000 (6
438000

The process is, to multiply the principal by double the rate, and that product by the number of days; then to divide the result by 73000, for the interest.

An estate of £280 per annum, to continue for ever, is sold so as to pay the purchaser 4 per cent.; what is its worth in ready money?

As £4 : £280 : : 100 : x = £7000, the purchase money.

EXERCISES 36.

1. Find the interest of £2500, at 5 per cent., for 1 year.
2. Find the interest of £295 5s. for 2 years, at 5 per cent.
3. Find the interest of £1050, for 3 years 2 months, at 5 per cent.
4. Find the interest of £245 12s. 6d., for 3 years, at 5 per cent.
5. What is the interest of £150 5s., for 2 years 6 months, at 6 per cent.?
6. Find the amount of £500, for 4¼ years, at 4¾ per cent.

7. Find the interest of £425 16s. 8d., for 2 years 6 months and 73 days, at 5 per cent.
8. What is the interest of £6159 7s. 6d., for 3 years 10 months, at 5 per cent.?
9. Find the interest of £1539 16s. $10\frac{1}{2}$d., for 1 year, and from the 15th of March to the 19th of December, at 4 per cent.
10. Find the interest of £3079 13s. 9d. from the 5th of March to the 23rd of December, at 5 per cent.
11. Find the interest of £1946 13s. 4d., for 1 year, and from the 10th of March to the 19th of January following, at 5 per cent.
12. Find the interest of £9239 1s. 3d. from the 3rd of March to the 17th of December, at 5 per cent.
13. What is the interest of £18478 2s. 6d. for 3 years 4 months, and from the 9th of April to the 19th of December, at 5 per cent.?
14. Find the interest of £1539 16s. $10\frac{1}{2}$d. for 2 years 4 months, and from the 7th of April to the 9th of October, at 4 per cent.
15. What is the interest of £6159 7s. 6d. for 3 years 4 months, and from the 15th of March to the 19th of November, at 5 per cent.?
16. Find the interest of £480 16s. 8d., from the 7th of April to the 9th of September, at 4 per cent.
17. What is the interest of £773 6s. 8d., from June 9th to December 6th, at 4 per cent.?
18. Find the interest of £387 17s. 9d., from the 3rd of April to the 17th of December, at 5 per cent.
19. What is the interest of £378 15s. 9d., for 2 years 5 months, and from the 3rd of March to the 25th of July, at 5 per cent.?
20. Find the interest of £387 13s. 9d., for 3 years 7 months, and from the 8th of April to the 13th of August, at 5 per cent.
21. Find the interest of £375 6s. 9d., for 2 years, and from the 9th of May to the 17th of September, at 5 per cent.
22. What is the interest of $118\frac{5}{8}$ guineas, for 2 years 10 months and 23 days, at $4\frac{3}{4}$ per cent.?
23. What is the interest of $917\frac{13}{16}$ guineas, from the 8th of May to the 17th of October, at $4\frac{3}{4}$ per cent.?
24. What is the interest of £4802 11s. 8d. for 1 year, and from the 11th of July to the 4th of December, at $3\frac{3}{8}$ per cent.?
25. If £315 15s. pay £15 15s. 9d. interest yearly, what rate per cent. is it?

26. Purchased a house for £600, which pays me £45 yearly rent; what rate per cent. is it paying?

As £600 : £100 : : £45 rent or interest : $x = 7\frac{1}{2}$.

27.* Bought a house for £750 which pays £60 a year rent, what rate per cent does it pay, and how many year's purchase did it cost?

28. Find the interest of £324·45 + 98 35 guineas + 19·65s. from the 4th of April to the 17th of November, at 4 per cent.

29. Out of a purse containing $64\frac{7}{9}$ guineas + £$18\frac{4}{7}$ + $7\frac{9}{14}$ crowns + $3\frac{3}{8}$s. + $5\frac{5}{6}$d. there are paid $21\frac{17}{18}$ guineas + £$13\frac{5}{7}$ + $16\frac{13}{14}$ crowns + $17\frac{4}{5}$s. + $5\frac{3}{4}$d.; find the interest of the remainder for 146 days at 5 per cent.

30. In what time would £500 amount to £675 at 5 per cent?

As £100 : £500 : : 5 : $x =$ £25 interest for 1 year.

675
500

As £25 : 175 entire interest : : 1 year : $x = 7$ years.

At what rate per cent. would £350 amount to £395 10s. in $3\frac{1}{4}$ years.

£395 10s.
350

£45 10s. = the entire interest.

and £45 10s. ÷ $3\frac{1}{4} = 14$, the interest for 1 year.

As £350 : £100 : : £14 : $x =$ £4 = rate per cent.

In what time would 10s. amount to £1 at 4 per cent.

As £100 : 10s. : : £4 : $x = \frac{2}{5}$s., the interest of 1 year.

Interest. Interest. Year.
As $\frac{2}{5}$s. : 10s. : : 1 : $x = 25$ years.

To what date must £1539 16s. $10\frac{1}{2}$d., lent on the 7th of April at 4 per cent., be left at interest to amount to £1571 1s. 3d.?

As £100 : £1539 16s. $10\frac{1}{2}$d. : : £4 : $x =$ £61 11s. $10\frac{1}{2}$d. Int. 1 year.

£1571 1 3
1539 16 $10\frac{1}{2}$

As £61 11s. $10\frac{1}{2}$d. : £31 4 $4\frac{1}{2}$ Interest. : : 365 : $x = 185$ days.

---

* As £60 is realized in one year, £750 will be realized in $12\frac{1}{2}$ years, and property worth £750, yielding an annual rental of £60, is said to be worth $12\frac{1}{2}$ year's rental or purchase.

31. At what rate per cent. would £400 amount to £440, in 1 year 8 months?

32. How long must £2053 2s. 6d. be lent, at 4 per cent., to pay £50 12s. 6d. interest?

33. For how many months must £583 17s. 10d., at 5 per cent., bear interest, to yield £111 18s. $3\frac{1}{80}$d. interest?

34. Lent at interest on the 4th of March 1858, £935 12s. 6d., at 5 per cent. per annum, and received for interest £74 1s. $7\frac{33}{73}$d.; what time was it bearing interest, and when did this interest become due?

At what rate per cent. will £1140 12s. 6d., from the 8th of March to the 19th of November, give £32 interest?

As 256 d. : 365 d. : : £32 : x = £45 12s. 6d. interest for a year.

As £1140 12s. 6d. : £100 : : £45 12s. 6d. : x = 4 per cent. *Ans.*

35. How long must £2053 13s. 9d. bear interest at 5 per cent., to give £67 10s. $4\frac{33}{73}$d. interest?

36. What sum must be lent for 181 days, at 5 per cent., to give £76 7s. $2\frac{1}{4}$d. interest?

As 181d. : 365d. : : £76 7s. $2\frac{1}{4}$d. : x = £153 19s. $8\frac{1}{4}$d. = 1 yr. int.

As £5 : £153 19s. $8\frac{1}{4}$d. : : £100 : x = £3079 13s. 9d. *Ans.*

37. At what rate per cent. will £480 16s. 8d., from the 7th of January to the 9th of September following, give £25 16s. $4\frac{60}{73}$d.?

38. Find the interest on the sale of $385\frac{18}{196}$ barrels of flour, at £1 9s. 8d. per barrel, left unpaid from the 5th of March to the 30th of August, at 5 per cent.

39. What is the interest of £875 12s. 9d., from the 14th of May to the 18th of October, at $5\frac{1}{2}$ per cent.?

40. Lent at interest at 5 per cent. for 228 days, the sale of 89 acres 3 roods 33 perches, at £7 16s. 6d. per acre; what is the interest due?

41. Find the interest of £835 3s. $10\frac{1}{2}$d. for 2 years 8 months, and from the 3rd of April to the 8th of September, at 5 per cent.

42. What sum at 5 per cent. will yield an annuity of £19 5s. $9\frac{3}{4}$d.?

43. In what time will £1539 16s. $10\frac{1}{2}$d. amount to £1583 0s. $10\frac{1}{2}$d., at 4 per cent.?

44. How long must £1539 16s. $10\frac{1}{2}$d. lent on the 1st of January, bear interest, to yield £160 2s. $10\frac{1}{2}$d.

Henry Smith's account with the Union Bank, in which he receives $2\frac{1}{2}$ per cent. for lodgments, and pays 5 per cent. on sums overdrawn.

1861.	HENRY SMITH,	*Dr.*	1861.	*Contra.*	*Cr.*
Mar. 8	To cash paid	£6750 0 0	June 10	By cash lodged	£1500 0 0
June 13	" " "	3700 0 0	Feb. 6	" " "	3700 0 0
Oct. 15	" " "	3960 0 0	May 14	" " "	3350 0 0
			Sept. 12	" " "	4700 0 0
			Decr. 13	" " "	1750 0 0

The account is closed on the 31st of December, 1861; is the interest for or against Smith?

$$
\begin{array}{lrcll}
 & & \text{days} & & \\
\text{As } 365 \times 100 : & (1500 \times & 27 & :: 2\frac{1}{2}) = 101250 & \text{product.} \\
 & (5200 \times & 30 & :: 2\frac{1}{2}) = 390000 & ,, \\
 & (1800 \times & 30 & :: 2\frac{1}{2}) = 135000 & ,, \\
 & (2800 \times & 33 & :: 2\frac{1}{2}) = 231000 & ,, \\
 & (590 \times & 18 & :: 2\frac{1}{2}) = 26550 & ,, \\
\hline
 & & & 883800 &
\end{array}
$$

And $£\frac{883800}{36500}$ = the interest in favour of Smith.

$$
\begin{array}{lrcll}
 & & \text{days} & & \\
\text{As } 365 \times 100 : & (1550 \times & 67 & :: 5) = 519250 & \text{product.} \\
 & 1900 \times & 91 & :: 5 = 864500 & ,, \\
 & 1160 \times & 59 & :: 5 = 342200 & ,, \\
\hline
 & & & 1725950 &
\end{array}
$$

And $£\frac{1725950}{36500}$ = the interest against Smith.

$£\frac{1725950 - 883800}{36500} = \frac{842150}{36500} =$ £23 1s. $5\frac{31}{73}$d., the balance of interest against Smith.

45. Lent at interest £538 13s. 7d. for 3 years 5 months, and from the 2nd of March to the 15th of October, at $5\frac{1}{2}$ per cent.; what is the amount of interest to be received?

46. Lent on the 10th of March, 1858, £865 13s. 8d., at 4 per cent.; how long must it be left at interest to receive £51 2s. $8\frac{2736}{9125}$d?

47. G. Hartley's account with Heywood's Bank, in which he receives $2\frac{1}{2}$ per cent. on sums lodged, and pays 5 per cent. on sums overdrawn.

1859.	G. HARTLEY,	*Dr.*		*Contra.*	*Cr.*
Mar. 3	To cash paid	£3375 0 0	Jan. 5	By cash lodged	£750 0 0
June 8	" " "	1850 0 0	Feb. 1	" " "	1850 0 0
Oct. 10	" " "	1980 0 0	May 9	" " "	1675 0 0
			Sept. 7	" " "	2350 0 0
			Decr. 8	" " "	875 0 0

The account was closed December 31; whether is the balance of interest for or against Hartley, and to what extent?

48. What is the interest of £837 18s. 9d., for 3 years 5 months, and from the 10th of March to the 18th of September, at 5 per cent.?

49. What would be the interest on £14875 3s. 6d., in the $3\frac{1}{2}$ per cents., from the 1st of April to the 10th of October?

50. Bought an estate for £4619 10s. $7\frac{1}{2}$d, which yields an annual rental of £184 15s. $7\frac{1}{2}$d.; what rate per cent. does the purchase money realize, and how many year's purchase is it?

51. What sum must be lent at 5 per cent. interest, to yield an annual income equal to the rent of 58 a. 2 r. 37 p., at £2 18s. 9d. per acre?

52. In what time will £140 double itself, at 5 per cent.?

53. In what time will £150 double itself, at 4 per cent.?

What sum must be lent at interest, on the 9th of March, 1856, to amount to £1232 18s., on the 26th December, 1859, at $3\frac{3}{4}$ per cent.?

As 1 y. : $3\frac{4}{5}$ y. : : £$3\frac{3}{4}$: $x =$ £$14\frac{1}{4}$ interest. From the 9th of March, 1856, to 26th December, 1859, gives 3 years 292 days, or $3\frac{4}{5}$ years.

£$14\frac{1}{4}$
100
———

As $114\frac{1}{4}$: £1232 18s. : : £100 : $x =$ £1080 *Ans.*

What sum must be lent at 4 per cent. interest, from the 6th April to the 17th of November, to amount to £2103 15s.?

From the 6th of April, to the 17th of November, gives 225 days.

As 365 d. : 225 d. : : £4 : $x = 2\frac{34}{73}$ interest.

£$2\frac{34}{73}$
100
———

As $103\frac{34}{73}$: £2103 15s. : : £100 : $x =$ £2053 2s. 6d. the *Ans.*

54. What sum should be lent, at 4 per cent. interest, to amount to £1172 12s. 6d., from the 8th of March to the 19th of November?

55. What sum, at 5 per cent. interest, must be lent from the 17th of March to the 9th of July, to receive £5 18s. $3\frac{24}{365}$?

At what rate per cent. will £125 double itself in $12\frac{1}{2}$ years?

As $12\frac{1}{2}$ years : 1 year : : £125 interest : $x =$ £10 interest for a year.

As £125 : £100 : £10 : $x =$ £8 the rate per cent.

56. At what rate per cent. will £120 double itself in 15 years?

57. What sum of money must be lent for 5 years, at 5 per cent., to produce the same interest as £3079 13s. 9d. for 4 years, at 4 per cent?

58. At what rate per cent will £1675 13s. 9d. yield £44 19s. $9\frac{324}{365}$d. interest, from the 4th of April to the 17th of October?

59. How long must £589 17s. 9d. be left at interest, at 5 per cent., to receive £21 13s. $1\frac{866}{1825}$d. interest?

60. An estate being purchased, realizes $3\frac{3}{4}$ per cent. when let for £275 a-year; what was the purchase money?

61. Find the interest of £689 15s. 6d. for 2 years 6 months, and from the 3rd of May to the 17th of September, at $3\frac{3}{4}$ per cent. per annum.

62. John Davis & Co.'s account with the Westminster Bank:—for lodgments they receive $2\frac{1}{2}$ per cent., and for sums overdrawn they pay 5 per cent. The account, which stands as follows, is closed on the 25th of July, 1860. What is the sum due, and to whom?

JOHN DAVIS & CO'S. ACCOUNT.

1860.		£	s.	d.	1860.		£	s.	d.
April 6	Drawn per receipt	584	0	0	Jan. 6	Lodged per receipt.	146	0	0
May 26	" " "	730	0	0	" 26	" " "	292	0	0
					Feb. 25	" " "	438	0	0
					July 25	" " "	876	0	0

EXAMPLE.

Borrowed from the Bank of Liverpool, on the 3rd of January, 1859, the sum of £7000, at 5 per cent.; of which I paid £2600 on the 7th of March; on the 12th of June, £2700; on the 10th of August, £600; and the remainder on the 20th of September; what is the entire interest?

		£ £ £	Interest.
		As 100 : 7000 : : 5 : x = 350	
Jan. 28	June 18	d. d. £	
Feb. 28	July 31	As 365 : 63 : : 350 : y =	$£60\frac{30}{73}$
Mar. 7	Aug. 10		
—	—	£ £ £	
63	59	As 100 : 4400 : : 5 : x = 220	
		d. d. £	
		As 365 : 97 : : 220 : y =	$£58\frac{34}{73}$
Mar. 24	Aug. 21		
April 30	Sept. 20	£ £ £	
May 31	—	As 100 : 1700 : : 5 : x = 85	
June 12	41	d. d. £	
—		As 365 : 59 : : 85 : y =	$£13\frac{54}{73}$
97			

£7000 principal
2600
4400 principal
2700
1700 principal.
600
Last 1100 principal.

$$\begin{array}{l} \text{As } \overset{\pounds}{100} : \overset{\pounds}{1100} :: \overset{\pounds}{5} : x = 55 \\ \text{As } \overset{\text{d.}}{365} : \overset{\text{d.}}{41} :: 55 : y = \pounds 6\tfrac{13}{73} \end{array}$$

Entire interest, $\pounds 138\frac{58}{73}$
or, £138 15s. $10\frac{50}{73}$d.

Otherwise,

$$\begin{array}{l} \text{As } \overset{\pounds}{100} \times 365 : 7000 \times 63 :: \overset{\pounds}{5} : x = \overset{\pounds}{60\tfrac{30}{73}} \text{ Interest.} \\ \text{As } 100 \times 365 : 4400 \times 97 :: 5 : x = 58\tfrac{34}{73} \\ \text{As } 100 \times 365 : 1700 \times 59 :: 5 : x = 13\tfrac{54}{73} \\ \text{As } 100 \times 365 : 1100 \times 41 :: 5 : x = 6\tfrac{13}{73} \end{array}$$

$$\textit{Ans. } \pounds 138\tfrac{58}{73} = \overset{\pounds}{138}\ \overset{\text{s.}}{15}\ \overset{\text{d.}}{10\tfrac{50}{73}}$$

Otherwise,

$$\begin{array}{r} \pounds 7000 \times 63 = 441000 \\ 4400 \times 97 = 426800 \\ 1700 \times 59 = 100300 \\ 1100 \times 41 = 45100 \end{array}$$

As $365 \times 100 : 1013200 :: 5 : x$.

$$\frac{1013200 \times 5}{36500}, \textit{ or, }$$

$$\frac{1013200 \times 10}{73000} = x = \pounds 138\ 15\text{s. } 10\tfrac{50}{73}\text{d.}$$

63. Lent £300 on the 13th of May, 1854, for one year at 5 per cent., on condition to pay off by instalments part of the principal; on the 9th of July £100 are received, and on the 17th of Sept., £75; how much principal and interest ought to have been received at the expiration of the year?

64. Lent £2000 Jan. 20, 1858, of which £350 were paid April 7th, £690 Sept. 28th, £420 Dec. 18th, and the balance, April 7th, 1859, at 4½ per cent.

65. If an estate of £1100 5s. yearly rent be purchased so as to pay 3½ per cent. on the purchase money, what is the amount?

Compound Interest.

107. If the interest be added yearly to the principal, and the sum accruing be considered as a new principal, the interest is said to be *compound*.

EXAMPLES.

(1.) Find the compound interest of £500, for 4 years, at 5 per cent.

£ £ s. d. £ s. d.
As 100 : 500 0 0 : : 5 : x = 25 0 0
25
As 100 : 525 0 0 : : 5 : x = 26 5 0
26 5 0
As 100 : 551 5 0 : : 5 : x = 27 11 3
27 11 3
As 100 : 578 16 3 : : 5 : x = 28 18 9¾
£107 15 0¾ = the compound interest for 4 years.

(2.) What is the compound interest on £640, for 5 years, at 5 per cent.?

£ £ s. d. £ s. d.
As 100 : 640 0 0 : : 5 : x = 32 0 0
32
As 100 : 672 0 0 : : 5 : x = 33 12 0
33 12 0
As 100 : 705 12 0 : : 5 : x = 35 5 7·2
35 5 7·2
As 100 : 740 17 7·2 : : 5 : x = 37 0 10·56
37 0 10·56
As 100 : 777 18 5·76 : : 5 : x = 38 17 11·088
£176 16 4·848 the compound interest.

Example (1) *otherwise*,

£ £ £
As 100 : 1 : : 105 : x = 1·05 = amt. of £1 in 1 yr.
As 100 : 1·05 : : 105 : x = 1·1025 or $1{\cdot}05^2$ „ £1 in 2 yrs.
As 100 : 1·1025 : : 105 : x = 1·157625 or $1{\cdot}05^3$ „ £1 in 3 yrs.
As 100 : 1·157625 : : 105 : x = 1·21550625 or $1{\cdot}05^4$ „ £1 in 4 yrs.

Now since the amount of £1 for 4 years, at 5 per cent, compound interest = 1·21550625; hence the amount of £500 for 4 years at 5 per cent. compound interest, will be 1·21550625 × 500 = 607·758125 or £607 15s. 0¾d., less £500, gives the interest, as above, £107 15s. 0¾d.

Example (2) *otherwise*,

$(1·05)^5$ = 1·2762815625 = the amount of £1 for 5 years, at 5 per cent. compound interest.
640

816·8202
640

£176·8202 = £176 16s. 4·848d. as above.

To avoid trouble, a table giving the amount of £1, at 3, 4, 5, and 6 per cent. compound interest, is constructed to 40 years, according to the plan educed in Examples (1) and (2) worked otherwise. Thus, Table I. is formed by raising 1·03, 1·04, 1·05, &c., to every power up to 40.

For instance,

opposite 1 year, and under 5 per cent. is 1·05
„ 2 „ „ „ 1·1025 = $(1·05)^2$
„ 3 „ „ „ 1·157625 = $(1·05)^3$
„ 4 „ „ „ 1·215506 = $(1·05)^4$

(3.) Find the compound interest of £500 for 3 years, at 4 per cent.

By Table I.

amount of £1 for 3 yrs., at 4 per cent. = 1·124864 that is $(104)^3$
500

562·432 = amount
500

Ans. £62·432 = £62 8s. 7·68d. int.

EXERCISES 37.

1. What is the amount of £100, in 5 years, at 4 per cent.?
2. Find the amount of £150, for 4 years, at 2½ per cent.
3. What is the compound interest of £450, for 5 years, at 4 per cent.?
4. Find the compound interest of £150 10s., for 10 years, at 5 per cent.
5 Find the compound interest of £400, for 6 years, at 5 per cent.

6. What is the amount of £500, for 4 years, at 4 per cent. compound interest?
7. Find the compound interest of £500, for 5 years, at 4 per cent.
8. If a person lends £7000, at 5 per cent. per annum, compound interest, and allows the interest to remain with the borrower, except £240, which he takes annually for household expenditure, how much will be due to him at the end of 8 years?
9. Find the difference between the simple and compound interest of £500, for 8 years, at 5 per cent.

EXAMPLES.

(1.) Find the amount of £800, for 50 years at 4 per cent. compound interest.

The amount of £1, by Table I, for 40 years = 4·801020
800
£1 in 10 years amounts to 1·480244
£3840·816 = what 800 will amount to in 40 years.

As £1 : £3840·816 : : 1·480244 : x = 5685·344889104 = the amount of £800 for 50 years. Or,

* 4·801020 × 1·480244 × 800 = £5685·344889104.

(2.) For a man just 80 years of age, the sum of £5 was invested at his birth, at 5 per cent. compound interest; what sum will be now due to him?

The amount of £1 for 40 years, by Table I. = 7·039989.

And the amount for 80 years = 7·039989 × 7·039989 = 49·561445.

And 49·561445 × 5 = 247·807225 = £247 16s. 1¾d.

† (3.) What is the amount of one farthing for 500 years, at 3 per cent. compound interest?

By Table I. the amount of £1 at 3 per cent., for 40 years = 3·262038.

---

* *When the amount of a sum to be determined, is beyond the limits of the table, as in finding the amount of* £800 *for* 50 *years, at* 4 *per cent.*—The amount of £1 for 40 years, is by Table I, £4·801020, and for 10 years £1·480244. Then by proportion, As £1 capital : £4·801020 capital, : : 1·480244 amount of £1 in 10 years : x, the entire amount of £1 for 50 years = 7·10668104888, which being multiplied by 800, gives £5685·344839104, the required sum.

† This and similar questions can be more readily solved by the aid of *Logarithms* thus: ¼d. = £·001041$\dot{6}$.

The amount of one farthing for 500 years = $(1·03)^{500}$ × ·001041$\dot{6}$
= 500 × log. 1·03 + log. ·001041$\dot{6}$ = £2730 8s. 3¾d.

The amount of £1 for 480 years = (3·262038) multiplied by itself 12 times, or $(3{\cdot}262038)^{12}$ and the amount of £1 for 20 years = 1·806111.

Hence the amount of £1 for 500 years = $(3{\cdot}262038)^{12}$ × 1·806111, which result being multiplied by £·0010416̇ the decimal of a farthing, gives the required amount £2730 8s. 3½d.

DISCOUNT.

108. Discount is the abatement made for receiving a sum of money before it is due, and is such, that the sum received, if put to interest at the given rate, would amount at the end of the time to the debt. The sum thus received is called the *present worth*.

Thus, if A has to pay B £420 at the end of a year, and the debt be discharged at once, B should receive a less sum than £420; it ought to be such a sum as if put to interest for one year, at the current rate, say 5 per cent., will amount to £420. It can be found thus:—

Since £100 at 5 per cent. amounts to £105 in 1 year, it follows that £105, due 1 year hence, is equivalent to £100 ready money.

Then, As £105 : £420 : : £100 : x = £400 the ready money value, or *present worth*—making the discount £20.

This method of calculating the present worth and discount, although the true one, is not that employed by bankers and merchants. The custom with them is to charge as discount, the interest on £420, which at 5 per cent. gives £21 discount; so that the banker's discount exceeds the true discount in this case by £1, and the sum in ready money which the bank pays, would be £399.

In practice, therefore, questions in discount are merely questions in simple interest.

Discount chiefly occurs in cashing bills of exchange and promissory notes. If any such bill is presented to a banker, and he accepts the party promising payment, or the party holding the bill, he will discount it by paying its present value at once, which is the whole amount less the interest for the time that must elapse before it becomes due.

In the United Kingdom, 3 days, called *days of grace*, are allowed after the time that a bill is due, before it is legally due. Thus, a three month's bill, dated the 1st of April, does not become due till the 4th of July, and the interest or discount is calculated to the 4th of July.

TRADE DISCOUNT.

109. If a tradesman selling goods, whose net cash price is fixed, has to sell them to customers requiring discount, then such a sum should be added to that fixed price, as will render it, after deducting the discount, the same as before.

To find the sum so increased.

Rule.—To the net sum add that fraction of it, which has the rate for numerator, and 100 less rate for denominator.

For 2 per cent.	or $\frac{1}{50}$,	Add	$\frac{2}{100-2}$	$=\frac{2}{98}$	$=\frac{1}{49}$
„ $2\frac{1}{2}$	„	$\frac{1}{40}$	„ $\frac{2\frac{1}{2}}{100-2\frac{1}{2}}$	$=\frac{5}{195}$	$=\frac{1}{39}$
„ 4	„	$\frac{1}{25}$	„ $\frac{4}{100-4}$	$=\frac{4}{96}$	$=\frac{1}{24}$
„ 5	„	$\frac{1}{20}$	„ $\frac{5}{100-5}$	$=\frac{5}{95}$	$=\frac{1}{19}$
„ 10	„	$\frac{1}{10}$	„ $\frac{10.}{100-10}$	$=\frac{10}{90}$	$=\frac{1}{9}$
„ $12\frac{1}{2}$	„	$\frac{1}{8}$	„ $\frac{12\frac{1}{2}}{100-12\frac{1}{2}}$	$=\frac{25}{175}$	$=\frac{1}{7}$
„ 20	„	$\frac{1}{5}$	„ $\frac{20}{100-20}$	$=\frac{20}{80}$	$=\frac{1}{4}$
„ 25	„	$\frac{1}{4}$	„ $\frac{25}{100-25}$	$=\frac{25}{75}$	$=\frac{1}{3}$
„ 30	„	$\frac{3}{10}$	„ $\frac{30}{100-30}$	$=\frac{30}{70}$	$=\frac{3}{7}$

EXAMPLE.

A manufacturer sells an equal quantity of goods, of the same kind and quality, to five different customers; the net cash selling price of each quality being fixed at £93 2s. and the discount allowed is as follows:—to A., 2 per cent.; to B., 4; to C., 5; to D., 10; and to E., 12½. What price must be charged to each, so that after deducting discount, the manufacturer may receive the net fixed sum of £93 2s.?

A.

	£	s.	d.
	93	2	0
$\frac{1}{49}$	1	18	0
Ans.	95	0	0
Proof. $\frac{1}{50}$	1	18	0

B.

	£	s.	d.
	93	2	0
$\frac{1}{24}$	3	17	7
	96	19	7
$\frac{1}{25}$	3	17	7

C.*

	£	s.	d.
	93	2	0
$\frac{1}{19}$	4	18	0
	98	0	0
$\frac{1}{20}$	4	18	0

D.

	£	s.	d.
	93	2	0
$\frac{1}{9}$	10	6	$10\frac{2}{3}$
Ans.	103	8	$10\frac{2}{3}$
Proof $\frac{1}{10}$	10	6	$10\frac{2}{3}$

E.

	£	s.	d.
	93	2	0
$\frac{1}{7}$	13	6	0
	106	8	0
$\frac{1}{8}$	13	6	0

EXERCISES 38A.

(4). When the net cash selling price of an article is a guinea, at what sum should it be sold, if 10 per cent. discount be allowed to the purchaser?

(2). What sum should be charged for cutlery, to allow 30 per cent. discount, when the net cash selling price of which is £4 7s. 6d.?

(3). When 25 per cent. discount is allowed on a diamond ring, whose net cash selling price would be £14 16s. 8d., what must be the selling price charged?

EXAMPLES.

(1.) What is the true and bank discount on 498, due 9 months hence, at 5 per cent.?

As 12 m : 9 m : : £5 : $x = 3\frac{3}{4}$.

As £$103\frac{3}{4}$: $3\frac{3}{4}$: : £498 : $y = 18$ the true discount.

As £100 : 498 : : 5 : $x =$ £24 18s.

As 12 m : 9 m : : £24 18s. : $y =$ £18 13s. 6d., the bank discount.

(2.) Find the true and bank discount of a bill of £968 14s. 6d., dated the 22nd of May, 1858, at 11 months, and discounted on the 29th of July, at 5 per cent.

The bill (including the 3 days of grace†) will be due on the 25th of April, 1859.

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* The discount on any sum, say at 5 per cent., is found by taking the $\frac{1}{20}$th of the proposed sum; that is, if the given sum be divided into 20 equal portions this will be one of them, which when deducted from it, will render the remainder equivalent to 19 portions; and hence if this remainder be divided by 19, one portion, or the discount is obtained.

† In every instance where discount is calculated for days, 3 days are added for *days of grace.*

As £100 : £968 14s. 6d. : : £5 : $x =$ £48 8s. $8\frac{7}{10}$d., interest for 1 yr.

As 365 d : 270 d : : £48 8s. $8\frac{7}{10}$d. : $y =$ £35 16s. $7\frac{34}{365}$d., bank disc.

As 365 d. : 270 d. : : 5 : $x = 3\frac{51}{73}$.

As $103\frac{51}{73}$: $3\frac{51}{73}$: : £968 14s. 6d. : $y =$ £34 11s. $0\frac{294}{757}$, true discount.

From £968 14 6 . . . £968 14 6
Take 35 16 $7\frac{34}{365}$ 34 11 $0\frac{294}{757}$

£932 17 $10\frac{331}{365}$, bank payment. £934 3 $5\frac{463}{757}$, true paymt.

(3.) What is the true and bank discount on £873 10s., for 292 days, at 5 per cent.?

As £100 : £873 10s. : : 5 : $x =$ £43 13s. 6d.

£ s. d.
As $\cancel{365}$: $\cancel{292}$: : £43 13s. 6d. : $y =$ 34 18 $9\frac{3}{5}$, the bank discount.
5 : 4 4

5) 174 14 0

34 18 $9\frac{3}{5}$

As 365 d. : 292 d. : : 5 : $x = 4$.

£ s. d.
As $\cancel{104}$: $\cancel{4}$: : 873 10 0 : $y =$ £33 11 $1\frac{1}{13}$d., the true discount.
26 1 1

26) 873 10 0

33 11 $1\frac{1}{13}$.

What is the present worth of an annuity of £500 to continue for 4 years, at 5 per cent. simple interest?

£	£	£	£	s.	d.			
As 105 :	100 : :	500 : $x =$	476	3	$9\frac{5}{7}$	value of	1st	year's annuity.
As 110 :	100 : :	500 : $x =$	454	10	$10\frac{10}{11}$	,,	2nd	,,
As 115 :	100 : :	500 : $x =$	434	15	$7\frac{19}{23}$	,,	3rd	,,
As 120 :	100 : :	500 : $x =$	416	13	4	,,	4th	,,
		Total -	£1782	3	$8\frac{796}{1771}$.			

A bill for £648, payable at the end of a year, is paid at sight; what is the true and bank discount, at 4 per cent.?

As £104 : £4 : : £468 : $x =$ £18, the true discount.

As £100 : £4 : : £468 ; $x =$ £18 18s. $4\frac{4}{5}$d. interest, or bank disct.

EXERCISES 38.

1. What is the present worth of an annuity of £840, to continue for 3 years, at 5 per cent., simple interest?
2. What is the true and bank present worth of a bill of £500, due 12 months hence, at 5 per cent.?
3. What is the difference between the true and bank discount of a bill of £640, due 3 years 6 months hence, at 4½ per cent.?
4. Find the true and bank discount on £800, due 9 months hence, at 4 per cent.
5. Find the true and bank discount on a bill of £755 5s. 9d., drawn March 17th, at 3 months, and discounted May 31st, at 6 per cent.
6. What is the true present worth of £170 10s. 6d., due 1½ years hence, at 3½ per cent. discount?
7. What is the true and bank discount on a bill of £512, due on the 16th of September, but paid on the 8th of May preceding, at 5 per cent.?
8. What is the present worth of £549 9s. 5½d., at the end of 32 days, according to the true and bank methods?
9. Find the true and bank value of a bill for £157 14s. 6d., dated the 27th of October, at 8 months, and discounted the 8th of February, at 5 per cent.
10. Find the true and bank discount on a bill of £693 13s. 9d., due 3 years 8 months and 101 days hence, at 5 per cent.
11. Required the true and bank value of a bill of £968 14s. 6d., dated the 19th day of May, at 11 months, and discounted the 29th of July following, at 5 per cent.
12. Find the true and bank discount on a bill due the 20th of December, but cashed the 4th of April, at 5 per cent.
13. What is the present worth of £270 10s. 6d., due 1½ years hence, at 3½ per cent.?
14. What is the true and bank discount on a bill of £1283 15s. 3d., dated the 9th of March, and discounted the 22nd of May, at 5 per cent.?
15. Find the true and bank discount on a bill of £987 12s. 6d., drawn on the 1st of January, 1859, for 8 months, and discounted the 14th of March, at 5 per cent.
16. Bought 5¾ brewings of ale, each consisting of 6 butts 2 bar. 33 gal. 3 qts., at £1 11s. 6d. per barrel, and 6 months credit; what ought to be deducted for present payment, at 5 per cent.?

DISCOUNT BY COMPOUND INTEREST.

110. If money be discounted at a certain rate of compound interest, the present value is such a sum as will, by being improved at once at the same rate, amount at the end of the given time to the sum due.

EXAMPLES.

(1.) What is the present worth of £1, to be received 3 years hence, discounting at 5 per cent., compound interest?

£1, at 5 per cent., for 3 years by Table I. = £1·157625, which is found by multiplying 1·05 × 1·05 × 1·05, as shown already in compound interest. Then £1 will be the present value of £1·157625, due 3 years hence. Therefore, by proportion,

$$\overset{\text{amt.}}{\text{As } 1{\cdot}157625} : \overset{\text{amt.}}{£1} :: \overset{\text{p. val.}}{£1} : x = \frac{1{\cdot}000000}{1{\cdot}157625} = \overset{£}{{\cdot}863838} = \overset{\text{s.}}{17}\ \overset{\text{d.}}{3\tfrac{1}{4}} =$$

present value of £1, due 3 years hence, at 5 per cent. compound interest.

(2.) What is the present value of £562·432, due 3 years hence, at 4 per cent., compound interest?

By Table I. the amount of £1, for 3 years, at 4 per cent. is £1·124864.*

$$\text{Present value} = \frac{£1}{1{\cdot}124864} \times 562{\cdot}432 = £500\ \textit{Ans.}$$

(3.) What is the present worth and discount of £607 15s. 0¾d., for 4 years, at 5 per cent., compound interest?

The amount of £1, for 4 years, at 5 per cent. (by Table I.), is $(1{\cdot}05)^4 = 1{\cdot}215506$.

$$\text{Present value} = \frac{£1}{(1{\cdot}05)^4} \times 607{\cdot}753125 = £500.$$

EXERCISES 38A.

1. What ready money will purchase £100, which will not become due until 10 years, at 5 per cent., compound interest?
2. What is the present worth of £816 16s. 4·848d., due 5 years hence, at 5 per cent., compound interest?
3. What is the present value of £100 to be received 5 years hence, at 5 per cent., compound interest?

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* Present value of £1, due 3 years hence, at 4 per cent., compound interest, will be $\frac{1}{(1{\cdot}04)^3} = {\cdot}888996$, and $\cdot888996 \times 562{\cdot}432 = £500$.

COMMISSION AND BROKERAGE.

111. **Commission** is the sum paid to an agent for buying or selling goods, or transacting any other business, and is estimated at a certain rate per cent.

112. **Brokerage** is a charge made by a broker, at so much per cent., for buying or selling stock, and executing bill-transactions for others. In the public funds the charge is 2s. 6d. per cent.

EXAMPLES.

(1.) What is a house-agent's commission on rents collected by him to the amount of £85 16s. 3d., at 5 per cent.? *

$$\begin{array}{llllllll}
 & £ & £ & \text{s.} & \text{d.} & & & £\ \text{s.}\ \text{d.} \\
\text{As} & \cancel{100} : & 85 & 16 & 3 :: & \cancel{5} : x = & & 4\ \ 5\ \ 9\tfrac{3}{4}\ \text{commission.} \\
 & 20 & \multicolumn{3}{c}{\overline{\qquad\qquad}} & 1 & & \\
 & & £4 & 5 & 9\tfrac{3}{4} & & &
\end{array}$$

(2.) A house-agent having sold a house for £675, what is his commission on the sale, at $2\frac{1}{2}$ per cent.?

$$\begin{array}{llllllll}
 & £ & £ & \text{s.} & \text{d.} & & & £\ \text{s.}\ \text{d.} \\
\text{As} & \cancel{100} : & 675 & 0 & 0 :: & \cancel{2\tfrac{1}{2}} : x = & & 16\ \ 17\ \ 6. \\
 & 40 & \multicolumn{3}{c}{\overline{\qquad\qquad}} & 1 & & \\
 & & £16 & 17 & 6 & & &
\end{array}$$

(3.) What is the commission on the sale of goods to the amount of £487 12s. 6d., at 5 per cent.?

$$\begin{array}{llllllll}
 & £ & £ & \text{s.} & \text{d.} & & & £\ \text{s.}\ \text{d.} \\
\text{As} & \cancel{100} : & 487 & 12 & 6 :: & \cancel{5} : x = & & 24\ \ 7\ \ 7\tfrac{1}{2}\ \text{commission.} \\
 & 20 & \multicolumn{3}{c}{\overline{\qquad\qquad}} & 1 & & \\
 & & £24 & 7 & 7\tfrac{1}{2} & & &
\end{array}$$

(4.) A commission agent sells goods to the amount of £2075 12s. 6d., at 6s. 8d. per £100; what is his commission?

$$\begin{array}{llllllll}
 & £ & £ & \text{s.} & \text{d.} & & & £\ \text{s.}\ \text{d.} \\
\text{As} & 100 : & 2075 & 12 & 6 :: & \tfrac{1}{3} : x = & & 6\ \ 18\ \ 4\tfrac{1}{2}. \\
 & & & & \tfrac{1}{3} & & & \\
\hline
 & & 6{,}91 & 17 & 6 & & & \\
 & & 20 & & & & & \\
\cline{3-3}
 & & 18{,}37 & & & & & \\
 & & \ \ 12 & & & & & \\
\cline{3-3}
 & & \ \ 4{,}50 & & & & & \\
 & & \quad 2 & & & & & \\
\cline{3-3}
 & & \ \ 1{,}00 & & & & &
\end{array}$$

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* If the commission be 5 per cent., that will be 1s. in the £; if $2\frac{1}{2}$ per cent., it will be 6d.

EXERCISES 39.

1. What is a house-agent's commission on rents collected by him, amounting to £153 17s. 6d., at 5 per cent.?
2. What is the commission on houses sold for £665 5s., at $2\frac{1}{2}$ per cent.?
3. What is the brokerage on the purchase of £65,000 stock, at £$\frac{1}{8}$ per cent.?
4. A broker's charge at 2s. 6d. per cent., in the purchase of stock, amounted to £7 16s. $10\frac{1}{2}$d.; what was the amount of stock purchased?
5. What is the commission on £5878 12s. 6d. at $\frac{3}{8}$ per cent.?
6. Find the commission on the purchase of $7598\frac{3}{8}$ lb of cotton, at $7\frac{3}{8}$d. the lb., and $6957\frac{1}{2}$ lb., at $7\frac{11}{16}$d. the lb., at $\frac{3}{4}$ per cent.
7. Purchased for J. Elms and Co. $753\frac{7}{8}$ bushels of wheat, at 7s. 9d. the bushel, and $843\frac{16}{196}$ barrels of flour, at £1 12s. 6d. the barrel; what is the purchase money and the commission thereon, at $2\frac{3}{8}$ per cent.?
8. What is the commission at $\frac{3}{8}$ per cent. on the purchase of
   $8759\frac{1}{4}$ lb. of sugar, at $5\frac{1}{4}$d. the lb.
   $6958\frac{3}{8}$ lb. „ „ $6\frac{1}{8}$d. the lb. ?
9. What is the commission on the purchase of 80 bales of cotton, each weighing 2 cwt. 2 qr. $17\frac{1}{2}$ lb., at $5\frac{5}{16}$d. the lb., and 42 packs of wool, each weighing 5 cwt. 2 qr. 17 lb., at £5 16s. 7d. the cwt., at $\frac{1}{8}$ per cent.?
10. What is the commission at $\frac{7}{16}$ per cent. on the sale of
    $5387\frac{13}{16}$ lb. of cotton, at $5\frac{11}{16}$d. per lb.
    $4965\frac{7}{8}$ lb. „ „ $5\frac{13}{16}$d. „
    1785 lb. „ „ $5\frac{7}{16}$d. „ ?
11. Find the commission at $\frac{1}{2}$ per cent. on the purchase of
    $5968\frac{3}{4}$ lb. of cotton, at $7\frac{11}{16}$d. the lb.
    $6915\frac{1}{4}$ lb. „ „ $7\frac{5}{8}$d. „
    $4893\frac{1}{2}$ lb. „ „ $7\frac{13}{16}$d. „ .
12. What is the commission on the purchase of $325\frac{1}{2}$ bales of cotton, each 2 cwt. 2 qr. $17\frac{1}{2}$ lb., at $7\frac{3}{4}$d. per lb., at $\frac{5}{8}$ per cent.?
13. What is the commission, at $\frac{3}{4}$ per cent., on the sale of
    $53\frac{3}{8}$ pieces of cloth, each $37\frac{1}{2}$ yards, at 3s. $8\frac{1}{2}$d. per yard.
    $21\frac{3}{8}$ „ „ „ $35\frac{3}{8}$ „ „ 4s. $5\frac{1}{2}$d. „ .
14. What is the commission, at $1\frac{1}{2}$ per cent., on the purchase of 875 barrels of flour, at £1 18s. $7\frac{1}{2}$d., and 639 barrels at £1 18s. $5\frac{1}{4}$d. per barrel?
15. What is the brokerage at $\frac{1}{8}$ per cent. on 6978 lb. of cotton, at $6\frac{5}{8}$d. per lb., and 4576 lb., at $7\frac{7}{8}$d. per lb.?

STOCK OR STOCKS.

118. STOCK, though a term used to denote the capital of banks and trading companies, is also applied to the money borrowed at different times by the government, when the public revenue did not suffice to meet the exigences of the country. These stocks or debts which the government has contracted are called the *funds*, and their amount the *national debt*.*

Of this debt, the government, reserving to itself the option of paying off the principal, is bound only to pay the interest, but if the creditor wants his principal, he has only to sell it out to another, who has money to lend at interest. This person who purchases, then becomes the creditor, and to him thenceforth must be paid the interest on the amount purchased.

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* The practice of borrowing money for a perpetuity, was begun in the reign of William III.; previous sovereigns indeed were borrowers, but their loans were repaid. William, however, and his immediate successors borrowed without any intention of repaying, and began a debt, which in 1860, amounted to £804,295,238. In the same reign, the interest given for loans was very various, being regulated by the state of the money market; but in the reign of George II., instead of varying the interest upon the loan, the rate of interest was generally fixed at 3 per cent., and the necessary variation was made in the sum funded. For sometimes, for £80 lent to the state, £100 were given in the funds, paying £3 annually; hence, in consequence of this practice, the debt now amounts to nearly $\frac{2}{5}$ths more than the actual sum advanced by the lenders.

The mass of the debt having been contracted at 3 per cent., the different 3 per cents. were, in 1751, consolidated into one stock, known ever since as the 3 *per cent. consols*, and amounted, in 1860, to a debt of £406,915,832. The funds have undergone great changes. The market value of the 3 per cents. fell from 107, (in 1737), their highest value, to $47\frac{7}{8}$, (in 1797) their lowest value, so that selling in 1737, at 107, and buying in 1797 at $47\frac{7}{8}$, would be equivalent to clearing $124\frac{2}{3}$ per cent.,—a rise and depression calculated to make or lose fortunes. In 1749 and 1852, the 3 per cent. funds were at *par*, that is, £100 stock would bring £100 in the market.

The interest on the National Debt has undergone several reductions, amounting, from the years 1716 to 1854, to £4,188,000; still the interest and charges for its management, at the end of 1859, were £28,372,416. This sum has been reduced to the extent of £1,250,000 by the termination of the long annuities in January, 1860. The annual charge of the debt, as it now stands, will be about £27,122,416,—a sum still sufficiently large to remind the country of one of the evils of war.

The number of persons receiving interest from the funds is about 270,000. Of these many are holders of scrip, entitling to interest for banking and insurance companies, which therefore makes the number of persons interested in the stability of the funds considerably more than the number stated.

The purchase or sale of stocks is made through brokers, who charge £⅛ per £100, as brokerage or commission upon the stock bought or sold. Thus, if the price of any stock, such as the 3 per cent. consols, be 92⅜, the purchaser pays (£92⅜ + ⅛) or £92½ for £100 of this stock, while the seller receives (£92⅜ — ⅛) or £92¼, for this same £100 stock.

The price of stocks varies according as money is scarce or plentiful. If money be scarce, any sum lent will bring a high rate of interest—much higher than the same amount will bring in the funds; consequently, there is no inducement to purchase stock. The funds therefore go down in value, and the more so, in proportion as there are more persons willing to sell than to buy. Whereas, when money is plentiful, a low rate of interest is the consequence; then many are willing to buy stock, the funds go up, and that the more readily, if only few fundholders are willing to sell.

EXAMPLES.

(1.) What sum will purchase £1600 in the 3 per cent. consols, at 92⅜ per cent.?

As 100 : £1600 : : £(92⅜ + ⅛) : x = £1480 *Ans.*

The following were the particulars of the National Debt, on the 1st of April, 1860:—

Funded debt.

	£
New 2½ per cents. - - - - -	2,981,038
The 3 per cent. consols - - -	406,915,833
The 3 per cent. reduced - - -	115,081,699
New 3 per cents. - - - - -	246,245,910
New 3½ per cents. - - - -	240,746
New 5 per cents. - - - - -	432,603
	£771,897,829

Unfunded debt.

	£
Exchequer bills and bonds - - -	16,646,600
Debt due to the Bank of England -	11,015,100
" " " Ireland -	2,630,769
	£30,292,469

Terminable annuities.

	£
Life annuities - - - - - -	1,099,674
Annuities expiring April 5th, 1867 -	585,740
" " April 5th, 1885 -	116,000
" " at various periods -	293,526
	£2,094,940

(2.) What sum does a person receive who sells £1600 in the 3 per cent. consols, at 92$\frac{3}{8}$ per cent.?

As £100 : £1600 :: £(92$\frac{3}{8}$ — $\frac{1}{8}$) : x = £1476 *Ans.*

(3.) What interest does the purchaser, in Example (1), receive for his money?

£ £ £ s. d.

As 100 : 1600 :: 3 : x = 48 0 0 the interest.

As 1480 : 100 :: 48 : x = 3 4 10$\frac{14}{37}$ per cent. *Ans.*

(4.) A guardian has £600 which are to be invested for his ward, in the 3$\frac{1}{4}$ per cents. the price being 95$\frac{7}{8}$; how much stock can his broker buy for the money?

As (95$\frac{7}{8}$ + $\frac{1}{8}$) : 600 :: £100 stock : x = 625. *Ans.*

EXERCISES 40.

1. The building of the Brown Free Library, Liverpool, cost about £65000; what annual interest would this sum bring, if it had been invested in the 3 per cent. consols, at 93$\frac{5}{8}$ per cent.?
2. The cost of St. George's Hall, Liverpool, was about £320680; what sum would this amount purchase in the 3$\frac{1}{2}$ per cent. funds, at 95$\frac{5}{8}$ per cent., and what would be the annual interest?
3. What income tax does £5843 17s. 6d. in the 3 per cent. consols, pay at 7d in the £?
4. What income tax must be paid on £13500 in the 3$\frac{1}{4}$ per cents., at 10d. in the £?
5. The cost of the Crystal Palace (as it stood in April, 1860), with its contents and grounds, was £1459000; what sum in the 3$\frac{1}{4}$ per cent. funds, would this amount purchase, and what would be the annual interest accruing from it?
6. A person who has £3200 in the 3 per cents., at 80$\frac{1}{2}$, transfers his capital to the 3$\frac{1}{2}$ per cents., at 95$\frac{7}{8}$; what is the alteration in his income?
7. What is the purchase money of £600, in the 3 per cent. reduced annuities at 91$\frac{5}{8}$?
8. What sum must be invested in the purchase of 3 per cent. consols, at 93$\frac{1}{8}$ per cent. to entitle the purchaser to an annual interest of £180?
9. A person who has £2854 in the 3 per cents., at 80$\frac{1}{2}$, transfers it into the 4 per cents. at 98; what difference does it make in his income?

10. Sold £3875 in the 3 per cent. consols, at 89½; and with the money received purchased in the 3½ at 92¾; what difference does it make in the income?

INSURANCE.

114. Insurance is a contract by which one party engages, upon being paid a certain sum, to indemnify another party against loss by fire, or accidents at sea.

(1.) The legal document binding the parties to the contract, is termed the *policy*.

(2.) The sum paid for protection is termed the *premium of insurance*.

(3.) The party protected, the *insured*.

(4.) The parties who agree to indemnify or protect, are the *insurers*, and in case of Fire ships, *underwriters*.

Fire.

Fire insurance offices charge annually—

	1s. 6d. per £100,	on common insurances
	2s. 6d. ,,	on hazardous
From	5s. to 10s. ,,	on doubly hazardous.

And still higher on special insurances, which comprise all risks of extraordinary hazard, together with an annual duty * of 3s. per cent. on the sum insured, which is paid to government. There is also 1s. duty on each policy, which is usually paid by the company.

Premium	On an ordinary dwelling house, 1s. 6d. per £100.
,,	Hotels, public-houses, and bread-baking houses 2s. 6d. per £100.
,,	Pawn-brokers, soap makers, and thatched-houses, 5s. per £100.
,,	Distillers of spirits, boilers of oil, drug grinders, &c., 10s. 6d. to 21s. per £100.

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* Farm stock, embracing crops, implements of husbandry, and live stock on the farm, are duty free; but the farm builings are chargeable with the same duty as other property. Public hospitals are also exempt from duty; but no other charitable institutions. The premium charged for 1 month is ⅛th of the annual; for 3 months ⅜; for 6 months ⅝; and for more than 6 months the entire annual premium. The duty on any fractional part of a £100, if for £5, 2d.; for £20, 7d.; for £50, 1s. 6d.; for £70, 2s. 1d., and for £99, 2s. 8d.

EXAMPLES.

(1.) * What is the total expense of insuring a dwelling house for £500; premium 1s. 6d., and duty 3s. per cent.?

As 100 : 500 :: 4s. 6d. : x = £1 2s. 6d., total expense.

(2.) A gentleman insures his house for £1200, his furniture for £700, and his library for £100; what is the annual expense, when he pays the usual insurance of 4s. 6d. (duty included) per cent.?

As £100 : £2000 :: 4s. 6d. : x = £1 10s. *Ans.*

(3.) A pawnbroker insures his premises for £2850; the premium being 5s. and the duty 3s. per £100; what is the expense?

	£		£		s.		£ s. d.	
As	100	:	2850	::	5	: x =	7 2 6	premium.
As	100	:	2850	::	3	: x =	4 5 6	duty.
							£11 8 0	total expense.

(4.) What is the expense of insuring an hotel for £3875; premium 2s. 6d., and duty 3s. per cent.?

	£		£		s. d.		£ s. d.	
As	100	:	3875	::	2 6	: x =	4 16 10½	premium.
As	100	:	3875	::	3 0	: x =	5 16 3	duty.
							£10 13 1½	total expense.

EXERCISES 41.

1. A distillery is insured for £3500; premium 10s. 6d., and duty 3s. per cent.; what is the expense of insuring?

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* The duty on fire insurance was first levied in 1694, in the reign of William III. and Mary. In 1782 the rate of duty charged was 1s. 6d. per cent. on the sum insured. In 1797, it was raised to 2s. per cent.; in 1804, to 2s. 6d. per cent; and in 1815, to 3s. per cent., which is the duty charged throughout the United Kingdom at the present time.

The total amount insured in 1859-60 was,

In England	£889,521,000	duty paid	£1,834,282
In Scotland	65,452,000	"	98,179
In Ireland	47,518,000	"	71,277
Total	£1,002,491,000		£2,003,738

2. A steamboat, while plying, is insured for £7500 against fire; premium 10s. 6d., duty 3s. per cent.; what is the expense of insuring?
3. For what amount was a thatched cottage insured, when the expense was £1 18s., and the premium 5s., duty, as usual?
4. What insurance must be paid at 1¼ per cent., which covers all expenses, for conveying 38 lb. 2 oz. 0 dwt. 2½ grs. bullion, from Liverpool to New York, when valued at £3 17s. 10½d. per ounce?

MARINE.

115. The duty on marine policies is by Act of Parliament of 1844, regulated by the rate of premium charged, according to the following scale.*

	s.	d.
If the premium is not above 10s. per cent., the duty is -	0	3
If above 10s., and not above 20s. " " - - -	0	6
" 20s. " " 30s. " " - - -	1	0
" 30s. " " 40s. " " - - -	2	0
" 40s. " " 50s. " " - - -	3	0
" 50s. - - - - - - - - -	4	0

If insured for any certain term or period of time.

On every £100 insured for a period not exceeding 6 calendar months. - - - - - - - -	2	6
On every £100 insured for a period exceeding 6 calendar months. - - - - - - - -	4	0

The duty upon any part of £100, is the same as upon £100.

EXAMPLE (1).

What is the expense of insuring a cargo, valued at £2991, from Liverpool to Hamburg; premium 25s. per cent, policy duty as per scale, 1s. per cent., and broker's commission on † premium 8⅓ per cent.

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* The premium for the same voyage will vary according to the seasons, the quality of the vessel, the sort of goods with which the ship is laden, and the known character and capacity of the captain.

† The commission, which is calculated on the premium, ranges from 5 to 15 per cent., according to stipulation.

As £100 : £2991 : : £1¼ : x = £37 7s. 9d. premium.

1¼

2991
747 15

37,38 15
20

7,75
12

9,00

As £100 : £37 7s. 9d. : : 8⅓ : x = £3 2s. 3¾d. commission.
12
12) 37 7 9

£3 2 3¾

Policy duty on £3000, at 1s. per cent. = £15 0s. 0d.

Ans. £55 10 0¾ entire expense.

EXERCISES 41. (*continued*).

5. What is the expense of insuring £5000 worth of goods from London to Bombay, at 50s. per cent. premium; policy duty, as per scale, 3s. per cent; broker's commission on premium 10 per cent.?
6. A vessel is insured on a voyage from Hull to New Orleans for £2400; premium 40s. per cent.; policy duty, as per scale, 2s. per cent., and broker's commission, 7½ per cent.; what is the entire expense?
6. Find the expense of insuring £2400 on goods, from London to Dublin, at 50s. premium per cent.; policy duty, as per scale, 4s. per cent., and broker's commission 12 per cent. on premium.

EXAMPLE (2).

What sum must be insured on goods worth £2910, on a voyage from Liverpool to Melbourne, the premium being 50s, per cent. policy duty 4s. per cent., and broker's commission 12 per cent., that in case of loss, the full value of the goods may be received?

	s.
Premium - -	50
Policy duty - -	4
Commission on 50s.	6
20)	60
	£3

100
3

As 97 : £2910 : : 100 : x = £3000

8 What was the expense, in March 1861, of insuring a cargo of cotton, valued at £2900, from New Orleans to Liverpool; the premium, instead of being as usual 30s., was then 60s., per cent., owing to the warlike aspect of affairs between the Northern and Southern States of the American Union; policy duty, as per scale, 4s. per cent., commission 10 per cent.?

PROFIT AND LOSS.

116. Those questions in Arithmetic which relate to gain or loss are placed under the head of *Profit* and *Loss.*

In order to compare the different gains or losses of several individuals employing different capitals in business, a common standard capital of £100, or 100 of any money denomination, is chosen, by which each person's gain or loss on this standard capital is ascertained.

EXAMPLES.

(1.) Bought 1 cwt. of tea, at 4s. 2d. per lb., and sold it at 4s. 6d. per lb.; what is the gain?

£	s.	d.	
0	4	2	
		4	
0	16	8	
		4	
3	6	8	
		7	
£23	6	8	cost.

£	s.	d.	
0	4	6	
		4	
0	18	0	
		4	
3	12	0	
		7	
25	4	0	selling price.
23	6	8	cost.
Ans. £1	17	4	gain.

Otherwise,

£	s.	d.	
0	4	6	
0	4	2	
		4	gain.
		4	
0	1	4	
		4	
0	5	4	
		7	
£1	17	4	gain.

(2.) Bought 1 cwt. of tea, at 4s. 2d. per lb., and sold at 4s. 6d. per lb.; what is the gain on 100, or the gain *per cent.?*

s. d.
4 6 selling price.
4 2 cost.
cost. cost. —— gain.
As 4s. 2d. : 100 : : 0 4 : $x = 8$, or 8 per cent.;

that is, 8d. on 100d., 8s. on 100s., or £8 on £100; so that the 100 is used to represent either £100, 100s, or 100d.

(3.) Bought tea at 4s. 2d. per lb.; at what price must it be sold per lb. to gain 8 per cent.?

cost. cost. selling price.
As 100 : 4s. 2d. : : 108 : x = 4s. 6d. the selling price.

(4.) If I bought tea at 4s. 2d. per lb.; at what price per lb. did I sell it when I lost 8 per cent.?

cost. cost. selling price.
As 100 : 4s. 2d. : : 92 : x = 3s. 10d. the selling price.

(5.) What is the cost of a lb. of tea, which, when sold for 4s. 6d., realized a profit of 8 per cent.?

selling price. cost.
As 108 : 4s. 6d. : : 100 : x = 4s. 2d. the required cost.

(6.) What is the buying price of a lb. of tea, which, when sold for 3s. 10d., entailed a loss of 8 per cent.?

selling price. cost.
As 92 : 3s. 10d. : : 100 : x = 4s. 2d. the required cost.

(7.) If by selling tea at 3s. 10d. per lb. I lose 8 per cent.; what must it be sold for per lb. to clear 8 per cent.?

selling price. selling price.
As 92 : 3s. 10d. : : 108 : x = 4s. 6d. the required selling price.

Otherwise,

selling price. cost.
As 92 : 3s. 10d. : : 100 : x = 4s. 2d. = cost.

cost. cost.
As 100 : 4s. 2d. : : 108 : x = 4s. 6d. = selling price.

Remarks.—100, which remains a fixed number, stands for capital or cost; when the selling price is 92, there is a loss of 8 per cent.; but with a gain of 8 per cent. the selling price is 108.

If 100 be termed the *imaginary cost,*

92 or 108 will be termed the *imaginary selling price*, according as there is a loss or gain of 8 per cent.

8 will be termed the *imaginary gain* or *loss*, according to the case.

(8.)* By selling tea at 3s. 10d. per lb., 8 per cent. is lost; what is the gain or loss per cent., when the same tea is sold at 4s. 6d. per lb.?

s. d. s. d. selling price
As 3 10 : 4 6 : 92 : x = 108 and (108 — 100) = 8 per cent. gain.

(9.) By selling tea at 4s. 6d. per lb., which cost 4s. 2d. per lb., there is gained £1 17s. 4d.; what quantity was sold?

s. d.
4 6 = selling price.
4 2 cost.

As 0 4 : £1 17s. 4d. : : 1 lb. : x = 112 lb.

(10.) If a person gain $12\frac{1}{2}$ per cent. by selling 8 eggs for $6\frac{1}{2}$d., how much does he gain per cent., by selling 8 eggs for $2\frac{1}{2}$d.?

As $112\frac{1}{2}$: $6\frac{1}{2}$d. : : 100 : x = $5\frac{7}{9}$d. the buying price of 8 eggs.

eggs.
As 8 : 3 : : $5\frac{7}{9}$: x = $2\frac{1}{6}$d. the buying price of 3 eggs.
$2\frac{1}{2}$ selling price of 3 eggs.
$2\frac{1}{6}$ cost of 3 eggs.

As $2\frac{1}{6}$d. : 100 : : $\frac{1}{3}$ gain on 3 eggs : x = $15\frac{5}{13}$ gain per cent. *Ans.*

Otherwise,

As 8 : 3 : : $6\frac{1}{2}$d. : x = $2\frac{7}{16}$ $\left\{\begin{array}{l}\text{the selling price of 3 eggs, at the}\\ \text{rate of 8 for } 6\frac{1}{2}\text{d.}\end{array}\right.$

As $2\frac{7}{16}$: $2\frac{1}{2}$d. : : $112\frac{1}{2}$: x = $115\frac{5}{13}$ $\left\{\begin{array}{l}\text{selling price per cent., at the}\\ \text{rate of 3 for } 2\frac{1}{2}\text{d.}\end{array}\right.$
100

Ans. $15\frac{5}{13}$.

(11.) By selling cloth at 11s. 3d. a yard, there is a loss of 10 per cent.; is there a gain or loss, and how much, when the same cloth is sold for 12s.?

s. s. l. s.
As 11s. 3d. : 12s. : : 90 : x = 96 and 100 — 96 = 4 per cent. loss.

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[*] In the foregoing examples the proportional arrangement can be expressed in words as follows:—

Ex. (2.) As cost : imaginary cost : : gain : imaginary gain.
,, (3.) As imaginary cost : cost : : imaginary selling price : selling price.
,, (4.) As imaginary cost : cost : : imaginary selling price : selling price.
,, (5.) As imaginary selling price : selling price : : imaginary cost : cost.
,, (6.) As imaginary selling price : selling price : : imaginary cost : cost.
,, (7.) As imaginary selling price : selling price : : imaginary selling price : selling price.
,, (8.) As selling price : selling price : : imaginary selling price : imaginary selling price, less imaginary cost = imaginary gain.

Otherwise,

l. s. s. cost.
As 90 : 11s. 8d. : : 100 : x = 12s. 6d. cost price.

s. d.
12 6
12
———
cost. l. cost.
As 12s. 6d. : 100 : : 6 : x = 4 per cent. loss.

(12.) A huckster bought 180 eggs, at 2 a penny, and 180 more at 3 a penny, and sold the entire at 5 for 2d.; was there a gain or loss, and how much per cent.?

e. e. d. d.
As 2 : 180 : : 1 : x = 90
As 3 : 180 : : 1 : x = 60
———
150 = cost price.
e. e. d.
As 5 : 360 : : 2 : x = 144 = selling price.
———
c. c. loss. 6 loss.
As 150d. : 100 : : 6 : x = 4 per cent. loss.

EXERCISES 42.

1. Bought 135 lb. of sugar, at 5d. per lb., and sold it at 6½d. per lb.; what is the gain?
2. Bought 325 lb. of cheese, at 8¾d. per lb., and sold it at 9¼d. per lb.; what is the gain?
3. Bought 256 lb. of soap, at 2½d. per lb., and sold it at 4¼d. per lb.; what is the gain?
4. Bought 428 yards of cloth, at 14s. 8d. per yard, and sold it at 16s. 8d. per yard; what is the gain?
5. Bought 697 lb. of soap, at 3½d. per lb., and sold it at 4¼d. per lb., what is the gain?
6. Bought 3 cwt. 2 qr. 5 lb. of bacon, at 6½d. per lb., and sold it at 8d. per lb.; what is the gain?
7. Bought 12 tons 27 lb. of sugar, at 4¾d. per lb., and sold it at 7½d. per lb.; what is the gain?
8. Bought 2 tons 18 cwt. 3 qr. 18 lb. of tea, at £214 13s. 4d. per ton, and sold it at 2s. 1½d. per lb.; what is the gain?
9. Bought 170000 ounces of sugar, at £2 13s. 8d. per cwt., and sold it at 6¾d. per lb.; what is the gain?
10. Bought 758000 ounces of coffee, at 1s. 1¼d. per lb., and sold it at £6 5s. 4d. per cwt.; what is the gain?

11. Bought 169 cwt. 3 qr. 26⅓ lb. of cheese, at £3 14s. 8d. per cwt., and sold it to gain £79 6s. 6½d.; what is the selling price of 1 cwt.?

12. Bought 6893 lb. of sugar, at 3¼d. per lb., and sold it at £1 12s. 8d. per cwt.; what is the gain?

13. Bought 18 cwt. 2 qr. 27 lb. of butter, at £93 6s. 8d per ton and sold it at 11⅓d. per lb.; what is the gain?

14. Bought 4⅜ bags of rice, each weighing 5 tons 14 cwt. 3 qr. 16 lbs., at £1 12s. 8d. per cwt., and sold it at 3¾d. per lb.; what is the gain?

15. Bought 8⅜ chests of tea, each weighing 18 cwt. 2 qr. 16 lb., at £13 1s. 4d. per cwt., and sold 24 cwt. 2 qr. 18 lb., at 2s. 8d per lb., and the remainder at £13 10s. 8d. per cwt.; what is the gain?

16. Bought 17 cwt. 2 qr. 26 lb. of tea, at £298 13s. 4d. per ton, and by selling it, lost £8 5s. 6d.; what was it sold at per cwt.?

17. Purchased 18 cwt. 2 qr. 27 lb. of butter, at £186 13s. 4d. the ton, and sold it so as to gain £18 2s. 4½d.; what was it sold at per cwt.?

18. Purchased 7¼ hhds. of sugar, each weighing 5 cwt. 1 qr. 12 lb., at £1 12s. 8d. the cwt.; sold 18 cwt. 2 qr. 23 lb., at £1 17s. 4d. per cwt., and the remainder at 3¾d. the lb.; what was the gain?

19. Bought 93 cwt. 3 qr. 12 lb. of iron, at 9s. 4d. the cwt., and paid as charges thereon £3 4s. 6d. What is gained by selling 27 cwt. 2 qr., at 12s. 4d. the cwt.; 29 cwt. 1 qr. 20 lb., at 12s. 8d., and the remainder at 12s. 9d. per cwt.?

20. Bought 37⅝ chests of tea, each weighing 7 cwt. 2 qr. 18 lb., at £13 1s. 4d. the cwt.; sold 113 cwt. 3 qr. 25½ lb., at 2s. 7½d. per lb., and the remainder for £2335 10s. 10½d.; what is the gain, and at what price did the remainder sell per cwt.?

21. Bought $189\frac{45}{196}$ barrels of flour, at £1 18s. 9d. the barrel, and sold the flour, so as to gain 10⅜ guineas; what was it sold at per barrel?

22. Bought flour at 6 lb. for a shilling, and sold it at 5 lb. for a shilling; what is the gain per cent.?

23. Bought 5·75 hhds. of sugar, each weighing 6·435 cwt., at £1·645 the cwt, and sold 3·4375 cwt., at 3½d. the lb., and the remainder at £1·7435 the cwt.; what is the gain?

24. Bought tea at 5s. 6d. the lb., which in consequence of a fall in price, was sold at 4s. 9d. the lb; what was the loss per cent.?

If I buy cloth at 9s. 4d. the yard, and sell it at 10s. 6d., what is my gain per cent?

Bought tea at 3s. 4d. the lb. which, being damaged, is sold at 3s. the lb.; what is the loss per cent.?

Bought 37 cwt. 3 qr. 14 lb. + 7·2735 cwt. + 6 3/16 cwt. + 47·32 cwt. + 32⅜ cwt. + 27 15/32 cwt. of pork, at £2·0625 per cwt., and sold it at 5¼d. the lb.; what is the gain?

Bought 148 87/196 barrels of flour, at £1 16s. 9d. the barrel, and sold it to gain 18⅜ guineas; what was it sold at the barrel?

Bought 648 yards of cloth at 7s. 7d. the yard, 68 at 11s. 6d., 98 at 4½d., 487 at 10½d., and 98 at 9s. 4d.; the whole was sold at 15 per cent. loss; what was the entire quantity sold for, and the average selling price of 1 yard?

A spirit dealer bought at an excise sale 46⅓ gallons of Hollands at 18s. the gallon, and 36¾ gallons of rum at 16s. the gallon. After reducing these spirits by 1 gallon of water to every 5 of spirits, he sold the Hollands at 16s. 6d. At what rate per gallon must he sell the rum so as to clear 20 per cent. on the whole?

Bought 49·45 lb. of tea at 2·65s. the lb., 4·38 cwt. at 2s. 8·75d. the lb., and 16·325 qr. at 3s. 1·65d. the lb.; what is the average price of 1 lb., and at what must it be sold per lb. to gain 2½ per cent.?

What is the cost of 37·675 tierces of beef, each weighing 8·745 cwt. + 4 cwt. 2·65 qr. + 3 cwt. 3 qr. 18·48 lb. + 2 cwt. 1 qr. 14·672 lb. at £14 16·875s. for 5·375 cwt.?

Bought 31·875 tierces of beef, each weighing 8·735 cwt. + 4 cwt. 2·45 qr. + 5 cwt. 3 qr. 18·75 lb. + 2 cwt. 1 qr. 24·675 lb., at £18 17·875s. for every 5·275 cwt.; what is the cost?

Purchased 5·375 cwt. of butter at £4·2 the cwt., 7·25 cwt. at £3·75, and 2·75 cwt. at £3·875 the cwt.; what is the average price of 1 cwt.?

Purchased 34·35 casks of sugar, each weighing 4·87 cwt. + 3·435 cwt. + 6·895 cwt. + ·1875 cwt. — 2·6475 cwt., at £23 15·45s. the ton, and sold it at 3·25d. the lb.; what is the gain?

Bought 34·375 chests of tea, each weighing (3·175 + 2·1875 + 1·632) cwt. + (2·875 + 5·63) qr., at £245·675 the ton, and sold the tea at 2s. 10d. the lb.; what is the gain?

Purchased 7¾ barrels of beef, each weighing 2 cwt. 3 qr. 20 lb. at £2 6s. 8d. the cwt., and sold 15 cwt. 1 qr. 10 lb. at 5¼d.

the lb., and the remainder at £2 13s. 8d. the cwt. what is the gain?

38. Bought $\left\{\begin{array}{l} 839\frac{15}{16} \text{ yards at 5s. 9d. the yard,} \\ 285\frac{5}{16} \text{ yards at 5s. 8d. the yard,} \end{array}\right\}$ and sold the cloth to gain £17 5s. $9\frac{1}{2}$d.; what was the average selling price of a yard?

39. If 8·375 + 9·9475 cwt. of beef be sold at $6\frac{1}{4}$d. the lb., and there is a gain of £1 9s. 0·1275d. by the sale; what is the buying price per cwt.?

40. Sold 179 cwt. 3 qr. 10 lb. of butter at £6 15s. 4d. per cwt., and gained thereby £62 18s. $10\frac{1}{2}$d.; at what price was it bought per cwt.?

41. If 79 cwt. 2 qr. 18 lb. of tallow cost £175 2s. 6d., how must it be sold per cwt. to clear 8 per cent.?

42. Bought 71 cwt. 3 qr. 18 lb. of coffee at £4 13s. 8d. the cwt., and 37 cwt. 2 qr. 25 lb. at £4 15s. 9d. per cwt.; what must be the average selling price of 1 cwt. so as to clear $7\frac{1}{2}$ per cent.?

43. Bought $\frac{2}{3}$ of $8\frac{3}{4}$ bags of wheat, each containing 5 qr. 4 bushel 1 peck 1 gal., of which $1\frac{1}{3} + 3\frac{3}{5}$ bushels cost $1\frac{4}{5}$ guineas, but are sold for $1\frac{5}{6}$ guineas; what is the entire gain?

44 Bought 21·5 packs of cloth, each pack containing 21·35 pieces, of 21·6 yards, at 3s. $7\frac{1}{2}$d. the yard, and sold it so as to gain $10\frac{1}{2}$ guineas; at what was it sold per yard?

45. Shipped to Bordeaux 600 quarters of wheat at 46s. 10d. per quarter; 300 of barley at 25s. 6d. per quarter; and 200 yards of cloth at 15s. $3\frac{3}{4}$d. per yard. Insured the cargo for £1100 at 2 guineas per cent., policy duty being 5s. per cent., and other charges amounting to £59 5s. The sales yield a net proceeds of £2000; is there a gain or loss, and how much per cent.?

46. Bought 85·35 barrels of flour at £1 16s. $3\frac{1}{2}$d. per barrel, and sold it at $2\frac{3}{4}$d. per lb.; what is the gain, and the gain per cent.?

47. Bought 69·375 cwt. + 14·45 cwt. of rice at $2\frac{1}{2}$d. per lb., and sold it at £1 5s. $3\frac{1}{2}$d. per cwt.; what is the gain, and the gain per cent.?

48. Bought 18 cwt. 2 qr. 27 lb. of coffee, at £95 13s 10d. per ton, and sold 10 cwt. 3 qr. $17\frac{1}{2}$ lb. at 1s. $1\frac{1}{2}$d. the lb. and the remainder at £4 18s. 9d. per cwt.; what is the gain, and the gain per cent.?

49. Bought 9 butts of rum, each containing 20 hhds. 41 gals. 3 qts. 1 pint, at £52 12s. $7\frac{1}{2}$d. per hhd., and sold it at 15s. 9d. the gallon; is there gain or loss, and to what extent?

50. Purchased 68.875 cwt. + 18·4625 cwt. of beef, at £2 18s. 3½d. per cwt., and sold it at 5¾d. the lb.; what is the loss, and the loss per cent.?

51. Bought 1400 casks of tallow, at £2 5s. per cask, and sold one half at £2 15s. per cask; the rest being damaged, is sold at such a price per cask as to realize the purchase money—what was it sold at per cask?

52. Purchased 78 cwt. 3 qr. 18 lb. of tea at £14 17s. 6d. per cwt.; but 18 cwt. 1 qr. 5 lb. being damaged, were sold at such a price as to incur a loss of 20 guineas; at what price per cwt. was the remainder sold in order to clear £30 9s. 7d. on the entire quantity?

53. A spirit dealer purchased at an excise sale, 46⅜ gallons of gin at 18s. per gallon, and 33⅓ of rum at 16s. After adding 1 gallon of water to every 5 of spirits, he sold the gin at 16s. 6d. per gallon; at what rate must he sell the rum per gallon to clear 20 per cent. on the whole?

54. Sold 25 cwt. 3 qr. 10 lb. of beef, at £2 17s. 10½d. per cwt., and thereby gained $7\frac{11}{16}$ guineas; what is the buying price per cwt.?

55. A merchant bought 1260 gallons of wine, half of which being damaged, he sold for £330, which was at a loss of 12 per cent., and the remainder for £397; did he gain or lose by the whole, and how much per cent.?

56. By selling tobacco at 4s. 4½d. per lb., I clear cent. per cent.; what is the gain on 17 cwt. 3 qr. 25½ lb., at £18 15s. 8d. per cwt., and the gain per cent.?

57. Sold tea at 8s. 9d. per lb., which was at 5 per cent. profit, and gained thereby £16 13s. 4d.; what quantity did I sell?

58. Bought 138 gallons of brandy and 112 gallons of rum, all at 8s. 6d. per gallon—the brandy is sold at 11s. 8d. per gal.; at what price must the rum be sold in order to gain 20 per cent. on the whole?

59. Purchased $189\frac{13}{196}$ barrels of flour, at £1 16s. 8d. per barrel, and sold the entire at 2¾d. per lb.; what is the gain, and the gain per cent.?

60. Bought 89·964285 cwt. of butter, at £·05729 per lb., and sold it to clear 5 per cent.; what was the entire selling price?

61. Bought 18 cwt. 1 qr. 19·25 lb. of tea, at £·287 5 per lb., and sold it so as to clear 5 per cent.; what was the entire selling price?

62. If 104·1255 oz. of gold be bought, at £2 18s. 7½d. for ·6 oz, and be sold to clear 12½ per cent.; what is the whole selling price?

63. Bought 9¾ loads of lime, each weighing 13 cwt. 1 qr. 19·25 lb., at £3 18s. 6d. per cwt., and sold it at 5 per cent. profit; what was the selling price?

64. Bought 3 pieces of cloth, each containing 67·5625 yards, at 6s. 9½d. per yard, and sold the whole to clear 12½ per cent.; what was the whole selling price?

65. Purchased 21⅔ pieces of cloth, each containing 85⅓ yards, at £15 13s. 9d. per piece, and sold it at 9s. 9d. per yard; what is the gain, and the gain per cent.?

66. Bought 5⅔ pieces of silk, each containing 87⅜ yards, at 7s. 10½d. per yard, and sold it to gain 7 1/16 guineas; what was the silk sold at per yard?

67 Sold 9 hhds. sugar, each weighing 17 cwt. 3 qr. 18 lb., at £1 5s. 10d. per cwt., which was at a loss of 4 per cent.; what was the cost, and the loss?

68. Bought 13 cwt. 2 qr. 25 lb. of coffee, at £75 8s. 9d. per ton, and sold 8 cwt. 3 qr. 19 lb. at 7¼d. per lb., and the remainder at £3 11s. 10½d. per cwt.; was there a gain or loss, and how much?

69. Bought 5·375 parcels of oats, each containing 7282 bushels, at 5s. 3¾d. per bushel, and sold the whole to gain 8 per cent.; what was the selling price and the gain?

70. Purchased 10·5 packages of tea, each weighing 6004·88625 lb., at 13s. 6¾d. for 3·5 lb., and sold it to gain 7½ per cent.; what was the selling price and gain?

71. If 6·75 pieces of satin, each measuring 7282 yards, were sold for £16755 3s. 11·835d., by which 8 per cent. was cleared what was the whole gain, and the cost per yard?

72. Bought 3⅝ bags of coffee, each weighing 18 cwt. 2 qr. 16 lb., at £6 10s. 8d. per cwt., and sold 24 cwt. 2 qr. 14 lb., at 1s. 4s. per lb., and the remainder at £6 15s. 4d. per cwt.; what was the gain, and the gain per cent.?

73. Bought 357 qr. 5 bus. 3 p. of wheat, at 58s. per quarter, and sold 85 qr. 2 b. 1 p., at 63s.; 185 qr. 1 b. 3 p., at 64s. 6d.; and the remainder at 64s. per quarter; what was the gain, and the gain per cent.?

74. Sold wool at 13s. per stone, and lost thereby 15 per cent.; how many stones were there, when the purchase money was £75 8s. 3d.?

75. Sold tallow at 7s. 7d. per stone, which was at 12½ per cent. loss; what was the cost of 7 cwt. 3 qr. 18 lb. sold?

76. Sold 23 hhds. of sugar, each weighing 17 cwt. 3 qr. 18 lb.; tare

16 lb. per cwt., at £1 9s. 9d. per cwt., and thereby gained 8 per cent. ; what was the cost?

77. Bought $7\frac{4}{5}$ nuggets of gold, each weighing 5 lb. 10 oz. 12 dwt., at £41 12s. 6d. per lb., and sold them to clear 5 per cent.; what was the gain?

78. Bought 97 a. 3 r. 25 p. of land, at £2 5s. 6d. per acre, and sold it so as to gain £27 16s. $10\frac{13}{125}$d.; what was the gain per cent.?

79. A grocer bought 14 cwt. 1 qr. of coffee, for £61 10s. 3d., of which he sold 2 cwt. 1 qr. 14 lb. at cost price; at what rate per cwt. must he sell the remainder, so as to clear £7 10s. by the bargain?

80. Bought $7\frac{2}{3}$ bags of coffee, each weighing $2\frac{1}{3} + 2\frac{1}{4}$ cwt., at 1s. $2\frac{1}{2}$d. per lb.; what must the whole be sold for, in order to clear 8 per cent.?

81. Sold butter at £4 10s. per cwt., which was at $22\frac{1}{2}$ per cent. profit; what was the cost price?

BARTER.

117. When one man buys goods of another, and pays for them, not in money, but in other goods, the transaction is called *barter*.

As the value of the goods received, must be equal to the value of those given, therefore, if the cash price of one be ascertained, the cash price of the other (being the same) becomes known; and thence the price of one, or the quantity can be obtained according as either is required.

EXAMPLES.

(1.) How many lbs. of tea, at 3s. 9d. per lb., must be given for 75 yards of linen, at 2s. 6d. per yard?

```
       s. d.
       2  6
          75
       -----
   £9  7  6 the price of the linen, and also the price of the tea.
                £  s. d.
       3s. 9d.) 9  7  6 (
       12       20
       ----     -------
       45       187
                  12
                -----
              ) 2250 ( 50 lb. of tea must be given in barter.
                2250
```

Otherwise,

Since the price of one yard of linen multiplied by the number of yards, is equivalent to the price of one lb. of tea multiplied by the number of lbs.; therefore if these equal products be put into a proportional statement, the question will stand thus:*

As 3s. 9d. : 2s. 6d. : : 75 yds. : x lbs. = 50 lbs. of tea.

(2.) Received 75 yards of linen, for 50 lb. of tea, at 3s. 9d. per lb.; what is the price of the linen per yard?

s. d.
3 9
50

75) £9 7 6 the price of the tea, and also the price of the linen.

2 6 the price of one yard.

Otherwise, by proportion,

As 75 : 50 : : 3s. 9d. : x = 2s. 6d.

(3.) Bought 63 gallons of brandy, at £1 4s. 6d. per gallon; paid in cash £25 19s. 9d., and the remainder in cloth, at 5s. 3d. per yard; how many yards of cloth were given?

1 4 6
63

77 3 6 = { price of brandy and = cash and price of the cloth together.

25 19 9 cash.

5s. 3d.) 51 3 9 = the price of the cloth.
12 20

63 1023
12

63) 12285 (195 yards.
63

598
567

315
315

---

* *See* note to proportion, where equality of products is made the subject of consideration.

EXERCISES 43.

1. How many yards of linen at 1s. $3\frac{1}{2}$d. per yard, must be given to pay for 3720 lb. of cotton wool, at $8\frac{1}{4}$d. per lb. ?
2. Exchanged 48 yards of cloth for 12 cwt. 3 qr. 24 lb. of sugar at £2 6s. 8d. per cwt.; what price per yard was received for the cloth?
3. A servant woman was hired on the 5th of May, at £12 18s. $6\frac{1}{2}$d. a year, and left on the 17th of August following. With the wages received she bought stockings at $6\frac{1}{2}$d. per pair to sell again. How many pairs had she for her money?
4. How many yards of cloth at 8s. 6d. the yard, will pay for 9 cheeses, each weighing 2 cwt. 3 qr. 18 lb. at £2 18s. 4d. per cwt.?
5. For 33 cwt. 0 qr. 9 lb. of coffee I exchange 35 cwt. 2 qr. 14 lb. of tobacco, at £6 18s. 8d. the cwt.; what is the cost of the coffee per lb. ?
6. Bought $34\frac{3}{16}$ pieces of cloth at £$13\frac{3}{8}$ the piece, and paid in cash $135\frac{7}{8}$ guineas, and the rest in wool, at $7\frac{3}{5}$ shillings per stone; how many stones of wool were given?
7. For 251 pieces of linen, each measuring $50\frac{1}{4}$ yards, at $17\frac{3}{4}$d. the yard, and 365 pieces of Nankeen, each containing 28 yards, at $15\frac{3}{4}$d. the yard, I received bags of cotton, each weighing 3 cwt. 1 qr. 5 lb., and valued at $15\frac{3}{4}$d. the lb.; how many bags did I receive?
8. Sent to Barbadoes a chest of tea worth £250; two parcels of silk worth £362; a crate of china which cost £155 15s. 5d., and 57 hams weighing 8 cwt. 3 qr. 14 lb., at 8d. the lb. The freight was £32 5s.; charges of shipping, &c., £2 5s. 6d.; insurance on the above, cost and expenses, at 15s. per cent. How much cotton ought to be received in return, at $11\frac{1}{8}$d the lb.?
9. How many yards of cloth at 5s. 8d. the yard, will pay for $3\frac{5}{8}$ bags of coffee, each weighing 6 cwt. 3 qr. 18 lb., at £6 10s. 8d. per cwt. ?
10. How many gallons of brandy at £1 5s. 9d. the gallon, will pay for $21\frac{1}{3}$ pieces of cloth, each containing $21\frac{1}{2}$ yards, at 5s. 10d. per yard ?
11. Sold 89 cwt. 3 qr. 25 lb. of coffee at £5 16s. 8d. per cwt., and received in payment rum at 15s. a gallon, and brandy at 30s. a gallon, of each an equal quantity. How many gallons of each did I receive?

12. How many yards of cloth at 9s. 3d. the yard, together with £13 16s 3d. in cash, will pay for 17 cwt. 3 qr. 18lb. of rice, at £1 18s. 8d. per cwt.?

PARTNERSHIP AND PROPORTIONAL PARTS.

118. When two or more persons unite in partnership to carry on some commercial or industrial undertaking, and the profit or loss, as the case may be, is to be distributed among the partners, it must be divided into parts which shall be proportional to the shares or capital of the respective partners, which, in other words, means division of a given quantity into parts, which shall be to each other in the ratio of certain numbers.

Rule.—As the sum of the given numbers is to any one of them, so is the whole quantity to be divided, to a part of it corresponding to the term used.

EXAMPLES.

(1). Two merchants, A. & B. join in trade; A.'s capital is £500, and B.'s £300, and their profits are £120; what is each person's share?

£500
300

As 800 : £500 : : £120 : $x =$ £75 = A.'s profit.
As 800 : 300 : : 120 : $x =$ 45 = B.'s profit.

£120 sum.

(2). Divide 180 into 3 parts which shall have the ratio of 3, 4, and 5.

3
4
5

As 12 : 3 : : 180 : $x = 45$
12 : 4 : : 180 : $x = 60$
12 : 5 : : 180 : $x = 75$

180 sum.

(3). A reservoir is filled by one pipe in 12 hours, by a second in 8 hours, and by a third in 6 hours. In what time will it be filled by the 3 pipes when running together?

The 1st fills $\frac{1}{12}$ of the reservoir in 1 hour.
„ 2nd „ $\frac{1}{8}$ „ 1 „
„ 3rd „ $\frac{1}{6}$ „ 1 „

As $\frac{3}{8} : 1 :: 1$ hour : x hours $= 2\frac{2}{3}$ hours.

EXERCISES 44.

1. Divide £60 among A. B. and C., so that their portions may be as 3, 5, and 7, respectively.
2. A. has $\frac{1}{2}$, B. $\frac{1}{3}$, and C. $\frac{1}{6}$ of a ship, whose freight, after deducting all expenses, amounts to £750; what is each person's share?
3. Pure water is composed of two gases, oxygen and hydrogen, in such proportion that 9 lb. of water will yield 8 lb. of oxygen and 1 lb. of hydrogen; what weight of each gas can be had from 1 cwt. of water?
4. A bankrupt, whose property realises only £875, owes three creditors, A. B. and C., £350, £450, and £600 respectively; what should each receive, and how much in the pound?
5. British gold coinage consists of 11 parts of pure gold, and 1 part of alloy, the composition being termed *standard* gold; what is the value of 3 lb. of *pure* gold, when the value of 1 lb. of standard gold is £46 14s. 6d.?
6. Limestone, or carbonate of lime, consists in the union of 28 parts of lime, to 22 parts of carbonic acid. What weight of lime will 100 tons of limestone yield, when freed from its carbonic acid by being burned in a kiln?
7. A father divides his estate among his four sons; to the eldest he gives $\frac{1}{3}$, to the second $\frac{1}{4}$, to the third $\frac{1}{5}$, and to the youngest the remainder, consisting of 182 acres; how many acres are there in the estate, and what is the share of each?
8. Suppose bell-metal to be composed of 3 parts of copper and 1 of tin; how much of each will be required to make a bell weighing 150 lbs.?
9. A. owned $\frac{1}{3}$, B. $\frac{1}{4}$, and C. $\frac{5}{12}$ of a ship, which was insured for £4200; the ship being lost, what sum will each be entitled to?
10. Two persons, A. and B., enter into partnership; A., as manager, and B., as sleeping partner; A. contributing to the firm £3000, and B. £3000. Now the former is to receive 20 per cent. of the profits for management; and at the end of the year their clear gain is £1350; what is each person's share?

EXAMPLES.

(4.) A., with a capital of £1500, commenced business on the 1st of January, 1860; and on the 1st of March following, B. joined, with a capital of £1800; three months afterwards, C. was admitted upon subscribing £2100. On the 1st of January, 1861, the profits amounted to £3250; what was each person's share?

$$\begin{array}{lrcrcr} & \pounds & & \text{mo.} & & \pounds \\ \text{A.} & 1500 & \times & 12 & = & 18000 \\ \text{B.} & 1800 & \times & 9 & = & 16200 \\ \text{C.} & 2100 & \times & 6 & = & 12600 \\ \end{array}$$

$$\begin{array}{rcccl} & & \pounds & & \pounds \\ \text{As } 46800 : 18000 & :: & 3250 : x & = & 1250 = \text{A.'s.} \\ 46800 : 16200 & :: & 3250 : x & = & 1125 = \text{B.'s.} \\ 46800 : 12600 & :: & 3250 : x & = & 875 = \text{C.'s.} \end{array}$$

11. Cane sugar is composed of carbon, oxygen, and hydrogen, in the proportions of 72, 88, and 11 respectively. How many cwt. of each can be obtained from 57 tons of sugar?

12. Two persons, A. and B., are partners in trade; A. with a capital of £1600, and B. with £2000; A. is to receive 12½ per cent. of the profits for managing the business, and the remaining profits are to be divided according to each person's capital. The entire profit at the year's end is £960; how much of it ought each receive?

13. Divide a common of 200 acres between A., B., C., and D., whose estates on which their claims are founded, are respectively valued at £500, £350, £220, and £90; and find the income accruing to each from this common, at 12s. 6d. per acre.

THE TERM *per cent.*—In interest, stock, commission, and profit and loss, when the term *per cent.* occurred it referred to 100 of money, but this term is also used when 100 is an abstract or concrete number of any kind.

By the census of 1861, the population of England and Wales consisted of 9,758,852 males, and 10,302,873 females; what are the number of females in each hundred of the entire population?

In 1851 the population of England and Wales was 17,927,609, and in 1861, 20,061,725; find the increase per cent. in this interval?

Standard silver is composed of 37 parts of silver and 3 parts of copper; how much per cent. is there of silver?

Find how much per cent. 9 is of 16?

The population of the United Kingdom in 1851 was 27,511,862, and in 1861, 29,193,319; what was the increase per cent. in the 10 years?

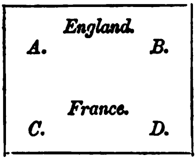
EXCHANGE.

119. **Exchange**, in arithmetic, is the method of finding what quantity of money of one country is equivalent to a given sum of another country.

The *par of Exchange* is the standard value of the money of one country in terms of another, deduced from a comparison of their weight and fineness, and is always fixed. According to the Mint regulations of England and France, £1 sterling is equal to 25 francs 22 centimes, which is, therefore, the *par of Exchange* between London and Paris.

The *course of Exchange* means the sum of money of any place which is given at a particular time for a fixed sum in another; because in foreign exchanges one country receives from another, for a fixed sum, an equivalent sum, which is liable to variation from the circumstances of trade. As when London receives 25 francs 10 centimes for £1 sterling at one time, and 25 f. 40 c. at another.

120. Exchange and its fluctuations have their chief foundation in trade. For suppose *A.* and *B.* to represent two merchants in London; *C.* and *D.* two others is Paris. Let *A.* import £1000 worth of goods purchased from *C.* for 25000 francs, while *D.* imports £1000 worth of goods purchased from *B.* and valued at 25000 francs. Instead of £1000 being sent across the channel by *A.* to *C.*, *A.* pays over to *B.* the £1000 which *D.* owes him, and in return *B.* gives *A.* a bill on *D.*, which *A.* *indorses* and transmits to *C.*, who presents it to *D.*, and receives its amount. By this means *A.* and *D.* discharge their respective debts, without any coin having left either country; the bill or document fulfilling this object being called a *Bill of Exchange.* But when the commercial transactions between both countries are very numerous, as, for instance, in 1858, when our imports amounted to £13,911,401, and our exports to £4,861,558, then bills or specie, or both, must be sent to France to pay off the surplus. But from the great competition in the London market for bills on Paris, *B.* will not let *A.* have the bill on *D.* of 25000 f. for £1000, because he can get more for it from others; and as *A.* knows that if he sends the £1000 in specie, he must pay carriage and insurance, he does not, therefore, hesitate to give more than £1000 for



B.'s bill; nevertheless, the increase he pays in *price* can never exceed the expense attending the transmission of the specie. In such a case as this, it is usual, in the language of commerce, to say, that the exchange is *against* England and *in favour* of France.

Course of Exchange.

England receives from—

	Belgium[*] *France* *Switzerland*	± 25 francs 22 cents., for £1 sterling.
	Austria	± 10 florins 85 kreutzers, for £1 sterling.
Germany.	*Saxony* *Hanover* *Prussia*[†] *Brunswick* *Hesse Cassel*	± 6 thalers, or dollars, 28 silver groschen, for £1 sterling.
	Bavaria[‡] *Wurtemberg* *Baden* *Frankfort on the Maine*	± 12 florins (golden) 0 kreutzer, for £1 sterling.
	Hamburg[§]	± 13 marks banco 10 schillings, for £1 sterling.
	Holland	± 12 florins (or guilders) 0 cents., for £1 sterling.
	Sardinia *Genoa*[‖]	± 25 lira 22 centisimi, for £1 sterling.

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[*] Belgium and Switzerland use the French system of money.

[†] In Hanover and Brunswick the *thaler* has the same value as in Prussia, but, instead of being divided into 24, it is divided into 30 groschen.

[‡] By the currency treaty, established in January, 1857, between Austria, Prussia, Bavaria, Saxony, Hanover, and several of the smaller states, the *Cologne mark* of fine silver weighs 7 oz. 10 dwt. 9 grains (English weight), and the different kinds of money used in Germany are valued by this standard.

Out of this Cologne mark (3609 grains) of fine silver, are coined— - -

- 14 thalers, or dollars, *standard of Prussian money.*
- 21 florins, *standard of Austrian money.*
- 24½ florins, *standard of Suddeutsche Wahrung money.*
- 27½ marks *Banco standard* } *of Hamburg money.*
- 34 marks *current* „ }

[§] In Hamburg there are two kinds of money,—*banco* and *currency*, which are as 111 banco to 136 currency.

[‖] The decimal system was introduced, in 1827, into Sardinia and Genoa. The weight and fineness of the coins are the same as those of France.

Rome ± 4 scudi 75 bajocchi, for £1 sterling.

Sweden } ± 4 riks dalers (silver specie) 54 ore, for £1 sterling.
Norway }

Denmark ± 9 rigsbank dollars 8 skillings, for £1 sterling.

Greece ± 28 drachma 20 lepta, for £1 sterling.

Turkey ± 108 piastres, for £1 sterling.

United States * ± 4 dollars 86 cents., for £1 sterling.

British America † ± 24 shillings and 4 pence, Halifax currency, for £1 sterling.

Jamaica ± 33 shillings and 4 pence, currency, for £1 sterling.

England gives to—

Russia about 37½ pence for 1 rouble.
Portugal about 56 pence for 1 Milreà.
Spain about 50 pence for 1 hard, or Spanish dollar, or pistole.
East Indies ‡ about 22¼ pence for 1 Company's rupee.
China § about 75 pence for 1 tael.

France.	Value in English money, nearly.		
	£	s.	d.
10 centimes = 1 decime or 2 sous - -	0	0	1
10 decimes = 1 franc - - - -	0	0	9·69
Austria.			
100 kreutzers = 1 florin - - - -	0	1	11¼

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* In 1837, an Act of Congress fixed the gold and silver coinage of the United States to be comprised of $\frac{9}{10}$ pure metal and $\frac{1}{10}$ alloy; and in 1842, the same Congress fixed 4·84 dollars to be the legal value of the English sovereign, thereby making the value of the dollar 4s. 1·6d.

† In British America, accounts were kept in what has been usually called Halifax currency,—the £1 sterling being reckoned at £1 4s. 4d. currency. But, in 1859, a new description of currency was issued on the decimal plan. The pound currency is divided into 100 centimes, containing 101·321 grains of standard gold, or 92·877 grains of pure gold. The new gold coins are for 100 and for 50 centimes, with silver coins for 20, 10, and 5 centimes. The 20 centimes piece is nearly of the value of the United States dollar.

‡ In 1835 the currencies of the three Presidencies were assimilated, and, since then, only one kind of rupee has been minted, called the *Company's rupee*.

§ Foreign coins, and more especially Spanish dollars, circulate in Canton,—720 taels being considered equivalent to 1000 dollars. The dollar, for the purpose of making small payments in Canton, is cut into sub-divisions, the weight of each being ascertained by scales, called *dotchin*, and valued accordingly.

		Value in English money, nearly.		
Saxony, Prussia, Hesse, Cassel, &c.[*]				
12 pfennings	= 1 silver groschen	0	0	1¼
30 silver groschen	= 1 thaler or dollar	0	2	10¾
Bavaria, Wurtemberg, Baden, Frankfort on the Maine, &c.				
4 pfennings	= 1 kreutzer			
60 kreutzers	= 1 florin suddeutsche	0	1	8
Hamburg.				
12 pfennings	= 1 schilling	0	0	1
16 schillings	= 1 mark banco	0	1	5½
8 marks	= 1 rix dollar	0	4	4½
Holland.				
100 cents.[†]	= 1 florin or guilder	0	1	8
16 pfennings (formerly)	= 1 stiver.	0	0	1
20 stivers	= 1 florin or guilder	0	1	8
Sardinia and Genoa.				
100 centisimi	= 1 lira nuova[‡]	0	0	9½
Rome.				
	1 bajoccho	0	0	0½
10 bajocchi	= 1 paolo	0	0	5
10 paoli	= 1 scudo or silver crown	0	4	2½
Sweden and Norway.[§]				
100 Ore	= 1 riksdaler-riksmynt	0	1	1¼
4 riksdalers-riksmynt	= 1 silver specie riksdaler	0	4	5

Note.—The riksdaler banco is a paper currency, issued by the National Bank, which is only ⅜ths of the riksdaler specie.

Denmark.[‖]				
16 skillings	= 1 mark	0	0	4½
6 marks	= 1 rigsbank dollar	0	2	2½

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[*] "The Germanic Confederation are about to introduce a uniform *decimal system* of *weights* and *measures*, based upon the *metrical system* of France. The unit of length is to be the *metre*, subdivided into 100 *centimetres*."—*London Review*, of Feb. 23rd, 1861.

[†] In 1820, the decimal system of 100 cents., equal to 1 florin, was introduced. The other coins, besides the guilder (or florin), are the 3 florin piece, the rix-dollar of 50 stivers, or 250 cents., with pieces of 25, 10, and 5 cents each.

[‡] Since the French system of coinage, which was adopted in Sardinia and Genoa in 1827, now prevails throughout the whole of Italy, (Rome excepted), the money calculations of both countries are consequently identical.

[§] A decimal system of coinage has been recently adopted in Sweden and Norway.

The *Cologne Mark* of 3609 grains makes 18½ Rigsbank dollars.

	Value in English money, nearly.		
Greece.			
100 lepta = 1 drachma	0	0	8½
10 drachma piece	0	7	0
Turkey.[*]			
40 paras = 1 piastre	0	0	2¼
100 piastres = 1 sequin	0	18	0
United States.[†]			
10 mills = 1 cent	0	0	0½
10 cents = 1 dime	0	0	5
10 dimes = 1 dollar	0	4	2
10 dollars = 1 eagle	2	1	1
Russia.[‡]			
100 copees = 1 rouble	0	3	1½
Portugal.			
1000 reas = 1 milrea	0	4	8
400 reas = 1 old crusado of silver	0	1	10½
480 reas = 1 new crusado of silver	0	2	3
5 milreas = 1 gold crown	1	3	4
15 milreas = 1 dobraon	3	10	0
Spain.			
34 maravedis vellon or 100 centavas = real vellon	0	0	2½
20 reals vellon = 1 hard dollar	0	4	2
80 " " = 1 pistole	0	16	8
100 " " = 1 Isabella, issued in 1848	1	0	6
320 " " = 1 doubloon	3	5	8
East Indies.			
12 pies or pice = 1 anna	0	0	1½
16 annas = 1 company's rupee	0	1	10¼
16 rupees = 1 gold mohur	1	9	8
China.			
1 tael = 10 mace = 100 candareens = 1000 cash§	0	6	3

720 taels = 1000 Spanish dollars.

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[*] The *piastre* of Turkey, which at one time was worth 2s. (English), is now so debased as to be only valued at 2¼d.

[†] The weights and measures of the United States are the same as those of Great Britain, with the exception of the measures of capacity.
Thus the unit of dry capacity is the old Winchester bushel of 2150·4 cubic inches.
" " of liquid capacity is the old wine gallon of 231 cubic inches.

[‡] The *gold* coins of Russia are the *ducat* = £0. 9s 6d.
" " " " *imperial* = £1 12s. 3d.

§ The dollar, in this instance, is valued at 4s. 6d.

British West Indies.

Accounts are kept in pounds, shillings and pence, of a depreciated currency, and until lately the value of that currency varied in different islands. In 1838, however, the legislature of Jamaica fixed £166 13s. 4d. Jamaica currency, to represent £100 sterling, which is in the proportion of 5 to 3; at the same time, the American dollar was fixed at 4s. 2d., and the doubloon at 64 shillings.

To reduce English money to French money, and the contrary.

EXAMPLE.

(1.) How much French money, of 25f. 30c. per £1 sterling, is equal to £300 10s. English.

```
          £     £   s.      f. c.          f.  c.
     As 1 : 300 10 : : 25 30 : x = 7602 65
               20         100                  or thus.
           ———————      ——————                 f.  c.
             6010        2530                  25  30
              2530                                300
           ———————                            ————————
            180300                            7590   0
           300500                  10s. ½d.     12  65
          120200                              ————————
          ————————                            7602  65
   2,0 ) 1520530,0
          ————————
  1,00 )  7602,65
          ————————
     Ans. 7602f. 65c.
```

EXERCISES 45.

France.

1. In £250 5s. sterling, how much French money at 25f. 20c. per £1 sterling?
2. What is the value, in English money, of 8970f. 50c, at 25f. 45c. per £1 sterling?
3. In £750 12s. 6d. sterling, how much French money, at 23f. 40c. per £1 sterling?
4. In 2246f. 27c., how much English money, at 23f. 35c. per £1 sterling?

Austria.

In £180 English, how many Austrian florins, at 9fl. 55k. per £1 sterling?

f. k.
As £1 : £180 : : 9 55 : x = 1785 florins.
60
595
180
6,0) 10710,0
Ans. 1785 florins.

Or thus,
fl. k.
9 55
180
Ans. 1785fl. 00

5. In £750 10s. sterling, how many Austrian florins, at 10 fl. 4 k. per £1 sterling?
6. How much sterling money is there in 8453 fl. 20 k. Austrian money, at 10 fl. per £1 sterling?

Prussia.

In £120 5s. sterling, how many Prussian thalers, at 6 th. 20 grosc. per £1 sterling?

£ s. th. gros.
As 1 : 120 5 : : 6 20 : x = 801th. 20 gros.
20 20 30
20 2405 200
200
2,0) 48100,0
3,0) 2405,0
801th. 20 gros.

Or thus,
th. gros.
6 20
120
800 0
5s. ¼d. 1 20
801 20

7. In £463 sterling, how many Prussian dollars, at 6 thal. 25 gros. per £ sterling?
8. In 3331 dollars 14 gros. Prussian, how much English money, at 6 thal. 28 gros. per £1 sterling?

Baden, Bavaria, Frankfort on the Maine, &c.

9. Exchange £482 10s. 5d. sterling, for Suddeutsche florins, at 1s. 8d. per florin.
10. Exchange £574 5s. 5⅜d. sterling, for Suddeutsche florins, at 1s. 7¾d. each.
11. Exchange 17370 florins 45 kreutzers Suddeutsche, for sterling money, at 1s. 8d. each.
12. Exchange 20935 florins 30 kreutzers Suddeutsche, for sterling, at 1s. 7¾d. each.

Hamburg.

13. What is the value of £420 sterling in Hamburg money, at 18 m. banco 10 sh. per £1 sterling?
14. In 7429 marks b. 9 sh. 4 pfennings, how much English money, at 13 m. 12s. per £1 sterling?

Holland.

15. In £387 English, how much Flemish money, at 11 florins 60 c. per £1 sterling?
16. In 12092 florins 5 st. Flemish money, how much English, at 11 florins 10 st. per £1 sterling?
17. In 4428 florins 45 cents Flemish money, how much English, at 11 florins 70 cents per £1 sterling?

Genoa.

18. In £463 13s. 4d. English, how much Genoese money at 25 liras 20 centisimi per £1 sterling?
19. In 15841 liras 85 centisimi of Tuscany, how much English money at 25 liras 25 centisimi per £1 sterling?

Rome.

20. In £387 English, how much Roman money, at 4 scudi 7 paoli 5 bajocchi per £1 sterling?
21. In 3470 scudi 95 bajocchi Roman money, how much English, at 4 sc. 70 baj. per £1 sterling.

Sweden and Norway.

22. In £450 English, how much Swedish money, at 4 riksdalers silver 54 ore per £1 sterling?
23. 3242 riksdalers (silver specie) 25 ore, how much English money, at 4 riksdalers silver per £1 sterling?

Denmark.

24. Exchange £96 15s. 7¼d. sterling, for rigsbank dollars, at 2s. 2½d. per dollar.
25. Exchange £546 0s. 11¼d. sterling, for rigsbank dollars, at 2s. 3d. each.
26. Exchange 2629 rigsbank dollars 3 marks for sterling, at 2s. 2½d. each dollar.

Greece.

27. In £327 English, how much Greek money, at 28 drachma 25 lepta per £1 sterling?
28. In 10456 drachma 85 lepta, Greek money, how much English, at 28 dr. 30 lep. per £1 sterling?

United States.

29. Find the value of £387 10s., English, at 4 dollars 86 cents. per £1 sterling.

30. Find the value of £568 15s., English, at 4 dollars 84 cents. per £1 sterling.

31. Find the value of 5649 dollars 75 cents., at 4 dollars 86 cents. per £1 sterling.

32. Find the value of 5505 dollars 50 cents., at 4 dollars 84 cents. per £1 sterling.

33. Find the value of 479 dollars 25 cents., at 4s. 2d. sterling per dollar.

Russia.

34. Exchange £84 7s. 6d. sterling, for roubles, at 3s. 1½d. each.

35. „ £136 15s. 11¼d. sterling, for roubles, at 3s. 1½d. each.

36. Exchange 1620 roubles for sterling, at 3s. 2d. per rouble.

37. „ 2626 roubles 50 copecks for sterling, at 3s. 1½d. per rouble.

Portugal.

38. Exchange £105 8s. 6d. sterling, for milreàs, at 4s. 8d. each.

39. „ £81 19s. 6¾d. sterling, for milreàs, at 4s. 7½d. each.

40. „ 1852 milreàs 250 reas, for sterling, at 4s. 8d. each.

41. „ 1068 milreàs 500 reas, for sterling, at 4s. 7½d. each.

Spain.

42. Exchange £78 8s. 6½d. sterling, for Spanish hard dollars, at 4s. 2d. each.

43. Exchange £98 5s. 6¾d. sterling, for Spanish hard dollars, at 4s. 1½d. each.

44. Exchange 753 hard dollars 7 reals 17 maravedis, for sterling, at 4s. each dollar.

45. Exchange 879 hard dollars 15 reals, for sterling, at 4s. 1d. per dollar.

East Indies.

46. Exchange £827 11s. 10d. sterling, for rupees, at 1s. 11½d. each.

47. „ £686 11s. 5½d. sterling, for rupees, at 1s. 11¾d. each.

48. „ £594 15s. 6¾d sterling, for rupees, at 2s. each.

49. „ 20,814 rupees, for sterling, at 1s. 11¾d. each.

50. „ 11,895 rupees 9 annas, for sterling, at 2s. each.

China.

51. Exchange £284 7s. 6d. sterling, for taels, at 6s. 6d. each.

52. „ £213 19s. 8¼d. sterling, for taels, at 6s. 3d. each.

53. Exchange 437 taels 5 mace, for sterling, at 6s. 6d. per tael.
54. „ 2054 taels 2 mace 5 candareens, for English money, at 6s. 3d. per tael.

Exchanged £250 5s. for French money, at 25 francs 40 centimes per pound sterling.

$$\begin{array}{lcccccccc} & \text{£} & & \text{£ s.} & & \text{f. c.} & & & \text{f c.} \\ \text{As } 1 & : & 250\ 5 & :: & 25\ 40 & : & x & = & 6356\ 35\ \textit{Ans.} \end{array}$$

Or thus,

$$\begin{array}{lr} & \text{f. c.} \\ & 25\ 40 \\ & 250 \\ \hline & 6350\ \ 0 \\ \text{5s. }\tfrac{1}{4}\text{d.} & 6\ 35 \\ \hline \end{array}$$

Fr. 6356 35 cents. *Ans.*

EXERCISES 45A.

1. Convert £375 6s. 8d. into French money, at 24 francs 90 cents. per £1 sterling.
2. How many £s sterling must I pay in Liverpool, to receive 822f. 40 centimes in Paris, at 25 francs 70 c. per £1 sterling?
3. For 7295 francs 40 centimes, I receive £284 19s. 6¼d., how much is that for £1 sterling?
4. What is the value in English money of 7950 francs 72 cents, at 24 francs 30 centimes per £1 sterling?
5. If for £247 15s. 9d. I receive 6293 francs 80 centimes, how much is that per £1 sterling?
6. In £7505 5s. 6d., how many American dollars, of 4s. 5½d. each?
7. How many Spanish dollars of 4s. each, can I have for £701 10s ?
8. What is the value of £759 18s. 9d. in French money, at 23f. 45c. per £1 sterling ?
9. What is the sterling value of 15186 dollars 40 cents., at 4s. 7d. per dollar?
10. What is the cost of 15186·4 yards of cloth, at 4s. 7d. the yard?
11. Find the value of 5379 francs 65 centimes, at 24 francs 34 centimes per £1 sterling?
12. What is the value, in English money, of 9187 francs 85 centimes, at 23 francs 95 centimes per £1 sterling?
13. If for 7295 francs 40 centimes, I pay £284 19s. 6⅜d., what is the exchange per £ sterling?
14. What is the value in English money of 7454 Russian roubles 25 copecks, at 3s. 2d. per rouble?

15. Received from J. Renceir & Co., of Havre, 139 qr. 5 b. 3 pks. of wheat, to be sold for their account; paid in charges £15 8s. $7\frac{1}{2}$d.; sold the wheat at £3 7s. 9d. per quarter, commission $2\frac{1}{2}$ per cent.; and remitted them a bill for the net proceeds at 24 francs 50 cents. per £1 sterling. How many francs do they receive?

16. Sold $899\frac{118}{196}$ barrels of flour, at £1 17s. 9d. per barrel, for J. M'Henry, of New York, commission $2\frac{1}{2}$ per cent.; charges £12 13s. $5\frac{1}{2}$d.; how many dollars, at 4s. 6d. each, will he receive as his net proceeds?

17. Sold $877\frac{35}{196}$ barrels of flour, for J. Pinolf of Bordeaux, at £1 13s. $7\frac{1}{2}$d. per barrel; paid in charges, £10 13s. $8\frac{1}{2}$d.; commission, $2\frac{1}{2}$ per cent.; and remitted the net proceeds at 24 francs 75 centimes per £1 sterling to Bordeaux. How many francs does he receive?

18. In 2643 Spanish dollars 10 reals, how much English money, the Exchange being at 4s. $5\frac{1}{4}$d. per dollar?

19. In 5000 milreas 250 reas, how much English money, at 5s. 2d. per milrea?

20. How many £s sterling can I have for 1359 milreas 520 reas, at 5s. $2\frac{1}{2}$d. per milrea?

21. How many milreas can I have for £245 8s. $10\frac{1}{2}$d., at 5s. $8\frac{1}{4}$d. per milrea?

22. Purchased in Manchester, for John Dwyer, of New York, 12651 yards of silk, at 1s. 6d. per yard, and 12640 yds., at 1s. $1\frac{1}{2}$d. per yard; paid as charges, £5 5s. 2d., and commission 2 per cent., which amounts to £33 3s. 4d.; what is the cost in dollars, at 4 dol. 80 cents. per £1 sterling?

23. What will be the sum purchased in the 3 per cent. consols, at $94\frac{3}{8}$ per cent., by 48177 francs $67\frac{1}{2}$ centimes, at 25 francs 35 centimes per £1 sterling; and what is the annual interest?

24. Find the interest in sterling money, of 8379 American dollars 25 cents., at 4s. 4d. per dollar, for 2 years 7 months, at 5 per cent.?

25. What is the interest in English money, of 18566 francs 13 centimes, at 25 francs 40 centimes per £1 sterling, for 2 years 11 months, at 5 per cent.?

26. The imports from France to the United Kingdom amounted, in 1858, to £13·911401, while the exports to France were only £4·861558; what was the value of the balance to France, in French money, at 25 francs 50 centimes per £1 sterling?

27. Imported from France in 1858, 1·695405 quarters of grain, or its equivalent of flour, at £2 16s. per quarter; what is its value in French money, when the exchange is at *par?*

28. The Italian war of 1859 cost Austria 200,000,000 florins; what is the value of this sum in English money, if 9 florins 50 kreutzers be worth £1?

29. What is the value of the preceding, in French money, if 25 francs 20 centimes equal 9 florins 50 kreutzers?

ARBITRATION OF EXCHANGES.

121. In conducting the business of exchanges, that course is adopted in sending remittances abroad, which gives the greatest sum in Foreign money, and costs the least in sterling; and for remittances home, that course is taken, which costs the least Foreign money, and yields the greatest sum in sterling. To realize these objects a direct remittance is not always the course adopted: for when a merchant in England has to discharge a debt due to another in France, it will be his interest, ere he does so, to inquire whether it will be more advantageous to buy a bill on New York, Hamburg, Lisbon, &c., and direct his agent there to invest the proceeds in a bill on Paris, rather than purchase a bill on Paris direct.

The operation of finding the *proportional rate* between England and France, through any of the intermediate places of Lisbon, New York, Hamburg, &c., or all of them, is termed the *Arbitration of Exchanges.*

When there is only one intervening place, it is said to be a *simple* arbitration: when more than one, a *compound arbitration.*

EXAMPLE.

If £1 sterling be equal to 13 marks banco 8 schillings, Hamburg, or 25 francs 20 cents., Paris, and 1 mark Hamburg equal to 1 franc 87½ centimes, Paris; whether is it better in paying a debt of 5040 francs in Paris, to remit it directly to Paris, or send it indirectly through Hamburg.

As 1m. : 13m. 8s. : : 1f. 87½c. : x = 25f. 31¼c. { the arbitrated exchange for £1.

As 25f. 31¼c : 5040 : : £1 : x = £199 2s. 2⅔d. { the amount required to discharge the bill indirectly *viâ* Hamburg.

As 25f. 20c. : 5040f. : : £1 : x = £220 $\left\{ \begin{array}{l} \text{the sum required to purchase} \\ \text{the bill directly.} \end{array} \right.$

Hence there is an advantage of 17s. 9⅓d. in remitting the payment of this debt indirectly.

By the Chain Rule.

Let x = the arbitrated francs to £1

$$\begin{aligned} \text{then } x &= £1 \\ £1 &= 216 \text{ schillings} \\ 16 \text{ sch.} &= 187\tfrac{1}{2} \text{ centimes} \\ 100 \text{ cents.} &= 1 \text{ franc} \\ \hline 100 \times 16 \times x &= 187\tfrac{1}{2} \times 216 \\ 200\,x &= 5062\tfrac{1}{2} \\ x &= 25\text{f. } 31\tfrac{1}{4}\text{c.} = \text{the arbitrated exchange.} \end{aligned}$$

EXERCISES 45A—*continued.*

30. If the exchange between Liverpool and Amsterdam be 11 florins 60 cents. per £1, and the exchange between Amsterdam and Havre be 4 florins for 9 francs; required the arbitrated exchange between Liverpool and Havre, and the value of £635 remitted directly and indirectly, when the direct exchange is 25 francs 40 cents. per £1 sterling.

The firm of A. and B. of London have a debt due to them in Bourdeaux of 35420 francs, which they can draw directly at 25 francs 30 cents. per £1, or have it sent to St. Petersburg at 3½ francs per rouble; from St. Petersburg to Frankfort on the Maine, at 4 roubles for 7½ florins; from Frankfort on the Maine to Copenhagan, at 2½ florins for 2 rigsbank dollars; and from Copenhagan to London at 8 rigsbank dollars 72s. per £1 sterling; what is the arbitrated exchange, and the more profitable course?

By Chain Rule.

$$\begin{aligned} \text{Let the arbitrated exchange be } x &= £1 \\ £1 &= 835 \text{ schillings} \\ 96 \text{ schillings} &= 1 \text{ rigsbank dollar} \\ 2 \text{ rigsbank dollars} &= 2\tfrac{1}{2} \text{ florins} \\ 7\tfrac{1}{2} \text{ florins} &= 4 \text{ roubles} \\ 1 \text{ rouble} &= 3\tfrac{1}{2} \text{ francs} \\ \hline 15\,x &= 306\tfrac{1}{4} \\ x &= \frac{306{\cdot}25}{15} = 20\text{ f. } 41 \text{ cents} \end{aligned}$$

As 20f. $41\frac{2}{3}$c. : 35420f. : : £1 : x = £1734 17s. $1\frac{5}{7}$d. $\left\{\begin{array}{l}\text{the value by}\\\text{indirect route.}\end{array}\right.$

As 25f. 30c. : 35420f. : : £1 : x = £1400 the value by direct route.

Hence the indirect route is the more profitable by £334 17s. $1\frac{5}{7}$d.

ANNUITIES.

122. **Annuity** is the term applied to the periodic payment of money at fixed intervals; and according as the periodic payments are made yearly, half-yearly, quarterly, or monthly, the annuity is said to be a yearly, half-yearly, quarterly, or monthly annuity.

Annuities are termed *certain*, if payable for a definite number of years; *contingent*, if their duration depends on the existence of one or more lives. In the latter case they are called *life annuities.*

Annuities *certain* will be the subject of consideration here. An annuity not to be entered on for a number of years, is termed a *deferred* or *reversionary* annuity.

EXAMPLES.

(1). If an annuity of £250, paid annually, be left unpaid for 4 years, what sum is then due at 5 per cent. simple interest?

The 1st £250 bears int. for 4 yrs., and yields	£50	0	0
„ 2nd £250 „ 3 „ „	37	10	0
„ 3rd £250 „ 2 „ „	25	0	0
„ 4th £250 „ 1 „ „	12	10	0
	125	0	0
£250 income for 5 years	1250	0	0
	£1375	0	0 = the sum due at the end of 4 years.

(2). What sum would the foregoing annuity of £250 amount to, if left unpaid for 4 years, at 5 per cent. compound interest?

The amount of an annuity of £1 left unpaid for 4 years $= 1 + 1{\cdot}05 + 1{\cdot}05^2 + 1{\cdot}05^3 + 1{\cdot}05^4 = 5{\cdot}52563125$, which multiplied by $250 =$ £1381·4078125 = £1381 8s. $1\frac{7}{8}$d.

To save time, a table, containing the amount of an annuity of

1, at 3, 4, 5, and 6 per cent. compound interest, is constructed to 0 years, by geometrical progression.*

Or,

Table II. may be constructed from Table I.; thus:—

1·0000 = the first annuity and first year in Table II.
1·05 = the amount of £1 for 1 year, Table I.

2·0500 = the amount for 2 years in Table II.
1·1025 = second year in Table I.

3·1525 = third year in Table II.
1·157625 = third year in Table I.

4.310125 = fourth year in Table II.
1·215506 = fourth year in Table I.

5·525631 = fifth year in Table II. = $\left\{\right.$ the amount of £1 for 5 years, or left unpaid for 4 years.
250

1381·40775 = £1381 8s. 1·86d.

EXERCISES 46.

(A.)

Find the amount of an annuity of £160, left unsettled for 7 years, at 5 per cent. compound interest.

What is the amount of an annuity of £120 for 5 years, at 5 per cent. compound interest?

123. The *present value* of an annuity is a sum, such that if lent t compound interest would, in a given period, amount to the same s an annuity left unpaid for the same time, with the same rate of ompound interest charged upon it.

Or,

The present value of an annuity is equal to the present values of ach of the periodic payments, discounted at compound interest.

EXAMPLES.

(1.) What is the present value of an annuity of £1 for 3 years, t 5 per cent., compound interest?

---

* $1 + 1·05 + 1·05^2 + 1·05^3 + 1·05^4 =$ (by geometrical progression) $\frac{·05^5 - 1}{·05 - 1} = 5·52563125$. By this mode is constructed Table II. The whole eory of annuities cannot be sufficiently explained without the aid of some lgebraical investigations.

The amount of an annuity of £1 for 3 years, at 5 per cent. by Table II. $\}$ = 3·152500.
The amount of £1 for 3 years, at 5 per cent., by Table I. $\}$ = 1·157625.

The present value is $\frac{3{\cdot}152500}{1{\cdot}157625} = 2{\cdot}723248$, as given in Table III.*

Or,

$\frac{1}{1{\cdot}05} = {\cdot}952381$ the present value of £1, due 1 year hence.

$\frac{1}{1{\cdot}05^2} = {\cdot}907029$,, ,, £1, due 2 years hence.

$\frac{1}{1{\cdot}05^3} = {\cdot}863838$,, ,, £1, due 3 years hence.

2·723248 = the present value of an annuity of £1, for 3 years, at 5 per cent. compound interest.

(2.) What is the present worth of an annuity of £80, to continue 5 years, at 6 per cent. compound interest?

$$\left.\begin{array}{l}\text{The amount of an annuity of £1,}\\ \text{for 5 years, at 6 per cent.}\end{array}\right\} = 5{\cdot}637092 \qquad \left.\begin{array}{l}\text{The amount of £1, for 5 years,}\\ \text{at 6 per cent.}\end{array}\right\} = 1{\cdot}338226 \qquad \frac{5{\cdot}637092}{1{\cdot}338226} = 4{\cdot}212362,$$

the same as in Table III., opposite 5 years, and under 6 per cent.; then 4·212362 × 80 = £336 19s. 9½d. *Ans.*

EXERCISES 46.

(B.)

1. What is the present worth of an annuity of £170, to continue 9 years, discounting at the rate of 4 per cent. compound interest?
2. What is the present value of an annuity of £280, to continue 20 years, at 5 per cent. discount?

---

[*] Table III. is constructed by dividing the amount of an annuity of £1 for the given time and at the given rate, taken from Table II., by the amount of £1 for the same time, and at the same rate, taken from Table I. Making a comparison founded on this question, £2·723248 present payment is equivalent to £1 paid at the end of each year, for 3 years, and also equivalent to £3·1525 paid at the end of 3 years. "Thus, a debtor may clear off a given sum now due, by paying to his creditor an equivalent annuity for a certain number of years, or its amount at the end of that time; and a landholder, whose estate is charged with an annuity, can compound for it by giving a present sum in cash." —Scratchley on *Benefit Building Societies.*

An annuity of £300 is mortgaged for 7 years, and then reverts to the owner for 13 years, what is the present value of the 13 years' income, at 5 per cent.?

In Table III. opposite to 20 years in 5 per cent. column, is found } 12·462208 = { present value of an annuity of £1 for 20 years.

Subtract the value of £1 opposite to 7 years . . . . } 5·582381

and there remains the deferred worth of £1 for the 13 years . . } 6·879827

And this difference multiplied by the annuity . . . . } 300

Ans. £2063·9481 = £2063 18 11½

If a freehold estate of £150 per annum, to commence 3 years hence, is to be sold; what is its worth now, allowing the purchaser 5 per cent. for his money?

As 5 : 150 : : 100 : x = 3000 = { the value of the estate if purchased 3 years hence, or when it commences,

and $\frac{1}{1\cdot05^3} \times 3000$ = the present value of estate or £2591 10s. 3½d.

3. What is the purchase money of an estate of £250 per annum, to continue for ever, but not to commence till the end of 3 years, allowing 4 per cent. to the purchaser?

COMPOUND INTEREST CALCULATED MONTHLY.

124. *To find the amount of* £1 *for any number of years, at from* 5 *to* 10 *per cent. compound interest, calculated monthly.*

The amount of £1 at the end of the first month, at 5 per cent. will be $1\cdot\frac{05}{12}$; at the end of the 2nd month, $(1\cdot\frac{05}{12})^2$; at the end of the 3rd month, $(1\cdot\frac{05}{12})^3$; at the end of the 12th month, $(1\cdot\frac{05}{12})^{12} = 1\cdot051171$*; at the end of the 24th month, $(1\cdot\frac{05}{12})^{24} = 1\cdot104954$;

---

* The interest of £1 for 12 months, if calculated monthly at 5 per cent. compound interest, will be 1·051171, which is more than 5 per cent. per annum, but the monthly rate of interest to realize 5 per cent. per annum, instead of being $\cdot\frac{05}{12}$, would be $1\cdot05^{\frac{1}{12}} - 1 = \cdot040754$.

The time in which money will double itself at any rate per cent. calculated annually, may be found approximately by dividing 70 by the rate: thus at 5 per cent. 70 ÷ 5 = 14 years, or the time in which the interest will equal the principal.

at the end of the 36th month, $(1\cdot0\frac{5}{12})^{36} = 1\cdot161492$, &c. In a similar way the amount of £1 at the other rates may be calculated. In order to find the amount of £1 for any number of years readily, Table VI. has been constructed, giving the amount of £1 yearly, at from 5 to 10 per cent. compound interest, calculated monthly up to 12 years.

EXAMPLES.

(1). To find the amount of £1 for 20 years at 6 per cent. compound interest, calculated monthly.

The amount by Table VI. of £1 for 12 years at 6 per cent. . . . . = } 2·050707

The amount by Table VI. of £1 for 8 years at 6 per cent. . . . . . . = } 1·614119

As £1 : 2·050707 : : 1·614119 : x = £3·310085 = the amount of £1 for 20 years.

(2). What is the amount of £250 at 5 per cent. compound interest for 6 years, calculated monthly?

By Table VI. the amount of £1 for 6 years, at 5 per cent.=1·349062, which multiplied by 250 = £337·2655 the required amount.

(3). What is the amount of £20 for 12 years at 7 per cent. compound interest monthly?

By Table VI., the amount of £1 for 12 years at 7 per cent. is £2·310702, which multiplied by 20 = £46·21404. *Ans.*

(4). What is the amount of £80 for 24 years at 8 per cent. compound interest calculated monthly?

By Table VI. the amount of £1 for 12 years at 8 per cent. is 2·603659 and 2·603659 × 2·603659 × 80 = £542·323215 = the required amount.

(5). What is the present value of £1 and of £40 due at the end of 12 months, if compound interest be allowed at 6 per cent. monthly on present payment.

By Table VI. the amount of £1 at the end of 12 months at 6 per cent. is 1·061677; therefore the present value of £1·061677 due at the end of 12 months will be £1.

£ £ £ £ s. d.
As 1·061677 : 1 : : 1 : x : ·9419 = 18 10 the present value of £1,
and ·9419 × 40 = £37·676 = £37 13s. 6d. the present value of £40.

BENEFIT BUILDING SOCIETIES.

125. Building Societies are associations formed among a large proportion of the industrial classes of this country, the members of which pay monthly subscriptions into a common fund,—some with the view of borrowing money, others for the purpose of investing their savings advantageously. The members of every building society are thus composed of two classes, viz.:—the borrowers and the investers, or lenders; and from among these two classes are annually elected by the members themselves, a directory or committee, to whom is entrusted, during the term of their office, the entire management of the affairs of the society.

To explain more fully the functions, as well as advantages of building societies, let A. and B. represent members of one. Now, suppose A. to occupy a house, for which he pays £25 yearly rent, but which he could purchase for £300; should he wish to possess this house, he obtains the £300 from the society of which he is a member, and thus effects the desired purchase. The repayment of this sum, together with the premium or interest upon it, is secured to the society by mortgage on the house purchased, or on other approved property, and is made chiefly in monthly instalments by A., who now, instead of paying the rent of his house to the landlord, makes it available in assisting to pay his monthly subscriptions to the society.

When, in the course of some years, the claims of the society in this transaction are satisfied, the house comes into A.'s possession, possibly clear of all incumbrances. By pursuing a similar course, A. can become posssessed of a second house, or a third, &c. To B. on the other hand, whose monthly subscriptions constitute in part the loan to A., there has accumulated, in addition to his deposits, at the period when the obligations of A. became discharged, such interest, produced by premiums, fines, transfer and entrance fees, &c., as would not have been realized to him had he deposited his money in any of the ordinary savings banks.

Of building societies there are two kinds: one called the "Terminating," the other the "Permanent."

The "Terminating" Society is so called, because it is said to terminate when each share, consisting of a monthly subscription of 10s., amounts, with its profits or interest, to £120. The committee, however, with the consent of the members, can terminate the affairs of the society at an earlier period, and they also determine the amount of interest which the borrower, or "purchased member, shall pay the lender, or "unpurchased member," in addition to his deposited subscriptions.

The "Permanent" Society is so designated because it has no fixed time for terminating, and has within itself the elements of continuity; inasmuch as new members can enter it at any time without paying up either arrears or increase of entrance fee. This feature in its organization induces a continual accession of fresh members, and renders the existence of the society continuous, or permanent. The borrower, or purchaser of money in this society need not be a member to contract a loan, but after the loan is obtained he becomes a member and pays a certain monthly subscription for a fixed number of years; after which the debt, with its obligation, becomes liquidated. Meanwhile, the investors, or lenders, receive good interest, the amount being contingent on the prosperity of the society.

In England there are, it is estimated, at the present time, upwards of 2000 building societies, embracing 150,000 members, with a paid-up capital of not less than £8,000,000; so that any investigation having reference to a subject in which so large a portion of the industrial community are concerned, is well entitled to a place in a school arithmetic,

TERMINATING SOCIETY.

126. The monthly sums deposited for a period of years, by an unpurchased member, into the funds of a building society, are equivalent to an income of the same amount, due every month, but left unpaid for this period. The interest on these periodical payments is, therefore, found as in *annuities foreborne* or *unpaid.*

EXAMPLES.

(1). What interest will a share in a building society, consisting of a payment of 10s. a month, realize in a year, at 5 per cent. per annum simple interest?

. The interest on the 12th* payment of 10s. for a month, at 5 per cent., is ½d.; on the 11th payment of 10s. for 2 months, 1d.; on the 10th payment of 10s. for 3 months, 1½d.; on the 9th payment of 10s. for 4 months, 2d., &c. Hence the entire interest will be ½ + 1 + 1½ + 2 + 2½ + 3 + 3½ + 4 + 4½ + 5 + 5½ + 6 = 3s. 3d.

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* On every 12 payments made, there are 12 interests, inasmuch as the interest on the sum deposited will only be paid on, or after the first annual meeting of the preceding year.

The series ½, 1, 1½, 2, 2½, &c., is what is called an *arithmetical series*, and as such is readily summed.

(2). What is the compound interest on a share of 10s., paid monthly, in a year, at 5 per cent. per annum?

For convenience sake, let the monthly payment be £1.

As interest is calculated to the time of making the 13th payment, the amount of the 13th payment will be simply £1.; the amount of the 12th payment will be $1{\cdot}0\frac{5}{12}$; of the 11th $(1{\cdot}0\frac{5}{12})^2$; of the 10th $(1{\cdot}0\frac{5}{12})^3$, of the 9th $(1{\cdot}0\frac{5}{12})^4$, &c. Hence all the payments, with the interest, will amount to $£1 + 1{\cdot}0\frac{5}{12} + (1{\cdot}0\frac{5}{12})^2 + (1{\cdot}0\frac{5}{12})^3 + (1{\cdot}0\frac{5}{12})^4 + (1{\cdot}0\frac{5}{12})^5 + (1{\cdot}0\frac{5}{12})^6 + (1{\cdot}0\frac{5}{12})^7 + (1{\cdot}0\frac{5}{12})^8 + (1{\cdot}0\frac{5}{12})^9 + (1{\cdot}0\frac{5}{12})^{10} + (1{\cdot}0\frac{5}{12})^{11} + (1{\cdot}0\frac{5}{12})^{12}$ * $= \frac{(1\,0\frac{5}{12})^{13} - 1}{1{\cdot}0\frac{5}{12} - 1} = \frac{1{\cdot}05555 - 1}{1\,0\frac{5}{12} - 1} = 13{\cdot}332 =$ the amount of £1, made by 13 monthly payments, which, being multiplied by 10s. or £½, gives the amount of 13 monthly payments of 10s. each, or £6·666. From this deduct 13 payments, or £6·5, and there is left £·166, the compound interest required; and £6, the yearly subscription, added to it, makes the amount at the end of the first year, £6·166.

The results which follow from the use of the foregoing series are recorded in Tables IV., V., and VII.

Table IV. gives the simple interest on one share.
„ V. „ the compound interest „
„ VII. „ amount of one share calculated at compound interest.

(3.) What is the amount of 6 shares of 10s. a month each, in a Terminating Building Society for 5 years, at 8 per cent. compound interest?

By Table VII. the amount of one share for 5 years at 8 per cent. is £36·9883; and 36·9883 × 6 = 221·9298 or £221 18s. 7d. the required amount.

(4.) What sum would a member holding 10 shares in a society, terminating at the end of the 10th year, and paying 7 per cent. be entitled to receive?

By Table VII. the amount of one share for 10 years at 7 per cent. is £87·046179; and £87·046179 × 10 = £870·46179 or £870 9s. 3d.

EXERCISES 47.

(A).

1. What is the amount of 8 shares in a terminating society, for 9 years at 6 per cent.?

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* The series $1 + (1{\cdot}0\frac{5}{12})^1 + (1{\cdot}0\frac{5}{12})^2 + (1{\cdot}0\frac{5}{12})^3 + (1{\cdot}0\frac{5}{12})^4$, &c., is what is called a *geometrical series*, whose sum is equal to the product of the first term, multiplied by ratio raised to a power whose number is expressed by the number of terms diminished by the first term, divided by ratio less 1.

2. Find the amount of 16 shares in a terminating society, for 7 years at 8 per cent. ?

EXAMPLES.

(5.) What sum in ready money could a member of a terminating building society raise on one share of 10s. a month or £6 a year, to be paid for 14 years, and to realize 5 per cent. compound interest monthly?

By Table VII. the amount of one share in 14 years at 5 per cent. is 121·823529.

By Table VI. the amount of £1 at 5 per cent. compound interest monthly is 2·01981.

$$\text{And } \frac{121{\cdot}823529}{2{\cdot}010981} = 60{\cdot}55 = \pounds 60 \text{ 11s. the required sum.}$$

(6.) A person wishing to raise a sum of money in order to clear off a debt now due, undertakes to pay on 5 shares, of 10s. each month, for 9 years, into a building society,* guaranteeing 5 per cent. compound interest. What sum ought he to receive now in respect of his 9 years' payments?

By Table VII. the amount of 1 share in 9 years is £68·314348, and of 5 shares is £68·314348 × 5 = £341·57174.

And by Table VI. the amount of £1 for 9 years at 5 per cent. is £1·566924.

$$\text{And } \frac{341{\cdot}57174}{1{\cdot}566924} = \pounds 218 \text{ the sum to be received at once.}$$

Remark.—£218 at 5 per cent. compound interest for 9 years will amount to £341 11s. 9½d.; and 5 shares of 10s. each, paid monthly, will, in 9 years, at 5 per cent. compound interest, realize £341 11s. 5d., so that £218 at once, £2 10s. paid every month for 9 years, and £341 11s. 5d. at the end of 9 years. will all be equivalent.

EXERCISES 47A.—*continued.*

3. What sum ought one person, who owes another £218, pay monthly in order to clear off this debt in 9 years, at 5 per cent., compound interest ?

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* The facilities and advantages which these societies afford of raising or obtaining money, might well be made available in the purchase of small tracts of land by farmers and others of restricted means. Had such societies flourished in Ireland during the existence of the "Encumbered Estates' Court," as they have been flourishing in England, there can be little doubt, that many farmers in the former country would be the owners, instead of the tenants, of the land on which they are now residing.

A person about to reside abroad, takes 6 shares in a building society guaranteed to terminate in 9 years and pay 7 per cent. compound interest, calculated monthly; what sum should he pay at once, to satisfy his obligations to the society for this period?

The amount of 1 share for 9 years at 7 per cent. by Table VII. is 75·365677, which multiplied by 6 = £452·194062 the amount of 6 shares.

By Table VI. the amount of £1 for 9 years at 7 per cent. is 1·874164,

And £$\frac{452{\cdot}194062}{1{\cdot}874164}$ = £241·277744 = £241 5s. 6½d. required sum.

EXERCISES 47A.—*continued.*

4. What is the present value of £500 due 4 years hence, and discounted at 5 per cent compound interest, calculated monthly?
5. Find the present value of £160 due 10 years hence; present payment being calculated at 5 per cent. compound interest?

EXAMPLE.

Suppose in a society arranged to terminate in 12 years, and pay 8 per cent. compound interest, A. holds 4 shares, and wishes at the end of the 8th year to receive a sum equivalent to his past payments and future subscriptions, what sum ought he to receive?

By Table VII. the amount of 1 share for 8 years at 8 per cent. } = 67·390450

And 67·390450 × 4 . . . = 269·5618

The amount of 1 share for 4 years = 28·366402

The amount of £1 for 4 years at 8 per cent. compound interest, monthly . . = 1·375714

And $\frac{28{\cdot}366402 \times 4 \text{ shares}}{13{\cdot}75714}$ = 82·4776

£352·0394 the required sum.

PERMANENT SOCIETIES.

127. The value of one share in a "Permanant Building Society" s £10, which may be paid in advance, or in monthly payments of 10s. until the total of the subscriptions amounts to £10. As soon as the funds of the society amount to the sum of £100 (the value of 10 shares) this sum is lent, for which the borrower pays a premium of about £10: that is, he becomes responsible to the society for £100, although he receives but £90; and his repayments, to

discharge this, are made according to a scale somewhat similar to the following. This scale indicates that if a borrower of £100, though receiving but £90, pay 16s. 3d. per month for 16 years, his obligations to the society become discharged:—

SCALE OF PAYMENTS.*

Showing the *Minimum* amount of Principal and Interest to be paid *Monthly* by a Member, for every £100 received on Mortgage Security.

Years, seriatim, commencing with Date of Mortgage.	Principal per Month, commencing at the First Monthly Meeting after execution of Mortgage.	Interest per Month, commencing at the First Monthly Meeting, after execution of Mortgage.	Total per Month.
	£ s. d.	£ s. d.	£ s. d.
1	0 7 11	0 8 4	0 16 3
2	0 8 3	0 8 0	0 16 3
3	0 8 7	0 7 8	0 16 3
4	0 8 11	0 7 4	0 16 3
5	0 9 3	0 7 0	0 16 3
6	0 9 7	0 6 8	0 16 3
7	0 9 11	0 6 4	0 16 3
8	0 10 3	0 6 0	0 16 3
9	0 10 7	0 5 8	0 16 3
10	0 10 11	0 5 4	0 16 3
11	0 11 3	0 5 0	0 16 3
12	0 11 7	0 4 8	0 16 3
13	0 11 11	0 4 4	0 16 3
14	0 12 3	0 4 0	0 16 3
15	0 12 7	0 3 8	0 16 3
16	0 12 11	0 3 4	0 16 3

Note.—Any Member may at any time pay larger Monthly Instalments than those mentioned in the second column, thereby shortening the duration of the Mortgage, in proportion to the increased rate of Monthly Payments, and decreasing the Monthly Instalments of Interest.

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* This is one scale of payments of a Permanent Building Society established in March, 1860. Besides the interest received with the monthly payments in this scale, and the interest on re-lending these payments, there are other sources of profit, such as entrance fees, fines, and transfer fees.

SCALE OF PAYMENTS—*continued.*

Showing the *Minimum* amount of Principal and Interest to be paid *Annually* on every £100, by a Member making his payments according to the foregoing Scale.

Years, seriatim, commencing with Date of Mortgage.	Principal per Annum.			Interest per Annum.			Total per Annum.		
	£	s.	d.	£	s.	d.	£	s.	d.
1	4	15	0	5	0	0	9	15	0
2	4	19	0	4	16	0	9	15	0
3	5	3	0	4	12	0	9	15	0
4	5	7	0	4	8	0	9	15	0
5	5	11	0	4	4	0	9	15	0
6	5	15	0	4	0	0	9	15	0
7	5	19	0	3	16	0	9	15	0
8	6	3	0	3	12	0	9	15	0
9	6	7	0	3	8	0	9	15	0
10	6	11	0	3	4	0	9	15	0
11	6	15	0	3	0	0	9	15	0
12	6	19	0	2	16	0	9	15	0
13	7	3	0	2	12	0	9	15	0
14	7	7	0	2	8	0	9	15	0
15	7	11	0	2	4	0	9	15	0
16	7	15	0	2	0	0	9	15	0

Note.—This Scale is given to show Members who have executed mortgages what their monthly payments will amount to per annum; and likewise to enable them to calculate at any time the net amount of their debt, and the period that will be required to liquidate the same.

EXAMPLES.

(1.) Suppose £100 is borrowed in a Permanent Building Society, to be repaid in monthly payments of 16s. 3d., to continue for 16 years; what rate per cent. would be realized to the society in case the repayments were lent at 5 per cent. per annum interest?

The interest of 16s. 3d. for one month at 5 per cent. = $\frac{13}{16}$d.
„ of 16s. 3d. for two months at 5 per cent. = 1$\frac{5}{8}$d.
„ of 16s. 3d. for 192 months at 5 per cent. = 13s.

And, by arithmetical series,

$\frac{13}{16}$d. + $1\frac{5}{8}$d. + $2\frac{7}{16}$d. to 192 terms will amount to - } £62 8 0

Total of 16s. 3d. paid monthly for 16 years - } 156 0 0

218 8 0 total.
90 0 0 sum received.

16) 128 8 0 = interest for 16 years.

8 0 6 interest for 1 year.

As 90 : 100 : : £8 0s. 6d. : $x = 8{\cdot}91\dot{6}$; or £8 18s. 4d., the required rate per cent. *Ans.*

(2.) Suppose, in the foregoing question, the repayments be lent at 5 per cent. compound interest, what rate per cent. would be realised to the society?

By the process of geometrical series 16s. 3d. paid monthly for 16 years will, at 5 per cent. compound interest, amount to £238·2978
The actual sum lent . . . . . = 90

16) 148·2978

9·2686125

£ £
As 90 : 100 : : 9·2686125 : $x = 10{\cdot}2984583$, *or*, £10 5s. $11\frac{3}{4}$d., the required rate.

EXAMPLES.

(1.) What sum would £90 amount to in 16 years, at 6 per cent. compound interest, calculated monthly?

The amount of £1 for 4 years by Table V. = 1·270479
The amount of £1 for 12 years by do. = 2·050707
The amount of £1 for 16 years = 1·27049 × 2·050707 = 2·60538
And 2·60538 × 90 = £234·4842 the required amount.

(2.) What sum would £90 amount to for 16 years, at $6\frac{1}{4}$ per cent. compound interest, calculated yearly?

The amount of £1 for 16 years at $6\frac{1}{4}$ per cent. compound interest yearly, is £2·637933; and £2.637933 × 90 = £237·41397, the required amount.

Remark.—Hence, from the foregoing, it is seen that £100 lent in a Permanent Building Society, and the repayments lent at 5 per cent. compound interest, calculated monthly, realize, according to the above scale of payments, more than 10 per cent. simple interest;—

more than 6 per cent. compound interest, calculated monthly; and more than 6¼ per cent. compound interest, calculated annually.[*]

(3.) What rate per cent. will be realized to a Permanent Building Society on £100 lent, to be repaid in 12 years in monthly subscriptions of 18s. 9d. each,.at 5 per cent. simple interest?

By process of arithmetical series the interest on 18s. 9d. paid monthly for 12 years, at 5 per cent. . = £40 10 0
Total of 18s. 9d. paid monthly for 12 years = 135 0 0

175 10 0
The sum lent = 90 0 0

12) 85 10 0 interest.

£7 2 6

£ £ £ s. d. £ s. d.
As 90 : 100 : : 7 2 6 : x = 7 18 4 per cent. simple interest. *Ans.*

(4.) Suppose in the foregoing question, the repayments be lent at 5 per cent. compound interest, what rate per cent. would be realized to the society by this course?

By process of geometrical series, 18s. 9d. paid monthly for 12 years, will, at 5 per cent. compound interest, amount to £184·5
The actual sum lent . 90·

12) 94·5

7·875 simple interest for 1 year on £90.

As £90 : £100 : : £7·875 : x = £8·75. *Ans.*

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[*] To find the actual rate at which £90 must be lent to amount in 16 years to £238·2978 compound interest, calculated monthly.

Let x = the annual rate of £1.

$\frac{x}{12}$ = the monthly rate.

Then $\left(1+\frac{x}{12}\right)^{12\times} \times 90 = 238·2978$

$$\left(1+\frac{x}{12}\right)^{192} = 2·64775\dot{3}$$

$$1+\frac{x}{12} = \sqrt[192]{2·64775\dot{3}} = 1·00508\dot{4}$$

$$\frac{x}{12} = ·00508\dot{4}$$

$$x = ·061012$$

And 100 times x = 6·1012.

EXERCISES 47B.

1. What rate per cent. will be realized in a Permanent Building Society, on £100 lent to be repaid in 20 years, in monthly subscriptions of 13s. 4d. each; the repayments being lent at 5 per cent. simple interest, calculated monthly?
2. If, in the foregoing question, the repayments be lent at 5 per cent. compound interest, monthly, what rate per cent. will be realized to the society?

LOAN SOCIETIES.

128. **Loan Societies** are societies composed of a number of individuals, who raise a capital in shares among themselves, for the purpose of lending it out at interest, it being required that the sum lent should be repaid in periodical instalments. Thus, a person wanting the loan of £100, can obtain the sum upon two persons, approved of by the managers of the society, becoming his security for its repayment. The sum which the borrower receives is not £100, but £95—the difference £5 being deducted for interest. For this accommodation the borrower pays into the office of the society £2 a-week for 50 weeks, and then the transaction ends.

The following scale is copied from the business circular of a society, whose loans, according to the report issued in January, 1860, amounted to £232,617 during the year 1859.

Loan and Discount Company (Limited).

Scale of Charges for Loans extending over one Year, repayable by Weekly Instalments.

Loan of					Charge for Form of Application.		Entire charge including Form of Application.
£15	repayable	at	6s. 0d.	per week	2s. 6d.	——	£0 18s. 3d.
£20	,,	,,	8s. 0d.	,,	8s. 0d.	——	£1 4s. 0d.
£50	,,	,,	20s. 0d.	,,	6s. 0d.	——	£2 18s. 6d.
£100	,,	,,	40s. 0d.	,,	10s. 0d.	——	£5 15s. 0d.
£200	,,	,,	80s. 0d.	,,	20s. 0d.	——	£11 10s. 0d.
£300	,,	,,	120s. 0d.	,,	20s. 0d.	——	£16 15s. 0d.

And upwards in the same proportion.

If the weekly instalments are increased, the interest will be reduced in the same proportion.

The interest (which with other expenses, is deducted from the loan, when advanced) is as follows:—

If repaid by	weekly instalments,	5	per cent.	per annum.
,,	monthly	,,	$7\frac{1}{2}$	,,	,,
,,	quarterly	,,	10	,,	,,

BILLS DISCOUNTED.

Four months and under, at the rate of	10	per cent.	per annum.
Beyond four months . . .	$12\frac{1}{2}$	,,	,,

EXAMPLES.

(1.) What will be the interest on £95 received from a loan society for £100 borrowed, when repaid in 50 weekly instalments of £2 per week, at 5 per cent. simple interest?

The question is to find the total of the interest on £95, £93, £91 &c., to £1 for 1 week at 5 per cent. simple interest.

The interest of £95 for a week at 5 per cent. $= \frac{95 \times 5}{5200}$

,, £93 ,, ,, $= \frac{93 \times 5}{5200}$

,, £91 ,, ,, $= \frac{91 \times 5}{5200}$

&c. &c.

And the interest of £1 ,, ,, $= \frac{1 \times 5}{5200} =$ interest

[at the 48th payment when the sum borrowed is paid off.

By arithmetical progression the sum total of interest $=$

$$\left(\frac{95 \times 5}{5200} + \frac{1 \times 5}{5200}\right) \times 24 = \frac{11520}{5200} = 2\tfrac{14}{65}. \textit{ Ans.}$$

(2.) What rate per cent., simple interest, is obtained for the use of £95 in the foregoing question, when £5 is charged for interest thereon?

Now the actual interest at 5 per cent. would be £$2\frac{14}{65}$.*

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* The 48th payment clears off the entire sum borrowed, with £1 of the interest; still leaving £4 interest unpaid to be discharged by the 49th and 50th payments; the interest on which will be $\frac{(4 + 2) \times 5}{5200}$, which being added to $\frac{11520}{5200}$ (the interest at the period of the 48th payment) $= \frac{11550}{5200}$ $= 2\frac{23}{104} =$ the total interest on principal and unpaid interest at the time the transaction ends. But the actual interest paid for the loan is £5;

$$\begin{array}{l} \qquad\quad\ \text{£} \quad\ \text{£} \ \text{int. rate.} \\ \text{Hence, as } 2\frac{23}{104} : 5 :: 5 : x = 11\frac{59}{231} = \text{£}11\ 5s.\ 1\frac{23}{77}d. \text{ rate per cent.} \end{array}$$

£ int. £ int. rate per cent.
As $2\frac{14}{65} : 5 :: 5 : x = 11\frac{41}{144}$, *Ans.* the rate per cent. charged to pay £5 interest.

Proof. What is the total interest on £95, £93, £91, &c., to £1 for 1 week, at $11\frac{41}{144}$ per cent. simple interest?

The total interest will be

$$\frac{95 \times 11\frac{41}{144}}{5200} + \frac{93 \times 11\frac{41}{144}}{5200} \text{ \&c. } \frac{1 \times 11\frac{41}{144}}{5200} = \frac{96 \times 11\frac{41}{144}}{5200} \times 24$$
$$= £5. \textit{ Ans.}$$

(3.) What will be the interest on £92 10s. received from a loan society in lieu of £100 borrowed, when repaid in 12 monthly instalments of £8 6s. 8d. at 5 per cent. simple interest?

The interest of £92 10s. for a month at 5 per cent. $= \frac{92\frac{1}{2} \times 5}{1200}$

" £$84\frac{1}{6}$ " 5 " $\frac{84\frac{1}{6} \times 5}{1200}$

" £$75\frac{5}{6}$ " 5 " $\frac{75\frac{5}{6} \times 5}{1200}$

&c. &c.

And the interest of £$\frac{5}{6}$ $\frac{\frac{5}{6} \times 5}{1200}$

$$\text{The total interest} = \left(\frac{92\frac{1}{2} \times 5}{1200} + \frac{\frac{5}{6} \times 5}{12}\right) \times 6 = \frac{2800}{1200} = 2\tfrac{1}{3}$$

Ans.

(4.) What rate per cent. simple interest is charged for the use of £$92\frac{1}{2}$, which is diminished by monthly instalments of £$8\frac{1}{3}$, when £7 10s. is the interest charged thereon?

The actual interest on the several diminished sums at 5 per cent. is £$2\frac{1}{3}$.

£ int £ int. rate per cent.
Then, as $2\frac{1}{3} : 7\frac{1}{2} :: 5 : x = 16\frac{1}{14}$. *Ans.* the rate per cent. charged.

Proof. Find the total interest on £$92\frac{1}{2}$, $84\frac{1}{6}$, &c., diminished by £$8\frac{1}{3}$ each month, the 12th payment being £$\frac{5}{6}$, at $16\frac{1}{14}$ per cent. simple interest.

The total interest will be

$$\frac{£92\frac{1}{2} \times 16\frac{1}{14}}{1200} + \frac{84\frac{1}{6} \times 16\frac{1}{14}}{1200} + \frac{75\frac{5}{6} \times 16\frac{1}{14}}{1200} \cdots\cdot$$
$$\frac{\frac{5}{6} \times 16\frac{1}{14}}{1200} = \frac{93\frac{1}{3} \times 16\frac{1}{14} \times 6}{1200} = £7\tfrac{1}{2}, \textit{ Ans. the interest.}$$

(5.) What rate per cent., simple interest, is charged for £90 received for £100 borrowed from a loan society, when repaid in 4 quarterly payments of £25 each?

				£	s.	d.
The interest of £90 at 5 per cent. for	3 months	=		£1	2	6
,, £65 at 5 ,,	3 ,,	=		0	16	3
,, £40 at 5 ,,	3 ,,	=		0	10	0
,, £15 at 5 ,,	3 ,,	=		0	3	9
				£2	12	6

£ s. d. Int. rate per cent.

As 2 12 6 : 10 : : 5 : $x = 19\frac{1}{21}$ *Ans.* the rate.

Proof. Find the total interest on £90, £65, £40, and £15, each for 3 months, at $19\frac{1}{21}$ per cent.?

The interest of £90 for 3 months at $19\frac{1}{21}$ per cent. $= \dfrac{90 \times 19\frac{1}{21}}{400}$

,, £65 for 3 ,, $19\frac{1}{21}$,, $= \dfrac{65 \times 19\frac{1}{21}}{400}$

,, £40 for 3 ,, $19\frac{1}{21}$,, $= \dfrac{40 \times 19\frac{1}{21}}{400}$

,, £15 for 3 ,, $19\frac{1}{21}$,, $= \dfrac{15 \times 19\frac{1}{21}}{400}$

The total interest will be $\dfrac{90 \times 19\frac{1}{21}}{400} + \dfrac{65 \times 19\frac{1}{21}}{400} + \dfrac{40 \times 19\frac{1}{21}}{400} +$

$\dfrac{15 \times 19\frac{1}{21}}{400} = \dfrac{210 \times 19\frac{1}{21}}{400} =$ £10. *Ans.*

EXERCISES 48.

1. Find the interest on £95, had in lieu of £100 borrowed from a loan society, to be repaid in weekly instalments of £2 each at 6 per cent. per annum, simple interest.
2. If, when 6 per cent. is the rate, the total interest on £95, £93, £91, &c., to £1, the last of the weekly payments of £2 each, amounts to £2 13s. $2\frac{2}{65}$d.; what should be the rate when the total interest is £5?
3. What will be the total interest on £92 10s. had from a loan society in lieu of £100 borrowed at 7 per cent., when repaid in 12 monthly instalments of £8 6s. 8d. each?
4. If when 7 per cent. is the rate, the total interest of £$92\frac{1}{2}$, £$84\frac{1}{6}$, £$75\frac{5}{6}$, &c., to $\frac{5}{6}$ for a month, amounts to £3 5s. 4d., what must be the rate to make the total interest £7 10s.?
5. What will be the total interest at 8 per cent. on £90 had for £100 from a loan society, when repaid in quarterly instalments of £25 each?
6. If the total interest on £90, £65, £40, and £15 at 8 per cent. for 3 months be £4 4s., what should be the rate to make the total interest £10?

TABLE I. showing the amount of £1, at compound interest.

Years.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	1·030000	1·040000	1·050000	1·060000
2	1·060900	1·081600	1·102500	1·123600
3	1·092727	1·124864	1·157625	1·191016
4	1·125509	1·169859	1·215506	1·262477
5	1·159274	1·216653	1·276282	1·338226
6	1·194052	1·265319	1·340096	1·418519
7	1·229874	1·315932	1·407100	1·503630
8	1·266770	1·368569	1·477455	1·593848
9	1·304773	1·423312	1·551328	1·689479
10	1·343916	1·480244	1·628895	1·790848
11	1·384234	1·539454	1·710339	1·898299
12	1·425761	1·601032	1·795856	2·012196
13	1·468534	1·665074	1·885649	2·132928
14	1·512590	1·731676	1·979932	2·260904
15	1·557967	1·800944	2·078928	2·396558
16	1·604706	1·872981	2·182875	2·540352
17	1·652848	1·947900	2·292018	2·692773
18	1·702433	2·025817	2·406619	2·854339
19	1·753506	2·106849	2·526950	3·025600
20	1·806111	2·191123	2·653298	3·207135
21	1·860295	2·278768	2·785963	3·399564
22	1·916103	2·369919	2·925261	3·603537
23	1·973587	2·464716	3·071524	3·819750
24	2·032794	2·563304	3·225100	4·048935
25	2·093778	2·665836	3·386355	4·291871
26	2·156592	2·772470	3·555673	4·549383
27	2·221289	2·883369	3·733456	4·822346
28	2·287928	2·998703	3·920129	5·111687
29	2·356566	3·118651	4·116136	5·418388
30	2·427262	3·243398	4·321942	5·743491
31	2·500080	3·373133	4·538039	6·088101
32	2·575083	3·508059	4·764941	6·453386
33	2·652335	3·648381	5·003189	6·840590
34	2·731905	3·794316	5·253348	7·251025
35	2·813862	3·946089	5·516015	7·686087
36	2·898278	4·103933	5·791816	8·147252
37	2·985227	4·268090	6·081407	8·636087
38	3·074783	4·438813	6·385477	9·154252
39	3·167027	4·616366	6·704751	9·703507
40	3·262038	4·801021	7·039989	10·285718

EXAMPLE.—Find the amount of £40 at 5 per cent. compound interest for 32 years?

Amount of £1 for 32 years at 5 per cent. = £4·764941

Multiplied by 40

Ans. £190 11s. 11½d. £190·59764

E II. showing the amount of an annuity of £1 at compound nterest.

3 per cent.	4 per cent.	5 per cent.	6 per cent.
1·000000	1·000000	1·000000	1·000000
2·030000	2·040000	2·050000	2·060000
3·090900	3·121600	3·152500	3·183600
4·183627	4·246464	4·310125	4·374616
5·309135	5·416322	5·525631	5·637092
6·468409	6·632975	6·801912	6·975318
7·662462	7·898294	8·142008	8·393837
8·892336	9·214226	9·549108	9·897467
10·159106	10·582795	11·026564	11·491315
11·463879	12·006107	12·577892	13·180794
12·807795	13·486351	14·206787	14·971642
14·192029	15·025805	15·917126	16·869941
15·617790	16·626837	17·712982	18·882137
17·086324	18·291911	19·598631	21·015065
18·598913	20·023587	21·578563	23·275969
20·156881	21·824531	23·657491	25·672528
21·761587	23·697512	25·840366	28·212879
23·414435	25·645412	28·132384	30·905652
25·116868	27·671229	30·539003	33·759991
26·870374	29·778078	33·065954	36·785591
28·676485	31·969201	35·719251	39·992726
30·536780	34·247969	38·505214	43·392290
32·452883	36·617888	41·430475	46·995827
34·426470	39·082604	44·501998	50·815577
36·459264	41·645908	47·727098	54·864512
38·553042	44·311744	51·113453	59·156382
40·709633	47·084214	54·669126	63·705765
42·930922	49·967582	58·402582	68·528111
45·218850	52·966286	62·322711	73·639798
47·575415	56·084937	66·438847	79·058186
50·002678	59·328335	70·760789	84·801677
52·502758	62·701468	75·298829	90·889778
55·077841	66·209527	80·063770	97·343164
57·730176	69·857908	85·066959	104·183754
60·462081	73·652224	90·320307	111·434779
63·275944	77·598313	95·836322	119·120866
66·174222	81·702246	101·628138	127·268118
69·159449	85·970336	107·709545	135·904205
72·234232	90·409149	114·095023	145·058458
75·401259	95·025515	120·799774	154·761965

AMPLE.—What sum will an annuity of £50, left unpaid for ars, amount to at 6 per cent. compound interest?

nt of an annuity of £1 in 16 years at 6 per cent. = 25·672528
Multiplied by 50

Ans. £1283 12s. 6d. £1283·6264

Table III. showing the present value of an annuity of £1, discounted at the rates of 3, 4, 5 and 6 per cent., compound interest.

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	·970874	·961538	·952318	·943396
2	1·913470	1·886194	1·859410	1·833392
3	2·828612	2·775190	2·723248	2·673011
4	3·717099	3·629994	3·545950	3 465105
5	4·579708	4·451821	4 329476	4·212363
6	5·417192	5·242136	5·075691	4·917324
7	6·230284	6·002054	5·786372	5·582381
8	7·019693	6·732744	6·463211	6 209793
9	7·786110	7·435331	7·107820	6·801691
10	8·530204	8 110895	7·721733	7·360086
11	9·252625	8·760576	8·306412	7·886874
12	9·954005	9·385073	8·863249	8·383843
13	0·634956	9 985647	9 393570	8·852682
14	11·296074	10·563122	9 898638	9·294983
15	11·937936	11·118487	10·379655	9 712248
16	12·561103	11·652395	10·837767	10·105894
17	13·166119	12·165668	11·274064	10 477258
18	13·753514	12·659396	11·689585	10 827602
19	14·323800	13·133938	12·085319	11·158115
20	14·877476	13·590325	12·462208	11·469920
21	15·415025	14·029159	12·821150	11·764075
22	15·936918	14 451114	13·163000	12·041580
23	16·443610	14·856840	13·488571	12·303377
24	16·935544	15·246961	13·798639	12·550356
25	17·413150	15·622078	14·093942	12·783355
26	17·876845	15·982767	14 375183	13·003165
27	18·327034	16·329584	14·643031	13·210533
28	18·764111	16·663061	14 898125	13·406163
29	19.188457	16·983712	15·141071	13·590720
30	19·600444	17 292031	15·372448	13·764830
31	20·000431	17·588491	15·592807	13·929085
32	20·388768	17·873549	15·802073	14 084042
33	20·765794	18·147643	16·002546	14 230228
34	21·131839	18·411195	16·192901	14·368140
35	21·487222	18 664610	16·374191	14·498245
36	21·832254	18·908279	16·546848	14·620986
37	22·167237	19·142576	16 711284	14·736779
38	22·492463	19·367861	16·867889	14·846018
39	22·808217	19 584482	17·017037	14 949074
40	23·114774	19·792771	17·159083	15 046296

Example.—Required the present value of an annuity of £100 for 12 years, abating 4 per cent., compound interest.

The present value of £1 for 12 years, at 4 per cent. = 9·385073
Multiplied by 100

Ans. £938 10s. $1\frac{1}{2}$d. 938·5073

TABLE IV.

Table showing the interest at the end of every year to 12 years, upon 1 share of 10s. a month or £6 per annum, paid monthly into a "Terminating Building Society" at from 5 to 10 per cent. per annum, simple interest.

Year.	1 per cent.			6 per cent.			7 per cent.			8 per cent.			9 per cent.			10 per cent.		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d	£	s.	d.	£	s.	d
1	0	3	3	0	3	11	0	4	7	0	5	2	0	5	10	0	6	6
2	0	12	6	0	15	0	0	17	6	1	0	0	1	2	6	1	5	0
3	1	7	9	1	13	4	1	18	10	2	4	5	2	9	11	2	15	6
4	2	9	0	2	18	10	3	8	7	3	18	5	4	8	2	4	18	0
5	3	16	3	4	11	6	5	6	9	6	2	0	6	17	3	7	12	6
6	5	9	6	6	11	5	7	13	4	8	15	2	9	17	1	10	19	0
7	7	8	9	8	18	6	10	8	3	11	18	0	13	7	9	14	17	6
8	9	14	0	11	12	10	13	11	7	15	10	5	17	9	2	19	8	0
9	12	5	3	14	14	4	17	3	4	19	12	5	22	1	5	24	10	6
10	15	2	6	18	3	0	21	3	6	24	4	0	27	4	6	30	5	0
11	18	5	9	21	18	11	25	12	1	29	5	2	32	18	4	36	11	6
12	21	15	0	26	2	0	30	9	0	34	16	0	39	3	0	43	10	0

EXAMPLE.—Required the interest on 3 shares of 10s. a month each, for 12 years, at 9 per cent., simple interest.

The interest on 1 share for 12 years at 9 per cent.	=	£39 3
Multiplied by . . .		3
Ans.		£117 9

TABLE V.

Table showing the compound interest, calculated monthly, at the end of every year to 12 years, upon 1 share, or £6 per annum, paid monthly, into a Terminating Building Society, at from 5 to 10 per cent. per annum.

Year.	5 per cent.			6 per cent.			7 per cent.			8 per cent.			9 per cent.			10 per cent.		
	£	s.	d.	£	s.	d.	£	s.	d	£	s.	d.	£	s.	d.	£	s.	d.
1	0	3	4	0	4	0	0	4	8	0	5	4	0	6	0	0	6	8
2	0	12	11	0	15	7	0	18	4	1	1	1	1	3	10	1	6	8
3	1	9	2	1	15	4	2	1	7	2	8	1	2	14	7	3	1	3
4	2	12	5	3	3	8	3	15	4	4	7	4	4	19	6	5	13	7
5	4	2	11	5	1	2	6	0	1	6	19	9	7	15	10	9	0	10
6	6	1	3	7	8	2	8	16	10	10	6	6	11	17	2	13	9	3
7	8	7	7	10	5	11	12	6	3	14	7	3	16	13	0	18	19	6
8	11	2	6	13	14	4	16	9	5	19	7	10	22	9	0	25	18	5
9	14	6	3	17	14	6	21	7	4	25	5	0	29	7	2	33	14	11
10	17	19	6	22	6	11	27	0	11	32	1	11	37	9	7	43	5	5
11	22	2	7	27	12	6	33	11	5	40	0	2	46	18	6	54	8	4
12	26	16	2	33	12	0	41	0	0	49	1	6	57	16	3	67	7	3
13	32	0	6	40	6	2	49	8	1									
14	37	16	6	47	16	1												

EXAMPLE.—Required the amount of interest on 4 shares of 10s. a month each, for 8 years at 7 per cent. compound interest.

The interest on 1 share for 8 years at 7 per cent.	=	£16 9 5
Multiplied by . .		4
Ans.		£65 17

TABLE VI.

Showing the amount of £1 every year to 12 years at from 5 to 10 per cent. compound interest, calculated monthly.

Year.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1·051171	1·061677	1·072289	1·083010	1·093806	1·104710
2	1·104954	1·127156	1·149802	1·172908	1·196410	1.220382
3	1·161492	1·196675	1·232923	1·270269	1·308638	1·348167
4	1·220921	1·270479	1·322052	1·375714	1·431396	1·489333
5	1·283364	1·348838	1·417621	1·489910	1·565669	1·645280
6	1·349062	1·432029	1·520098	1·613585	1·712537	1·817556
7	1·418091	1·520351	1·629985	1·747528	1·873181	2·007871
8	1·490651	1·614119	1·747818	1·892587	2·048896	2·218113
9	1·566924	1·713673	1·874164	2·049689	2·241093	2·450373
10	1·647098	1·819365	2·009648	2·219831	2·451322	2·706950
11	1·731378	1·931577	2 154925	2 404099	2·681265	2·990393
12	1·819970	2·050707	2·310702	2·603659	2 932784	3·303511
13	1·913092					
14	2·010981					

EXAMPLE.—Required the sum which £60 will amount to in 10 years at 7 per cent. compound interest, calculated monthly.

The amount of £1 in 10 years at 7 per cent = 2·009648
Multiplied by 60
120·57888

Ans. £120 11s. 7d.

TABLE VII.

Showing the sum to which 10s. a month, or 1 share in a Terminating Building Society, will accumulate to at the rates of 5, 6, 7, 8, 9, and 10 per cent. compound interest, calculated monthly from 1 to 12 years.

Year.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	6·165508	6·198525	6·232409	6·267250	6·300507	6·33491
2	12 646900	12·779160	12·915280	13·054645	13·192222	13 33314
3	19·459677	19·765746	20·081224	20 405350	20·730300	21·06418
4	26·621352	27·182878	27·765220	28·366402	28·975520	29·6795
5	34·145458	35 058255	36·004725	36·988300	37·994000	39·0395
6	42·062125	43·412290	44·840140	46 325842	47·858730	49.4627
7	50·380000	52·295422	54·313800	56·363350	58·648666	60·97616
8	59·123440	61·719318	64·471420	67·390450	70 450800	73·9195
9	68 314348	71·724206	75·365677	79·251625	83·359450	87·7473
10	77·975440	82·346240	87·046179	92·097307	97·480303	103·27018
11	88·131040	93·623323	99 571000	106·008333	112·925000	120·41875
12	98·806329	105·596208	113·001298	121·076350	129·814266	139·3625
13	110·027920	118·307537	127·402463			
14	121·823529	131·802674				

EXAMPLE —Required the sum which 2 shares in a Building Society will accumulate to in 7 years at 8 per cent.

The amount of 10s. a month, or £6 a year for 7 years at 8 per cent. = 56 363350
Multiplied by 2
112·726700

Ans. £112 14s. 6d.

SQUARE AND SQUARE ROOT.

129. The SQUARE of a number, as already stated, is the product arising from multiplying a number by itself.

Thus the square of $4 = 4 \times 4 = 16$.
" $5 = 5 \times 5 = 25$.
" $13 = 13 \times 13 = 169$.
" $24 = 24 \times 24 = 576$.

EXERCISES 49.

(A).

1. 36^2
2. 284^2
3. 529^2
4. 11025^2
5. 121550625^2
6. 7960^2
7. 5043456^2
8. 2535^2
9. 2925^2
10. 7912^2
11. $4796{\cdot}35^2$
12. 230049733225^2
13. $785{\cdot}963^2$
14. $61773783{\cdot}7369^2$

SQUARE ROOT.

130. The SQUARE ROOT of a given number is a number which, when multiplied by itself, produces that given number.

Thus the square root of 16 equal 4, because $4 \times 4 = 16$
" $25 = 5$ " $5 \times 5 = 25$
" $169 = 13$ " $13 \times 13 = 169$.

	I.		II.		III.	
Roots - -	2	9	20	99	200	999
Squares - -	4	81	400	9801	40000	998001

By comparing columns I., II., and III., it will be seen, that if any figure be annexed to a given number, the square of the resulting number will exceed the square of the given number by *two* figures; also, if two figures be annexed, the square of this number will be *four* figures in excess of the square of given number; and so on, increasing by two figures for every figure annexed to the right. Hence the units of the root are contained in the first two figures on the right hand, the *tens* of the root in the next two figures, &c. From these observations arise the directions for the *pointing off* of numbers, in the following

Rule for extracting the square root.

Begin at the units' place, or decimal point, if any; point off the integers from right to left into periods of two figures each (as far as the number will admit), and the decimals from left to right in a similar way. The number of divisions will indicate the number of figures in the required root.

Next, find the number which, when squared, will give, as nearly as possible, the first, or left-hand period; place it in the quotient, and subtract its square from the first period, and to the remainder annex the second period for the dividend. Double the quotient figure for the first part of the divisor, which divide into the dividend, omitting the right-hand figure, and annex the result to the divisor and quotient. Then multiply the entire divisor by this figure, and subtract the product from the dividend; to this remainder annex the next period for a new dividend, double the figures in the quotient for the first part of the new divisor, and proceed as before; and thus continue till the periods are exhausted. If there be a remainder, the root may be carried into decimals by annexing two ciphers for each decimal figure.

$$\begin{array}{rl}
24 = & 20 + 4 \\
24 = & 20 + 4 \\
\hline
96 & 400 + 20 \times 4 \\
48 & \quad\quad\;\; 20 \times 4 + 16 \\
\hline
\end{array}$$

$$24^2 = 576 = 400 + 2(20 \times 4) + 16 \text{ or } 20^2 + 2(20 \times 4)$$

$$4^2 = 576$$

From this it is seen that the square of 24 is equal to the square of 20 and of 4, and twice the product of 20 by 4, and, conversely, that the square root of 576 will be 20 + 4; and hence by induction, the rule.

EXAMPLES.

(1.) Extract the square root of 576.

$$\begin{array}{rl}
 & 5'76 \;(\; 20 + 4 \text{ or } 24 \\
 & 400 \\
 & \overline{} \\
40 &)\; 176 \;(\; 4 \\
4 & \\
\overline{44} &)\; 176 \\
 & \overline{}
\end{array}
\qquad\qquad
\begin{array}{rl}
 & 5'76 \;(\; 24 \text{ root.} \\
 & 4 \\
 & \overline{} \\
44 &)\; 176 \;(\\
 & 176 \\
 & \overline{}
\end{array}$$

(2.) Extract the square root of 89794576.

```
            89'79'45'76      /9000          89'79'45'76 ( 9476
             81000000        | 400          81
           -----------       |  70          ----
18000 \     8794576(400      |   6     184) 879 (
  400  )                     | ---          736
------/     7360000          \9476 root.   ----
18400                                  1887) 14345 (
            -------  ( 70                    13209
18000 \     1434576                          ------
  800  )                              18946) 113676 (
   70  )    1320900                          113676
------
18870 /     -------

18000
  800 \      113676 ( ·6
  140  )
    6  )     113676
------
18946 /     -------
```

(3.) Extract the square root of 2996350129111329.

```
             29'96'35'01'29'11'13 29 ( 54738927 Ans.
             25                        ----------
             ---
   104 )      496
              416
             ----
  1087 )       8035
               7609
              -----
 10943 )        42601
                32829
               ------
109468 )         977229
                 875744
                -------
1094769 )       10148511
                 9852921
                --------
10947782 )        29559013
                  21895564
                 ---------
109477847 )       766344929
                  766344929
```

EXERCISES 49.

(B.)

Find the square roots of the following numbers:—

1. 1296.
2. 80656.
3. 27·9841.
4. 121550625.
5. 63361600.
6. 6426225.
7. 8555625.
8. 62599744.
9. 14774·554437890625.
10. 25436448423936.
11. 23004973·3225.
12. 1·4774554437890625.
13. 529228797568936·68900625.
14. 617737·837369.
15. 3816000357173290·92842161.

To Extract the square root of a vulgar fraction.

Rule.—If the denominator be a complete square, extract square root of both terms; but if the denominator be not a c plete square, multiply the numerator by the denominator, and square root of the product will be the numerator, which, divide the denominator, will be the required root. Or, reduce the f tion to a decimal, and then extract the square root.

EXAMPLES.

(1.) Find the square root of $\frac{169}{576}$. *Ans.* $\frac{13}{24}$.

(2.) $$\sqrt{\frac{9}{16}} = \frac{3}{4}$$

(3.) $$\sqrt{\frac{3}{4}} = \frac{1·73205}{2} = ·866025 \text{ Ans.}$$

(4.) $$\sqrt{\frac{5}{9}} = \frac{2·236068}{3} = ·745356 \text{ Ans.}$$

(5.) $$\sqrt{\frac{3}{7}} = \sqrt{\frac{21}{49}} = \frac{4·58257}{7} = ·65465 \text{ Ans.}$$

(6.) $$\sqrt{\frac{4}{5}} = \sqrt{\frac{20}{25}} = \frac{4·4721}{5} = ·8944 \text{ Ans.}$$

(7.) $$\sqrt{\frac{4}{5}} = \sqrt{·8} = ·8944 \text{ Ans.}$$

.) Extract the square root of ·4.*

```
              ·40'00'00'00 ( ·6324 Ans.
               36
              ----
     123   )  400
           /  369
              ----
     1262  )  3100
           /  2524
              -----
     12644 )  57600
           /  50576
              -----
               7024
```

EXERCISES 49B—*continued*.

Find the square root of ·04.

" " " ·5.

" " " ·05.

" " " ·8.

" " " ·0089794576.

" " " ·000144.

" " " ·00144.

" " " 1·6.

" " " ·16.

" " " ·016.

" " " 529·092004.

" " " 617737·837369.

" " " 58275238·4.

" " " 189863769·6.

" " " 851972

" " " ·1.

" " " ·01.

" " " ·001.

In extracting the square root of decimals, the decimal places must always ven; if not so, affix a cipher to the right, and then point off in periods of two ı left to right, as already directed. Thus $\sqrt{\cdot 4} = \sqrt{\frac{4}{10}}$, which, if left his condition, would require the square root of both numerator and denomi- ɔr to be taken; hence the utility of making $\sqrt{\frac{4}{10}}$ into $\sqrt{\frac{40}{100}}$; that is, ɩing the decimal places in the numerator even, so that the denominator r be an exact square, and the root a decimal fraction.

34. Extract the square root of 005184, and reduce the result to the corresponding equivalent fraction in its lowest terms.

35. A square field contains 86 a. 1 r. 38 sq. p. 9½ sq. yds.; find the length of a side.

131. *Any two sides of a right-angled triangle being given, to find the third side.*[*]

Rule (1).—*When the two legs are given, to find the hypothenuse.* Add the square of one leg to the square of the other, and the square root of the sum will be the hypothenuse.

(2).—*When the hypothenuse and one of the legs are given, to find the other leg.* From the square of the hypothenuse subtract the square of the given leg, and the square root of the remainder will be the required leg.

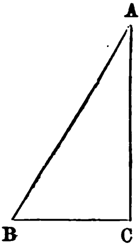
EXAMPLE.

In the right-angled triangle B A C, are given the base B C = 56 yards, and the perpendicular A C = 105 yards, to find the hypothenuse B A.

Here $56^2 + 105^2 = 3136 + 11025 = 14161$; and $\sqrt{14161} = 119$ yards, the hypothenuse B A.

When the hypothenuse B A = 55, and the perpendicular C A = 44 feet, to find the base, B C.

Here $55^2 - 44^2 = 3025 - 1936 = 1089$; and $\sqrt{1089} = 33$ ft. the length of base B C.



EXERCISES 49.

(C.)

1. A ladder 78 feet long, placed 30 feet from the bottom of a wall, reaches its top exactly; how high is the wall?
2. A steeple 96 feet high, casts a shadow 180 feet long; what must be the length of a staight line to reach from the extremity of the shadow to the top of the steeple?
3. A ladder 102 feet long, being placed in a street, reached a window 90 feet from the ground on one side, and on the other side it reached a window 48 feet high from the same spot. What is the breadth of the street?

---

[*] The base and perpendicular of a right-angled triangle are frequently called the *legs.*

4. A ladder 68 feet long, when placed upright against a wall, reached the top; how far would the top of the ladder fall if its other end were drawn out 32 feet from the bottom of the wall?

5. A rectangular field measures 348 yards in length and 261 yards in breadth; what is the length of the diagonal?

THE CUBE AND CUBE ROOT.

THE CUBE OF A NUMBER.

132. The *cube* of a number is the product which occurs from multiplying that number by itself, and then multiplying the result again by the same number.

Thus, the cube of $4 = 4 \times 4 \times 4 = 64$.
„ 5 „ $5 \times 5 \times 5 = 125$.
„ 13 „ $13 \times 13 \times 13 = 2197$.

EXERCISES 50A.

1. Cube 36 written 36^3.
2. „ 45^3.
3. „ 1452^3.
4. „ 8415^3.
5. „ 9999^3.
6. „ 26712^3.
7. „ 5·38321.

CUBE ROOT.

133. The cube root of a given number, is a number which, when multiplied into itself, and the result again multiplied by it, will produce the given number.

Thus, the cube root of $64 = 4$, because $4 \times 4 \times 4 = 64$.
„ „ 125 „ 5, „ $5 \times 5 \times 5 = 125$.
„ „ 2197 „ 13, „ $13 \times 13 \times 13 = 2197$

	I.		II.		III.	
Roots	2	9	20	99	200	999
Cubes	8	729	8000	970299	8000000	997002999

By comparing columns I., II. and III., it is seen that if any figure be annexed to a given number, the cube of the resulting number will exceed the cube of the given number by three figures; also, if two figures be annexed, the cube of this new quantity will be six figures in excess of the cube of the given number; and so on, increasing by three figures for every figure annexed to the right. Hence, the *units* of the root are contained in the three right-hand figures, the *tens* of the root in the three next, and so on. Therefore, in extracting the cube root of a number it is divided into periods of *three*, beginning at units' place.

To extract the cube root of a given number.

Rule I.—Place in succession 1, 0, 0, and the given number whose cube root is to be found; separate the given number into periods of three figures each, counting from the units' place towards the left. Find the greatest figure whose cube shall not exceed the first or left-hand period, and place it in the quotient for the first figure of the root. This quotient figure is used as a constant multiplier throughout the first stage of the operation; thus, 1 is multiplied by it, and the product added to the first 0; this sum is multiplied by the same root figure, and added to the second 0, and again, this last sum is multiplied by the same root figure, and subtracted from the first or left-hand period. Continue a similar process of multiplying and adding by the root figure till you have three additions in the first column, two in the second, and this completes the first step.

II.—To the sum in the *first* column annex one cipher, to the *second* two ciphers, and to the *third* the next period, or if the periods be finished, annex three ciphers. Then divide the number in the third column by that in the second (called the *trial divisor*), and the quotient will be the next figure of the root, with which proceed through the same course of multiplications and additions as before, and this completes the second step of the process. If the product should at any time exceed the number above it in the third column, the next lower root figure must be used.

III.—To find the next figure of the root, proceed exactly in the same manner as before, and so continue until all the figures are exhausted. If the process should not then terminate, the extraction of the root may be continued to as many decimal places as may be deemed fit, by annexing three ciphers for every period.

EXAMPLES.

(1.) Extract the cube root of 485587656.

```
1          0       0          485'587'656 ( 786 root.
           7       49         343
           —       —
           7       49         142587
           7       98         131552
           —
          14       14700        11035656        Proof.
           7        1744        11035656          786
End of 1st step —   ———                           786
         210       16444                         ———
           8        1808                         4716
         ———                                    6288
         218       1825200                      5502
           8         14076                     ———
         ———       ———                        617796
         226       1839276                       786
           8                                  ———
 „   2nd step ———                            3706776
        2340                                4942368
           6                               4324572
        ———                               ————
 „   3rd step 2346                        485587656
```

The greatest number (7), whose cube does not exceed the first period (485), is placed in the quotient for the first figure of the root. Multiply 1 by 7, and add the product to the first 0; this amount (7) multiply by 7, and add the product (49) to the second 0; and again, this amount (49) multiplied by 7, and subtract the product (343) from 485, which gives the remainder (142). Again, multiply 1 by 7 (the root figure), and add the product to the last sum in the first column, multiply the sum (14) by 7, and add the product (98) to the last sum in the second column, there results 147; and, lastly, multiply 1 by the same root figure (7), and add the product to the sum (14) of the first column. This concludes the first step in the process.

Annex one cipher to the number (21) in the first column, two to 147 in the second, and annex the next period (587) to the remainder (142). Find now the second figure of the root, by dividing the third column (142587) by the second (147000), which is called the *trial divisor*. The quotient would appear to be 9; this, however, on trial is found to be too large, and 8 is taken, which answers; then multiply 1 by 8 (the root figure), and to the sum (210) in the first column add this product; then the sum (218) multiply by 8, and to this product add the sum (14700) in the second column, and again multiply the sum (16444) by 8 and the product (131552) subtract from the third column, which gives the remainder (11035). Then multiply 1 by 8, and adding this product to the sum (218) in the first column, multiply the sum (226) by 8, and to the second column (16444) add this product, and there results 18252. Again multiply 1 by 8, and to the sum (226) in first column add the product, and there results 234, which completes the second step in the process.

For the third step, annex, as before, one cipher to 234 in the first column, two to the second (18252), and the next period (656) to the third (11035). For the third figure of the root, divide the third column (11035656) by the second (1825200); which gives 6 for the next figure in the root. Now, as before, multiply 1 by 6, and to 2340 in the first column add the product; multiply the sum (2346) by 6, and add the product (14076) to 1825200 in the second column; then multiply the sum (1839276) by 6 and subtract it from the number in the third column, when, as there is no remainder, the process terminates. Therefore the root is 786, and by raising it to the third power the original number is found.

(2.) Extract the cube root of 92775111183.*

```
        1       0          0              92′775′111′183 (4527
                4          16             64
                —          —      ┌───────────────────
                4          16     │       28775
                4          32     │       27125
                —    ┌────────────┘ ┌─────────────
                8    │     4800     │      1650111
                4    │      625     │      1220408
End of 1st step ─────┘     ────     │  ┌──────────────
              120          5425     │  │    429703183
                5           650     │  │    429703183
              ───    ┌──────────────┘  │    ─────────
              125    │     607500      │
                5    │       2704      │
              ───    │     ──────      │
              130    │     610204      │
                5    │       2708      │
 „   2nd step ───────┘  ┌──────────────┘
             1350       │  61291200
                2       │     94969
             ────       │  ────────
             1352       │  61386169
                2       │                              Proof.
             ────       │                                4527
             1354       │                                4527
                2       │                           ─────────
 „   3rd step ──────────┘
            13560                                    20493729
                7                                        4527
            ─────                                 ───────────
            13567                                 92775111183
```

---

* This mode of extracting the cube root is a particular application of the method of resolving numerical equations discovered by the late Mr. Horner, of

EXERCISES 50.

(B.)

1. Extract the cube root of 4096.
2. " " 592704.
3. " " 18609·625.
4. " " ·999700029999.
5. " " 10403062487.
6. " " 495289·174528.
7. " " 79·999477540346456.
8. " " 1999593535728747279183354752.
9. " " 918517247483640899.
10. " " 190598838576128.
11. " " 156.
12. " " 9·99999089147021850.
13. " " 45882712.
14. " " 110839903795372875.

134. *To find the cube root of a fraction.*

Rule.—If the denominator of the fraction be not a cube number, multiply the numerator by the square of the denominator, and the cube root of the product will be the new numerator for this denominator. Or, reduce the fraction to the form of a decimal, the cube root of which will be the required root.

EXERCISES 50.

(C.)

1. Find the cube root of $\frac{125}{512}$.
2. " " $\frac{64}{343}$
3. " " $\frac{2}{3}$
4. " " $\frac{7}{8}$
5. " " $\frac{5}{9}$
6. " " $\frac{11}{16}$
7. There are 3 boxes, whose contents are, respectively, 1000, 16656, and 20000 cubic inches; what will be the length of the side of a cubical box that will contain as much as the other three?
8. There are 3 balls of the same material, whose diameters are 6, 8, and 10 inches respectively; what is the diameter of a fourth ball of the same substance, which is equal in weight to the 3 others?
9. A cubical box contains 91125 solid inches; what is the cost of painting the outside, at 4½d a square foot?

MENSURATION.

135. **Mensuration** is that part of mathematics by which the lengths of lines, the areas* of surfaces, and the capacity or cubical contents of solids are determined.

In Surface Measure.

Inches × inches = square inches.
Inches × feet = parts.
Feet × feet = square feet.
Yards × yards = „ yards.
Perches × perches = „ perches.

Thus,

12 inches × 12 inches = 144 square inches.
3 feet × 3 feet = 9 „ feet.
$5\frac{1}{2}$ yards × $5\frac{1}{2}$ yards = $30\frac{1}{4}$ „ yards.

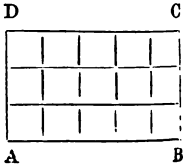
In Solid or Cubical Measure.

Inches × inches × inches = cubic or solid inches.
Feet × feet × feet = „ „ feet.
Yards × yards × yards = „ „ yards.
Perches × perches × perches = „ „ perches.

Thus,

12 inches × 12 inches × 12 inches = 1728 cubic or solid inches.
3 feet × 3 feet × 3 feet = 27 „ „ feet.
$5\frac{1}{2}$ yards × $5\frac{1}{2}$ yards × $5\frac{1}{2}$ yards = $166\frac{3}{8}$ „ „ yards.

To find the area of a rectangular figure.



Let A B C D be a rectangular figure, whose length is 5 inches and breadth 3 inches.

If A B be divided into 5 equal parts, and B C into 3 equal parts, an inch each, and through the points of division lines be drawn parallel to the

---

* The area of a figure is the quantity of surface contained in it. In the examples and exercises given on this subject, it is assumed that the pupil is already acquainted with their geometrical terms and qualities, and, therefore, no explanation of them is offered here.

(3.) **Extract the cube root of 41999·034176.**

0	0	41′999 ·034′176 (34·76.
3	9	27
3	9	14999
3	18	12304
6	2700	2695034
3	376	2477923
90	3076	217·111176
4	392	217·111176
94	3468	
4	71·89	
98	3539·89	
4	72·38	
102	3612·27	
·7	6·2496	
102·7	3618·5196	
7		
103·4		
7		
104·1		
·06		
104·16		

Proof.

34·76
34·76

1208·2576
34·76

41999·034176

---

ath, from whose plan is derived a method of extracting any root whatever. rofessor Young's books on Equations contain a clear and able exposition of is method of Mr. Horner's; and in the author's "Key to Rutherford's Edi- on of Hutton's Course of Mathematics," will be found a large variety of tercises in numerical equations solved by the same method.

(4.) Extract the cube root of 80.

1	0	0		80	4·3
	4	16		64	·008
					·0008
	4	16		16·	·00006
	4	32		15·507	
					4·3088693800637
	8	48		·493	
	4	3·69		·444586112	
	12	51·69		·048413888	
	·3	3·78		·044549545472	
	12·3	55·47		·003864342528	
	·3	·103264		·003341882874456	
	12·6	55·573264		522459653544	
	·3	·103328		501290458542	
	12·9	55·676592		21169195002	
	·008	·01033984		16709718017	
	12 908	55·68693184		4459476985	
	·008	·01034048		4455925197	
	12·916	55·69727232		3551788	
	·008	·0007755876		3341944	
	12·924	55·6980479076		209844	
	·0008	7755912		167097	
	12·9248	55 698823498	8	42747	
	·0008	116339	2	38989	
	12·9256	55·698939838	0	3758	
	·0008	116339	2	3341	
	12.9264	55·69905617	7	417	
	·00006	387	8	389	
	12 92646	55·69906005	5	28	
	·00006	387	8	28	
	12·92652	55 6990639	3	0	
	·00006	10	3		
	1,2.9 | 26 | 58	55·6990649	6		
		10	3		
		55· | 6 | 9 | 9 | 0 | 660			

17. How many boards, 18 ft. 4 in. long and 9 in. broad, will cover a floor 80 ft. long and 29 ft. 4 in. wide?

18. How many flags, 2 ft. 8 in. by 1 ft. 3 in., will cover an area 34 ft. 8 in. by 18 ft. 9 in?

19. How many yards of carpet, 4 ft. 10 in. wide, will cover a floor 48 ft. 4 in. long and 25 ft. 6 in. wide?

20. Bought 36 boards, each containing 23 ft. 9 in. by 1 ft. 4 in., at 7½d. per square foot; what is the cost?

21. How many boards, 14 ft. 8 in. by 1 ft. 5 in. will cover a floor 58 ft. 8in. long and 19 ft. 10 in. broad?

22. How many yards of carpet, 2 ft. 1 in. wide, will cover a room 468 ft. 9 in. long and 66 feet 8 in. broad; and what is its cost at 5s. 8¾d. per yard?

23. How many acres are occupied by a railway 210 miles long and 84 feet broad, supposing it a rectangle?

24. If a ploughman turns up a furrow of 18 in. wide, how far will he travel in ploughing 3 acres?

25. What is the cost of paving a street 468 ft. 9 in. long and 33 ft. 4 in. broad, at 8½d. per square yard?

26. A room is 60 ft. in circuit and 12 ft. high; how much paper, 2 ft. wide, will cover it, deducting the door, which is 8 ft. by 4 ft., and three windows, each 5 ft. by 3½ ft, and the chimney, 4 ft. square?

27. How many yards of paper, ¾ yard wide, will cover a room 24⅔ ft. long, 15⅚ ft. wide, and 9⅓ ft. high?

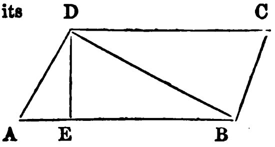
28. What is the expense of paving a street, ¾ mile long and 66½ ft. broad, at 6½d. per square yard?

SURFACES.

137. *To find the area of a parallelogram.*

Rule.—Multiply its length by its perpendicular breadth.

Thus A B × D E = area of parallelogram A. B C. D.



EXERCISES 52.

(A.)

1. What is the area of a parallelogram 8 ft. 6 in. long, and 4 ft. 10 in. high?

2. Find the area of a parallelogram 10 chains 52 links long, and 7 chains 63 links in perpendicular breadth.

3. What is the rent of a field in the form of a parallelogram, whose length is 18 chains 75 links and perpendicular breadth 14 chains 45 links, at £3 11s. 6d. per acre?

138. *To find the area of a triangle.*

Rule—Multiply the base by the altitude, and take half the product.

$$\text{Thus } \frac{\text{A B} \times \text{D E}}{2} = \text{area of triangle A D B.}$$

EXERCISES.

4. Find the area of a triangle whose base is 254 yards, and altitude 136 yards.
5. What is the area of a triangle whose base is 49 perches, and height 34 perches?
6. If the base of a triangle be 363 feet, what must be the altitude so that the area may contain an acre?

139. *To find the area of a triangle when the three sides are given.*

Rule.—From half the sum of the three sides subtract each side severally; multiply the half sum, and the three remainders continually together, and the square root of the product will be the area.

EXAMPLE.

What is the area of a triangle whose three sides are respectively 13, 14, and 15 yards?

13	21	21	21
14	13	14	15
15	—	—	—
——	8	7	6 remainder.
2) 42			
——			
21 = half sum.			

$$\text{Area} = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84 \text{ square yards.}$$

EXERCISES 52A—*continued.*

7. Find the area of a triangle whose sides are 354, 432, and 574 yards respectively.
8. Find the area of a triangle whose three sides are 518, 574, and 618 feet.

THE CIRCLE.

140. *To find the circumference and also the area of a circle, when the diameter is given.*

For the circumference—multiply the diameter by 3·1416.

For the area—multiply the square of the diameter by ·7854.

opposite sides, 15 equal squares are obtained, each of which has its side one inch; that is, the area is 3 × 5, or 15 square inches; hence the

Rule.—Multiply the length by the breadth, and the product will denote the area.

DUODECIMALS.

136. When numbers diminish from left to right at a ten-fold rate, it is called the decimal system of notation; but if the diminution were at a twelve-fold rate it would be the duodecimal, as when unity is divided into 12 parts, and each of these into 12 more, and so on, the relation of each of such divisions is called *duodecimal.*

The rule for performing duodecimals, or cross multiplication.

Write the denominations of the same name under each other. Multiply every term in the multiplicand, beginning at the lowest by the feet in the multiplier, divide each product by 12 and carry the quotient to the next product, and write the remainder under the term of the multiplicand just used. Then multiply by the inches in the multiplier, in the same manner, and write each remainder, after dividing by 12, a place further to the right; and so on with each denomination. The sum of the several lines will be answer.

EXAMPLES.

(1.) Multiply 38 feet 7 inches by 15 feet 9 inches.

ft. in.
38 7
15 9
578 9
28 11 3
607 8 3

ft. in. ft. in.
(38 7) × (15 9)
12 12
463 189
189
4167
3704
463
12) 87507 square inches.
12) 7292 0 3
607 8 3

Ans. 607 square feet, 8 parts, 3 square inches.

Or,

$(607 + \frac{8}{12} + \frac{3}{144})$ square feet.

(2.) Multiply 14 feet 7 inches 6 seconds by 9 feet 5 inches.

ft.	in.	sec.	
14	7	6	
9	5		
131	7	6	
6	1	1	6
137	8	7	6

$$(14 \text{ ft. } 7 \text{ in. } 6 \text{ sec.}) \times (9 \text{ ft. } 5 \text{ in.})$$

12   12

175   113

12

2106

113

6318

2106

2106

12) 237978

12) 19831 6

12) 1652 7 6

12) 137 8 7 6

137 square feet, 8 parts, 7 square inches $\frac{6}{12}$ square inch.

Or,

$$\left(137 + \tfrac{8}{12} + \tfrac{7}{144} + \tfrac{6}{1728}\right) \text{ square feet.}$$

EXERCISES 51.

1. Multiply 4 feet 6 inches by 3 feet 9 inches.
2. " 3 feet 8 inches by 3 feet 4 inches.
3. " 2 feet 8 inches by 2 feet 8 inches.
4. " 9 feet 3 inches by 3 feet 5 inches.
5. " 4 feet 3 inches by 3 feet 6 inches.
6. " 2 feet 6 inches by 3 feet 9 inches.
7. " 13 feet 9 inches by 8 feet 5 inches.
8. " 17 feet 3 inches 6 sec. by 12 feet 7 inches.
9. " 18 feet 3 inches by 5 feet 10 inches.
10. " 4 feet 7 inches by 9 feet.
11. " 15 feet 3 inches by 7 feet 9 inches.
12. " 9 feet 4 inches 7 sec. by 2 feet 9 inches 2 sec.
13. " 4 feet 2 inches by 3 feet 3 inches.
14. " 8 feet 7 inches 4 sec. by 5 feet 9 inches.
15. What are the contents of a plate glass window 2 ft. 11 in. broad and 4 ft. 10 in. high?
16. What are the contents of a carpet 5 ft. 2 in. wide and 24 ft. 9 in. long?

EXAMPLE.

What is the circumference of a circle whose diameter is 50 feet?

50 × 3·1416 = 157·08 feet. *Ans.*

EXERCISES 52A—*continued.*

9. Find the circumference of a circle whose diameter is 1764 yards.
10. If the circumference of a circle be 14971260·2424 feet; what is the diameter?

EXAMPLE.

What is the area of a circle whose diameter is 50 feet?

50 × 50 × ·7854 = 1963·5 square feet. *Ans.*

11. What is the area of a circle whose diameter is 31 yards?
12. What is the area of a ring of which the internal diameters are respectively 246 and 228 yards?
13. What is the diameter of a circle whose area is 201·0624 square perches?

SOLIDS.

CUBE AND PARALLELOPIPEDON.

141. *To find their solidity.*

Rule.—Multiply the length by the breadth, and that product by the depth.

EXAMPLE.

What is the solidity of a cube whose side is 5½ yards?

5½ × 5½ × 5½ = 166⅜ cube yards. *Ans.*

EXERCISES 52.

(B.)

1. Required the area of the surface of the foregoing cube.*
2. How many cubic feet are there in a rectangular piece of timber 15 in. broad, 10 in. thick, and 24 ft. long?
3. Required the cubical contents of a parallelopipedon whose length is 24 ft. 6 in., breadth 9 feet, and height 16 ft. 8 in.
4. What weight of water will a cistern, 4 ft. long, 3 ft. 3 in. broad, and 2 ft. 6 in. deep, contain? A cubic foot of water weighs 1000 ounces.

PRISM AND CYLINDER.

To find their solidity.

Rule.—Multiply the area of the base by the altitude.

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* The surface consists of six equal squares; the area of one of which is 5½ × 5½ = 30¼ square yards and 30¼ × 6 = 181½ square yards.

EXERCISES.

5. Required the solid contents of a triangular prism whose height is 9 ft., and each side of its base 34 in.

6. How many cubic feet are there in a cylindrical pillar 14 ft. high and 14 in. in diameter?

7. What is the expense of conveying three boxes from Calcutta to London, at 60s. per ton?—dimensions as follow:—

Box 1	5 feet 3 inches	8 feet 2 inches	2 feet 9 inches.
„ 2	5 „ 9 „	8 „ 0 „	3 „ 0 „
„ 3	5 „ 9 „	8 „ 0 „	2 „ 8 „

8. How many cubic inches in a round bar of iron 20 ft. long and $\frac{3}{4}$ in. in diameter?

PYRAMID AND CONE.

143. The solidity of the pyramid or cone is $\frac{1}{3}$ of the prism or cylinder which have respectively the same base and altitude. Hence,

$$\frac{\text{area of base} \times \text{altitude}}{3} = \text{solidity}.$$

EXAMPLE.

One of the Egyptian Pyramids, 480 ft. in altitude, stands on a square base whose side is $763\frac{2}{5}$ feet; what are the solid contents?

$$\frac{763\frac{2}{5} \times 763\frac{2}{5} \times 480}{3} = 93244729\tfrac{4}{5}.\ \textit{Ans.}$$

EXERCISES.

9. Suppose the mast of a ship to be a regular cone, 87 ft. long and 2 ft. in diameter at its base; what is its solidity?

10. Required the weight of a pyramid of marble, whose base is a hexagon, each side being 15 in., the vertical height 10 ft., and the specific gravity of the marble 2720.

GLOBE.

144. *To find its surface when the diameter is given.*

Rule.—Multiply the square of the diameter by 3·1416.

EXAMPLE.

Required the surface of a globe whose diameter is 18 inches.

Surface = 18 × 18 × 3·1416 = 1017·8784 square inches.

EXERCISES 52B—*continued.*

11. What is the surface of the moon, supposing it to be a globe, whose diameter is 2160 miles?

12. What must be the diameter of a ball, to have 1 square inch of surface?

13. A ball, 3 ft. in diameter, is covered with copper $\frac{1}{20}$ of an inch thick; what must be the value of the coating at 2s. $3\frac{1}{2}$d. per lb., the specific gravity of the copper being 9000?

14. A balloon contains 1000 yards of silk $\frac{3}{4}$ yard wide; what is its diameter, admitting it to be a perfect globe?

145. *To find the solidity of a globe when the diameter is given.*[*]

Rule.—Multiply the cube of the diameter by ·5236.

EXAMPLE.

Find the solidity of a sphere whose diameter is 17 inches.

Solidity = 17 × 17 × 17 × ·5236 = 2572·4468. *Ans.*

EXERCISES 52—*continued.*

15. Required the solidity of the moon, supposing it a perfect sphere whose diameter is 2160 miles.

16. What are the solid contents of the earth, supposing it a perfect sphere whose diameter is 7912 miles?

ALLIGATION.

146. By this rule are solved those questions occurring in business in which goods of the same kind are mixed together, either to improve an inferior kind by mixing with it a superior one, or to effect a ready sale for a superior quality whose high price has interfered with its disposal. It is divided into two cases.

CASE I.

147. *When several articles are mixed together, and the price of each sample is given, to find the mean or average price of one of the mixture.*

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[*] If the diameter of the base and the vertical height both of a cone and cylinder be respectively equal to the diameter of a globe, the solidity of the cone and globe together will be equal to the solidity of the cylinder (by solid geometry); but the solidity of the cone being $\frac{1}{3}$ of the cylinder (by solid geometry), the solidity of the globe must, therefore, be $\frac{2}{3}$ of the cylinder.

EXAMPLE.

A grocer mixes 30 lb. of sugar at 4d. per lb., 18 lb. at 6d., and 24 lb. at 5½d. together; what is the average price of the mixture per lb.?

30 lbs. at 4d. per lb. = 120d.
18 ,, at 6d. ,, = 108d.
24 ,, at 5½d. ,, = 132d.

As 72 lb. : 1 lb. :: 360d. : x = 5d. *Ans.*

EXERCISES 53.

1. A mixture being made of 16 lb. of tea at 4s. 10½d. per lb.; 10 lb. at 4s. 6d. per lb.; and 24 lb. at 2s. 2d. per lb.; what is 1 lb. of the mixture worth?
2. If 18 gallons of water be added to a mixture of 126 gallons of rum at 16s. 3d. per gallon, and 108 gallons at 17s. 6d. per gallon; what is the value of a gallon of the compound?
3. Of what fineness is a composition of 3 lb. of gold bullion of 9 oz. pure to the lb., with 5 lb. of standard gold of 11 oz. to the lb.?

CASE II.

148. *When the price of each of the ingredients to be mixed is given, and the mean or average price is known, to find how much of each sample must be taken to form the mixture.*

How much tea at 6s. and 3s. 8d. per lb. must be mixed together to make a compound worth 4s. 4d.?

$$\text{Mean } 52 \begin{cases} 72 \ldots \text{a loss of 20d. on every lb. of the 6s. tea.} \\ 44 \ldots \text{a gain of 8d. on every lb. of the 3s. 8d. tea.} \end{cases}$$

Now the object is to mix the teas in such proportion, that the loss resulting from one sample may be counter-balanced by the gain arising from the other; then to make such a mixture, if 8 lb. of the 6s. tea be taken, the loss thereon will be 160d.; and if 20 lb. of the 3s. 8d. tea be taken, the gain thereon will be 160d. Hence, 8 lb. of the 6s. tea, with 20 lb. of the 3s. 8d. will make the required compound.*

---

* Any two numbers having the ratio of 8 to 20 will also make the required mixture, as well as 2 and 5; as also will 12 and 30, &c. The questions under Case II. will admit of a variety of answers if there be no restriction as regards the quantity of the mixture.

EXAMPLES.

(1). A wine merchant has three sorts of brandy worth respectively 17s. 18s. and 22s. per gallon. Of these he wishes to make a mixture of 45 gallons to sell at 20s. per gallon; how many gallons of each must he take?

$$20\begin{cases} 17s\ldots 3s.\text{ gain} \times 2\text{ gallons.} & = 6s.\text{ gain.} \\ 18s.\ldots 2s.\text{ gain} \times 2\ \text{,,} & = 4s.\ \text{,,} \\ 22s.\ldots 2s.\text{ loss} \times (3+2)\text{ gals.} & = 10s.\text{ loss.} \end{cases}$$

$$\left.\begin{array}{llll} \text{That is} & 2 & \text{gals.} & \text{at } 17s. \\ \text{,,} & 2 & \text{,,} & \text{at } 18s. \\ \text{And} & 5 & \text{,,} & \text{at } 22s. \end{array}\right\}\text{ make a mixture.}$$

$$\begin{array}{l} \overline{\ 9\text{ gallons.}} \end{array}$$

$$\begin{array}{lllll} & \text{g.} & \text{gal.} & \text{g.} & \textit{Ans.} \\ \text{As} & 9 : & 45 :: & 2 : x = & 10\text{ gallons at } 17s. \\ \text{As} & 9 : & 45 :: & 2 : x = & 10\ \text{,, at } 18s. \\ \text{As} & 9 : & 45 :: & 5 : x = & 25\ \text{,, at } 22s. \\ & & & & \overline{45}\text{ gallons.} \end{array}$$

(2). A licensed victualler wishes to mix together 4 kinds of claret wine, respectively valued at 8s., 7s., 5s., and 4s. a gallon, in order to produce one wine of 6s. a gallon. What quantity of each must be taken to make the mixture?

$$6\begin{cases} 8\ldots 2\text{ loss} \times 3 = 6s.\text{ loss; that is } 3\text{ gals. of } 8s. = 24\ \textit{Proof.} \\ 7\ldots 1\text{ loss} \times 1 = 1s.\text{ loss;} \quad \text{,,}\ 1\ \text{,, of } 7s. = 7\ \text{,,} \\ 5\ldots 1\text{ gain} \times 1 = 1s.\text{ gain:} \quad \text{,,}\ 1\ \text{,, of } 5s. = 5\ \text{,,} \\ 3..3\text{ gain} \times 2 = 6s.\text{ gain;} \quad \text{,,}\ 2\ \text{,, of } 3s. = 6\ \text{,,} \end{cases}$$

$$\overline{7} \qquad\qquad \overline{42}$$

EXERCISES 53—*continued.*

4. In what proportions must 3 sorts of rum at 15s., 13s., and 8s., per gallon respectively, be mixed, in order to produce a mixture worth 12s. a gallon?
5. What quantity of tea worth separately, 4s., 3s. 9d., and 3s. 3d., per lb., must be mixed together to make a parcel containing 90 lb., worth 3s. 8d.?
6. How many gallons of water must be added to 30 gallons of brandy, worth 28s. per gallon, so that the mixture may be sold at 24s. per gallon?
7. How much alloy must be added to gold of 10 oz. fine, to bring it to $7\frac{1}{2}$ oz. fine?

8. A grocer having teas at 3s., 4s., 5s., and 6s. per lb., wishes to make a mixture of 64 lb. worth 4s. 6d. per lb.; how much of each would be required?

9. If 40 lb. of tobacco at 2s. 4d. per lb., be mixed with others at 2s. 6d., 3s., and 3s. 6d. per lb.; what quantity of each sort must be taken to sell the composition at 2s. 8d. per lb.?

EXAMPLES.

(1.) Hiero, King of Syracuse, suspecting that a crown of gold, which he had ordered, was debased by the workman, employed Archimedes to detect the supposed fraud.

Archimedes took a mass of pure gold and another of silver, each of the same weight as the crown (say 5 lbs.); and putting them separately into a vessel full of water, he accurately noted the quantity of water that flowed over. That expelled by the gold weighed ·26 lb., that by the silver ·46 lb., and that by the crown ·30 lb. What were the quantities of gold and silver in the crown?

$$\cdot30\begin{cases}\cdot26 \ldots \cdot04\text{ lb.} \times 16\text{ lbs.}\\ \cdot46 \ldots \cdot16\text{ lb.} \times \underline{\ \ 4\ \ }\end{cases}$$

Ans.

As 20 : 5 lb. : : 16 : x = 4 lb. of gold.

As 20 : 5 lb. : : 4 : x = 1 lb. of silver.

EXERCISES 53.—*continued.*

10. How many gallons of gin at 10s., 13s., and 15s. a gallon, must a spirit merchant take, to make a mixture of 120 gallons worth 12s. a gallon?

11. How much gold of 19 and 20 carats fine, must be mixed with 18 ounces of 23 carats fine, to bring the whole to 22 carats fine?

12. How many gallons of water must be added to 70 gallons of whiskey, worth 18s. per gallon, to make the value of the mixture 17s. 6d. per gallon?

13. A goldsmith wishes to melt together gold of 13, of 14, of 15, and 21 carats fine to the ounce, in order to make a composition of 35 ounces of 18 carats fine; how many ounces of each must he take?

14. How many gallons of water, and how many of gin, worth 12s. a gallon, must be added together to make a cask of 20 gallons, worth 9s. a gallon?

MENTAL ARITHMETIC.

149. The process of performing arithmetical calculations without the aid of artificial help, such as slate, paper, &c., is termed mental arithmetic.

To acquire the power of computing mentally with tolerable facility, it will be only necessary to commit to memory the multiplication table up to 19 times 19, to be well versed in the process of compound multiplication and division, and to understand the leading principles of arithmetic.

With this preparation, and with moderate exercise in the application of numbers generally, a youth with ordinary capacity may answer with quickness any reasonable question asked. Beyond the course suggested, there are no rules offered, save four, which are already known; nor is any "royal road" proposed to aid in the acquisition of this mental power. No doubt there are several good books written on this subject, containing a large number of rules, some as many as 90; but the advantage of many of these rules is very questionable, and the labour in committing so many of them to memory would far outweigh any benefit derived from their use and application.

MULTIPLICATION TABLE (*Continued*).

13 times	14 times	15 times	16 times	17 times	18 times	19 times
1 are 13	1 are 14	1 are 15	1 are 16	1 are 17	1 are 18	1 are 19
2... 26	2... 28	2... 30	2... 32	2... 34	2... 36	2... 38
3... 39	3... 42	3... 45	3... 48	3... 51	3... 54	3... 57
4... 52	4... 56	4... 60	4... 64	4... 68	4... 72	4... 76
5... 65	5... 70	5... 75	5.. 80	5... 85	5.. 90	5... 95
6... 78	6... 84	6... 90	6... 96	6...102	6...108	6...114
7... 91	7... 98	7...105	7...112	7...119	7...126	7...133
8...104	8...112	8...120	8...128	8...136	8...144	8...152
9...117	9...126	9...135	9...144	9...153	9...162	9...171
10...130	10...140	10...150	10...160	10...170	10...180	10...190
11...143	11...154	11...165	11...176	11...187	11...198	11...209
12...156	12...168	12...180	12...192	12...204	12...216	12...228
13...169	13...182	13...195	13...208	13...221	13...234	13...247
14...182	14...196	14...210	14...224	14...238	14...252	14...266
15...195	15.. 210	15...225	15...240	15...255	15...270	15...285
16...208	16...224	16...240	16...256	16...272	16...288	16...304
17...221	17...238	17...255	17...272	17...289	17...306	17...323
18...234	18...252	18...270	18...288	18...306	18...324	18...342
19...247	19...266	19...285	19...304	19...323	19...342	19...361
20...260	20...280	20...300	20...320	20...340	20...360	20...380

EXERCISES.

Addition.

Ans.[*]
4 + 11 + 17 = 32
16 + 8 + 21 = 45
93 + 9 + 35 = 137
146 + 18 + 40 = 204
69 + 31 + 37 = 137
38 + 156 + 37 = 231
13s. 3d. + 6s. 9d. = £1 0 0
16s. 9d. + 13s. 7d. = £1 10 4
£1 12 9 + £1 16 3 = £3 9 0
£1 17 7 + £1 4 10½ = £3 2 5½
£2 18 9 + £1 18 5½ = £4 17 2½

Subtraction. *Ans.*

199 — 53 = 144
738 — 124 = 614
68 — 19 = 49
156 — 79 = 77
48 + 69 — 90 = 27
112 + 78 — 50 = 140
212 — 36 — 156 = 92
10s. — 3s. 6d. = £0 6 6
12s. 8d. — 9s. 9d. = £0 2 11
£1 16 6 — £0 18 8 = £0 17 10
£1 0 0 — £0 9 4½ = £0 10 7½
£2 5 6 — £1 13 10½ = £0 11 7½
£3 0 0 — £0 0 7½ = £2 19 4½
£1 10 0 — £0 11 3¾ = £0 18 8¼
£4 0 0 — £0 0 1½ = £3 19 10½
£5 0 0 — £1 0 0¾ = £3 19 11¼
£3 13 5½ — £1 5 11½ = £2 7 6
3 cwt. — 23 lb. = 2 cwt. 3 qrs. 5 lb.
5 cwt. — 2 qrs. 20 lb. = 4 cwt. 1 qr. 8 lb.
2 tons — 13 cwt. 1 qr. = 1 ton 6 cwt. 3 qrs.
3 gallons — 1 quart = 2 gallons 3 quarts.
2 yards — 5 inches = 1 yard 2 feet 7 inches.
3 miles — 33 perches = 2 miles 7 fur. 7 perches.

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[*] As these questions are intended chiefly to be used by the teacher only at *viva voce* examinations, it is thought more convenient to annex the answers.

Addition and Subtraction.

	Ans.
£1 17 3$\frac{1}{4}$ + £1 2 8$\frac{3}{4}$ =	£3 0 0
£2 5 6 + £0 11 4$\frac{1}{2}$ =	£2 16 10$\frac{1}{2}$
£3 5 7$\frac{1}{2}$ — £1 18 3$\frac{1}{2}$ =	£1 7 4
$\frac{7}{12} + \frac{5}{12}$ =	1
$\frac{11}{16} + \frac{5}{16}$ =	1
$\frac{13}{18} + \frac{17}{18} + \frac{6}{18}$ =	2
$\frac{13}{16} + \frac{11}{16}$ =	1$\frac{1}{2}$
$\frac{15}{17} + \frac{10}{17} + \frac{9}{17}$ =	2
$\frac{1}{2} + \frac{1}{3}$ =	$\frac{5}{6}$
$\frac{1}{3} + \frac{1}{4} + \frac{1}{2}$ =	1$\frac{1}{12}$
$\frac{2}{3} + \frac{3}{4} + \frac{7}{12}$ =	2
$\frac{3}{4} + \frac{1}{5}$ =	$\frac{19}{20}$
10 — $\frac{5}{9}$ =	9$\frac{4}{9}$
15 — 1$\frac{5}{11}$ =	13$\frac{6}{11}$
11 — $\frac{17}{25}$ =	10$\frac{8}{25}$
14$\frac{1}{2}$ — 10$\frac{1}{4}$ =	4$\frac{1}{4}$
17$\frac{1}{2}$ — 10$\frac{3}{4}$ =	6$\frac{3}{4}$
10$\frac{1}{3}$ — 5$\frac{1}{4}$ =	5$\frac{1}{12}$
16$\frac{3}{5}$ — 12$\frac{2}{3}$ =	3$\frac{14}{15}$
$\frac{8}{15}$ — $\frac{9}{20}$ =	$\frac{1}{12}$
£$\frac{5}{8}$ + $\frac{3}{4}$ guinea =	£1 8 3
£$\frac{3}{8}$ + $\frac{3}{8}$ shilling =	£0 7 10$\frac{1}{2}$
£$\frac{7}{8}$ + $\frac{7}{8}$ shilling + 7$\frac{1}{2}$d. =	£0 19 0
£$\frac{2}{3}$ + $\frac{2}{3}$ guinea + $\frac{2}{3}$ florin =	£1 8 8
£$\frac{2}{3}$ + $\frac{2}{3}$ shilling =	£0 14 0
£$\frac{3}{5}$ — $\frac{1}{3}$ shilling =	£0 11 8
£$\frac{5}{6}$ — $\frac{2}{3}$ shilling =	£0 16 0
£$\frac{7}{8}$ — $\frac{3}{4}$ shilling =	£0 16 9
£1$\frac{2}{3}$ guinea — 3$\frac{1}{6}$ shilling =	£1 10 2
£4$\frac{3}{8}$ — 4$\frac{1}{2}$ shilling =	£4 3 0
£$\frac{2}{3}$ + £$\frac{3}{4}$ — $\frac{7}{8}$ shilling =	£1 7 5$\frac{1}{2}$
£1$\frac{3}{8}$ + £$\frac{5}{6}$ — 11$\frac{5}{8}$ shilling ≐	£1 17 3$\frac{1}{2}$

Multiplication.

£	s.	d.		£	s.	d.	*Ans.*
0	1	3	× 6 =	0	7	6	
0	2	3½	× 3 =	0	6	10½	
0	1	5¾	× 5 =	0	7	4¾	
0	1	8½	× 9 =	0	15	4½	
0	1	10½	× 11 =	1	0	7½	
0	2	8½	× 10 =	1	7	1	
0	4	5½	× 12 =	2	13	6	
0	5	7½	× 13 =	3	13	1½	
1	7	7½	× 14 =	19	6	9	
1	5	10	× 15 =	19	7	6	
1	13	5½	× 16 =	26	15	4	
1	11	8½	× 17 =	26	19	0½	
1	13	5¾	× 18 =	30	2	7½	
1	16	5¼	× 19 =	34	12	5¾	

Multiplication.

$\frac{1}{2} \times 2 = 1$ *Ans.*
$\frac{1}{2} \times 3 = 1\frac{1}{2}$
$\frac{1}{2} \times 4 = 2$
$\frac{1}{3} \times 5 = 1\frac{2}{3}$
$\frac{1}{3} \times 6 = 2$
$\frac{1}{3} \times 8 = 2\frac{2}{3}$
$\frac{2}{5} \times 2 = 0\frac{4}{5}$
$\frac{2}{5} \times 3 = 1\frac{1}{5}$
$\frac{2}{5} \times 5 = 2$
$\frac{2}{5} \times 13 = 5\frac{1}{5}$
$\frac{2}{5} \times 20 = 8$
$\frac{3}{8} \times 7 = 2\frac{5}{8}$
$\frac{5}{9} \times 16 = 8\frac{8}{9}$
$\frac{5}{12} \times 7 = 2\frac{11}{12}$
$\frac{13}{16} \times 13 = 10\frac{9}{16}$

Division.

£	s.	d.		£	s.	d.	
3	8	3	÷ 2 =	1	14	1½	*Ans.*
2	17	3	÷ 2 =	1	8	7½	
1	10	0	÷ 8 =	0	3	9	
1	13	6	÷ 4 =	0	8	4½	
0	11	6	÷ 8 =	0	1	5¼	
0	13	8	÷ 16 =	0	0	10¼	
0	13	6	÷ 18 =	0	0	9	
0	15	9	÷ 18 =	0	0	10½	
0	13	9	÷ 15 =	0	0	11	
0	15	4	÷ 16 =	0	0	11½	
5	0	6	÷ 18 =	0	5	7	
2	10	3¾	÷ 15 =	0	3	4¼	
6	17	8	÷ 56 =	0	2	5½	
8	1	4	÷ 44 =	0	3	8	
126	5	5	÷ 110 =	1	2	11½	
8	6	3	÷ 19 =	0	8	9	
14	14	0	÷ 84 =	0	3	6	
180	9	0	÷ 72 =	2	10	1½	
222	2	1	÷ 42 =	5	5	9	
1	7	6	÷ 11 =	0	2	6	
1	10	10½	÷ 19 =	0	1	7½	
40	11	6	÷ 36 =	1	2	6½	
6	7	6	÷ 45 =	0	2	10	
26	7	1	÷ 55 =	0	9	7	

Division.

$1 \div 2 = \frac{1}{2}$ *Ans.*
$1 \div 3 = \frac{1}{3}$
$1 \div 4 = \frac{1}{4}$
$\frac{1}{2} \div 2 = \frac{1}{4}$
$\frac{1}{2} \div 3 = \frac{1}{6}$
$\frac{2}{3} \div 2 = \frac{1}{3}$
$\frac{2}{3} \div 3 = \frac{2}{9}$
$\frac{2}{3} \div 5 = \frac{2}{15}$
$\frac{2}{3} \div 1 = \frac{2}{3}$
$\frac{3}{7} \div 3 = \frac{1}{7}$
$\frac{3}{7} \div 4 = \frac{3}{28}$
$\frac{8}{9} \div 4 = \frac{2}{9}$
$\frac{8}{9} \div 6 = \frac{4}{27}$
$\frac{9}{16} \times 3 \div 4 = \frac{27}{64}$
$\frac{9}{16} \times 4 \div 5 = \frac{9}{20}$
$\frac{9}{16} \times 5 \div 3 = \frac{15}{16}$
$\frac{9}{16} \times 6 \div 3 = 1\frac{1}{8}$
$\frac{9}{16} \times 12 \div 4 = 1\frac{11}{16}$

Multiplication, with Questions on the Prices of Articles.

										Ans.	
			Ans.						£	s.	d.
13 times	2	are	26 . . .	26	apples	at ¼d.	each	=	0	0	6½
”	5	”	65 . . .	65	”	at ½d.	”	=	0	2	8½
”	8	”	104 . . .	104	”	at ¼d.	”	=	0	2	2
”	11	”	143 . . .	143	”	at ½d.	”	=	0	5	11½
”	14	”	182 . . .	182	”	at ¼d.	”	=	0	3	9½
”	17	”	291 . . .	291	”	at ½d.	”	=	0	12	1½
14 times	14	”	196 . . .	196	”	at ¼d.	”	=	0	4	1
”	15	”	210 . . .	210	”	at ¼d.	”	=	0	4	4½
”	19	”	266 . . .	266	”	at 1d.	”	=	1	2	2
”	18	”	252 . . .	252	”	at ¾d.	”	=	0	15	9
”	17	”	238 . . .	238	”	at ¼d.	”	=	0	4	11½
15 times	13	”	195 . . .	195	”	at 1d.	”	=	0	16	3
”	15	”	225 . . .	225	”	at ¼d.	”	=	0	4	8¼
”	16	”	240 . . .	240	”	at ¾d.	”	=	0	15	0
”	17	”	255 . . .	255	”	at 1d.	”	=	1	1	3
”	18	”	270 . . .	270	”	at ¼d.	”	=	0	5	7½
16 times	13	”	208 . . .	208	”	at 1d.	”	=	0	17	4
”	14	”	224 . . .	224	”	at 2d.	”	=	1	17	4
”	16	”	256 . . .	256	”	at ¼d.	”	=	0	5	4
”	17	”	272 . . .	272	”	at ½d.	”	=	0	11	4
”	19	”	304 . . .	304	”	at 1d.	”	=	1	5	4
17 times	12	”	204 . . .	204	”	at ¾d.	”	=	0	12	9
”	8	”	136 . . .	136	”	at 1½d.	”	=	0	17	0
”	17	”	289 . . .	289	”	at ¼d.	”	=	0	6	0
”	18	”	306 . . .	306	”	at 1½d.	”	=	1	18	3¼
”	19	”	323 . . .	323	”	at ½d.	”	=	0	13	5½
18 times	13	”	234 . . .	234	”	at 1¼d.	”	=	1	4	4½
”	9	”	162 . . .	162	”	at 2½d.	”	=	1	13	9
”	11	”	198 . . .	198	”	at ¼d.	”	=	0	4	1½
”	16	”	288 . . .	288	”	at ¾d.	”	=	0	18	0
”	18	”	324 . . .	324	”	at 1½d.	”	=	2	0	6
19 times	13	”	247 . . .	247	”	at ½d.	”	=	0	10	3½
”	14	”	266 . . .	266	”	at ¼d.	”	=	0	5	6½
”	15	”	285 . . .	285	”	at ½d.	”	=	0	11	10½
”	11	”	209 . . .	209	”	at 1½d.	”	=	1	6	1½
”	18	”	342 . . .	342	”	at 3½d.	”	=	4	19	9
”	19	”	361 . . .	361	”	at ¾d.	”	=	1	2	6¾

Finding the Prices of the same Articles at various Rates.

		Ans.			Ans.				Ans.		
Yds.		s.	d.		£	s.	d.		£	s.	d.
91	at ½d. =	3	9½;	at 1d. =	0	7	7;	at 1s. =	4	11	0
143	at 1d. =	11	11;	at 1s. =	7	3	0;	at £1 =	143	0	0
208	at ¼d. =	4	4;	at 1d. =	0	17	4;	at 1s. =	10	8	0
234	at 1d. =	19	6;	at 1s. =	11	14	0;	at £1 =	234	0	0
196	at ½d. =	8	2;	at 1s. =	9	16	0;	at £1 =	196	0	0
238	at ¼d. =	4	11½;	at 1d. =	0	19	10;	at £1 =	238	0	0
252	at ¼d. =	5	3;	at 1s. =	12	12	0;	at £1 =	252	0	0
225	at ½d. =	9	4½;	at 1s. =	11	5	0;	at £1 =	225	0	0
270	at ¼d. =	5	7½;	at 1s. =	13	10	0;	at £1 =	270	0	0
285	at ½d. =	11	10½;	at 1d. =	1	3	9;	at 1s. =	14	5	0
272	at ¼d. =	5	8;	at 1d. =	1	2	8;	at 1s. =	13	12	0
304	at ¼d. =	6	4;	at 1d. =	1	5	4;	at 1s. =	15	4	0
289	at ½d. =	12	0½;	at 1d. =	1	4	1;	at 1s. =	14	9	0
306	at ¼d. =	6	4½;	at 1d. =	1	5	6;	at 1s. =	15	6	0
323	at ½d. =	13	5½;	at 1d. =	1	6	11;	at 1s. =	16	3	0
288	at ¼d. =	6	0;	at 1d. =	1	4	0;	at 1s. =	14	8	0
324	at ½d. =	13	6;	at 1d. =	1	7	0;	at 1s. =	16	4	0
198	at 1d. =	16	6;	at 1s. =	9	18	0;	at £1 =	9	18	0
247	at ½d. =	10	3½;	at 1d =	1	0	7;	at 1s. =	12	7	0
266	at ¼d. =	5	6½;	at 1d. =	1	2	2;	at 1s. =	13	6	0
342	at ½d. =	14	3;	at 1d. =	1	8	6;	at 1s. =	17	2	0
361	at ½d. =	15	0½;	at 1d. =	1	10	1;	at 1s. =	18	1	0

Reduction.

	Ans.
Reduce 23s. to sixpences.	46
„ 47s. to threepences.	188
„ £1 7s. to fourpences.	81
„ 3s. to farthings.	144
„ 45 sixpences to threepences.	90
„ 13 sixpences to halfpence.	156
„ 10s. to farthings.	480
„ 216 farthings to threepences.	18
„ 11s. 4½d. to farthings.	546
„ 324 sixpences to florins.	81
„ 320 sixpences to half crowns.	64
„ 312 crowns to pounds.	78
„ £1 2s. 9d. to threepences.	91
„ £2 10s. to florins.	25

		Ans.
Reduce	500 farthings to pence	125
„	£1 7s. 6d. to farthings	1320
„	5 cwt. to stones	70
„	2 cwt. 2 qr. 14 lb. to stones	21
„	3½ lb. to ounces troy	42
„	3½ lb. to ounces, avoirdupois weight	56
„	8 miles to perches	2560
„	324 inches to yards	9
„	4 leagues to furlongs	96
„	5½ yards to inches	198
„	400 ounces (avoirdupois) to pounds	25
„	180 quires to reams	9
„	3¾ score to units	75
„	17 quires to sheets	408

To find the price of any number of articles, when the price of one is given.

Rule.—Multiply the price of one article by the number of articles, and the price of the whole is found.*

						Ans.		
						£	s.	d.
Find the price of		45 lb.	of sugar at	3d.	per lb.	0	11	3
„	„	76	„	4d.	„	1	5	4
„	„	93	„	5d.	„	1	18	9
„	„	73	„	7d.	„	2	2	7
„	„	69	„	6d.	„	1	14	6
„	„	87	yards at	½d.	per yd.	0	3	7½
„	„	93	„	¾d.	„	0	5	9¾
„	„	683	„	1d.	„	2	16	11
„	„	845	„	1½d.	„	5	5	7½
„	„	19	„	10½d.	„	0	16	7½
„	„	17	„	1s. 3½d.	„	1	1	11½
„	„	18	„	1s. 5½d.	„	1	6	3
„	„	16	„	1s. 5¾d.	„	1	3	8
„	„	25	„	1s. 6d.	„	1	17	6
„	„	32	„	1s. 8d.	„	2	13	4
„	„	42	„	2s. 8d.	„	5	12	0
„	„	63	„	3s. 9d.	„	11	16	3
„	„	88	„	2s. 10½d.	„	12	13	0

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* By a little exercise of judgment, other processes besides those conveyed by the rule may suggest themselves, by which several of the questions proposed in this and the other sections will be more readily solved.

					£	s.	d.
Find the value of	[illegible]	yards at	2s. 3½d.	per yard	7	11	3 *Ans.*
- -	[illegible]	-	3s. 9d.	,,	0	1	10½
- -	[illegible]	-	5s. 7½d.	,,	0	2	9¾
- -	[illegible]	-	10s. 6d.	,,	0	2	7½
- -	[illegible]	-	12s. 6d.	,,	0	9	4½
- -	[illegible]	-	13s. 8d.	,,	0	5	1½
- -	[illegible]	-	3s. 6d.	,,	0	5	3
- -	[illegible]	-	3s. 9d.	,,	0	5	7½
- -	[illegible]	-	3s. 8d.	,,	0	4	7
- -	[illegible]	-	3s. 6d.	,,	0	4	4½
- -	[illegible]	-	3s. 10d.	,,	0	4	9½
- -	[illegible]	-	5s. 8d.	,,	0	14	2
- -	[illegible]	-	7s. 6d.	,,	0	16	10½
- -	[illegible]	-	6s. 8d.	,,	0	18	4
- -	[illegible]	-	4s. 6d.	,,	0	15	9
- -	[illegible]	-	5s. 6d.	,,	0	17	10½
- -	[illegible]	-	¼d.	,,	0	7	3½
- -	470	-	½d.	,,	0	19	7
- -	570	-	¼d.	,,	0	11	10½
- -	[illegible]	-	½d.	,,	0	15	10
- -	700	-	¾d.	,,	2	3	9
- -	[illegible]	-	1d.	,,	4	1	8
- -	[illegible]	-	¾d.	,,	2	16	3
- -	[illegible]	-	½d.	,,	2	5	10
- -	[illegible]	-	¾d.	,,	3	2	6
- -	[illegible]	-	1¼d.	,,	4	7	6
- -	[illegible]	-	1¼d.	,,	3	10	10
- -	540	-	1½d.	,,	3	7	6
- -	[illegible]	-	⅔d.	,,	1	1	3
- -	[illegible]	-	1¼d.	,,	4	15	10
- -	630	-	1½d.	,,	3	18	9
- -	560	-	2½d.	,,	5	16	8
- -	870	-	3½d.	,,	12	13	9
- -	365	-	1s. 0d.	,,	18	5	0
- -	365	-	1s. 1d.	,,	19	15	5
- -	350	-	5s. 0d.	,,	87	10	0
- -	365	-	6s. 8d.	,,	121	13	4
- -	365	-	4s. 0d.	,,	73	0	0
- -	365	-	1½d.	,,	2	5	7½
- -	860	-	1s. 8d.	,,	71	13	4
- -	950	-	1s. 4d.	,,	63	6	8

Find the	price of	490	yards at	2s. 6d.	per yd.	£61	5	0	*Ans.*
”	”	450	”	1s. 3d.	”	28	2	6	
”	”	18	”	£1 1s. 3½d.	”	19	3	3	
”	”	17	”	£1 5s. 10½d.	”	21	19	10½	
”	”	19	”	£1 13s. 7½d.	”	31	18	10½	
”	”	15	”	15s. 3½d.	”	11	9	4½	
”	”	14	”	£1 16s. 11½d.	”	25	17	5	
”	”	25	”	£1 11s. 4½d.	”	39	4	4½	
”	”	28	”	£1 13s. 5¾d.	”	46	17	5	

When the price of a number of articles is given, to find the price of e.

Rule.—Divide the price by the number of articles, and the price one article is obtained.

What is the cost of 1 lb. (or 1 of any denomination).

							Ans.	
When	5	cost	£0	6	3	£0	1	3
”	3	”	0	1	1½	0	0	4½
”	7	”	0	9	0½	0	1	3½
”	8	”	0	10	6	0	1	3¾
”	9	”	0	13	1½	0	1	5½
”	11	”	0	15	1½	0	1	4½
”	13	”	1	9	3	0	2	3
”	14	”	1	17	11	0	2	8½
”	15	”	1	19	4½	0	2	7½
”	16	”	2	4	8	0	2	9½
”	17	”	2	8	10½	0	2	10½
”	18	”	3	2	7½	0	3	5¾
”	19	”	3	4	1½	0	3	4½
”	27	”	3	1	10½	0	2	3½
”	30	”	6	3	9	0	4	1½
”	40	”	2	15	0	0	1	4½
”	50	”	11	19	7	0	4	9½
”	1½	”	0	5	3	0	3	6
”	1½	”	0	11	4½	0	7	7
”	2½	”	1	4	4½	0	9	9
”	1¼	”	1	0	7½	0	16	6
”	2¼	”	1	11	1½	0	13	10
”	3¾	”	2	3	1½	0	11	6
”	5½	”	0	19	11¼	0	3	7½
”	4¼	”	1	4	9½	0	5	10
”	4¾	”	1	16	0¼	0	7	7
”	5⅔	”	1	12	7	0	5	9

When	2⅜	cost	£1	4	11¼	£0	10	6	*Ans*
„	3⅛	„	4	2	9¾	1	6	6	
„	3⅜	„	0	18	6¾	0	5	6	
„	12½	„	9	16	10½	0	15	9	
„	20	„	133	14	2	6	13	8½	
„	112	„	7	9	4	0	1	7	
„	120	„	273	10	0	2	5	7	
„	140	„	202	14	2	1	8	11½	

When the price of a number of articles is given with the price of to find the number of articles.

Rule.—Divide the price of the number of articles by the pric one (both brought into the same denomination) and the quotient be the number.

What is the number of articles whose cost is

£	s.	d.		s.	d.	*Ans.*
0	6	3	when the cost of 1 is	1	3	5
0	1	1½	„ „	0	2¼	6
0	2	4½	„ „	0	4¾	6
0	4	4½	„ „	0	7½	7
0	9	0½	„ „	1	3½	7
0	11	8	„ „	2	11	4
0	17	10½	„ „	1	7½	11
1	13	0	„ „	3	8	9
1	17	4	„ „	1	4	28
2	6	8	„ „	0	5	112
7	18	8	„ „	1	5	112
1	0	10	„ „	0	0¼	1000
1	11	8	„ „	0	0¾	500
0	17	6	„ „	0	0¾	280
1	17	6	„ „	0	0½	900
2	12	6	„ „	0	0¾	840
1	14	1½	„ „	3	9½	9
4	13	4	„ „	0	0½	2240
7	0	0	„ „	0	0¾	2240
3	14	8	„ „	0	8	112
1	4	9	„ „	5	6	4½
0	1	3¾	„ „	0	3½	4½
0	5	9	„ „	11	6	½
0	5	1½	„ „	6	10	¾
0	16	4	„ „	4	8	3½
2	4	4½	„ „	11	10	3¾

INTEREST, COMMISSION AND COMMERCIAL DISCOUNT.

150. *To find the interest for* 1 *year, discount or commission, at* 5 *per cent.*

Take $\frac{1}{20}$ (since 5 is $\frac{1}{20}$ of 100), or reckon a shilling for every pound in the sum given.

At 4 *per cent.*—Find at 5 per cent., and deduct $\frac{1}{5}$th, because $5 - \frac{1}{5}$ of $5 = 4$.

At 6 *per cent.*—Find at 5 per cent. and add $\frac{1}{5}$th, because $5 + \frac{1}{5}$ of $5 = 6$.

At 2½ *per cent.*—Take $\frac{1}{40}$ (since 2½ is $\frac{1}{40}$ of 100), or reckon 6d. for every pound in the sum given.

Find the interest (for 1 year), commission or discount,

	£	s.	d.		£	s.	d.	
On	18	10	0	at 5 per cent.	0	18	6	*Ans.*
,,	37	12	6	,,	1	17	7½	
,,	45	17	6	,,	2	5	10½	
,,	150	7	6	,,	7	10	4½	
,,	189	3	4	,,	9	9	2	
,,	635	6	8	,,	31	5	4	
,,	1087	16	8	,,	54	7	9¾	
,,	845	8	9	,,	42	5	5¼	
,,	6387	18	9	,,	319	7	11¼	
,,	377	10	0	at 4 per cent.	15	2	0	
,,	378	6	8	,,	15	2	8	
,,	2175	12	6	,,	87	0	6	
,,	194	16	10½	,,	7	15	10½	
,,	685	12	6	,,	27	8	6	
,,	6387	18	4	at 6 per cent.	383	5	6	
,,	887	10	0	,,	53	5	0	
,,	1469	7	6	,,	88	3	3	
,,	467	7	11	,,	28	0	10½	
,,	902	1	8	,,	54	2	6	
,,	879	11	8	,,	52	15	6	
,,	53	10	0	at 2½ per cent.	1	6	9	
,,	48	15	0	,,	1	4	4½	
,,	168	10	0	,,	4	4	3	
,,	795	5	0	,,	19	17	7½	
,,	683	12	6	,,	17	1	9¾	
,,	864	17	6	,,	21	12	5¼	
,,	739	10	0	,,	18	9	9	
,,	965	7	6	,,	24	2	8¼	
,,	2875	17	6	,,	71	17	11¼	

		£	s.	d.				Amt. £	s.	d.
Find the amount of		29	10	0	for 2 years,	at 5 per cent.		2	19	0
-	-	87	15	0	for 3	,,	,,	13	3	3
-	-	488	10	0	for 3	,,	,,	73	5	6
-	-	693	15	0	for 4	,,	,,	138	15	0
-	-	287	12	6	for 5	,,	,,	71	18	1½
-	-	365	17	6	for 2	,,	,,	36	11	9
-	-	1187	10	0	for ½	,,	,,	29	13	9
-	-	1263	15	0	for ½	,,	,,	31	11	10½
-	-	947	10	0	for 4 months,		,,	15	15	10
-	-	638	15	0	for 4	,,	,,	10	12	11
-	-	869	12	6	for 4	,,	,,	14	9	10½
-	-	987	15	0	for 3	,,	,,	12	6	11¼
-	-	3875	10	0	for 3	,,	,,	48	8	10½
-	-	768	15	0	for 2	,,	,,	6	8	1½
-	-	633	10	0	for 1	,,	,,	2	12	9½
-	-	180	0	0	for 6 months,	at 6 per cent.		5	8	0
-	-	187	10	0	for 6	,,	,,	5	12	6
-	-	389	11	8	for 6	,,	,,	11	13	9
-	-	350	16	8	for 3	,,	,,	5	5	3
-	-	893	19	2	for 4	,,	,,	17	17	7
-	-	953	8	9	for 4 months,	at 4 per cent.		12	14	3
-	-	678	15	0	for 6	,,	,,	13	11	6
-	-	853	2	6	for 4	,,	,,	11	7	6
-	-	870	0	0	for 6 months,	at 2½ per cent.		10	17	6
-	-	784	10	0	for 4	,,	,,	6	10	9
-	-	683	10	0	for 4	,,	,,	5	13	11
-	-	675	0	0	for 3	,,	,,	4	4	4½

DECIMAL COINAGE.

172. The propriety of establishing a system of decimal coinage in this country, as in France and the United States of America, has of late years been a subject of considerable discussion.

In April 1853 a Committee of the House of Commons was appointed to consider the subject, and on the 1st of August following, this committee presented its report, which recommended:—

That the pound sterling should be the unit coin of account, and that it should be divided into 1,000 parts.

That the tenth, the hundredth, and thousandth part of the pound should be the other coins of account, to be called the *florin*, *cent*, and *mil*.

The table would stand thus—10 mils = 1 cent.
10 cents = 1 florin.
10 florins = £1.

The sovereign then = 1000 Mils.
„ half-sovereign = 500 „
„ 5 shillings = 250 „
„ 2s. 6d. = 125 „
„ 2s. = 100 „
„ 1s. = 50 „
„ 6d. = 25 „

152. With the exception of the silver threepenny and fourpenny pieces, all the present gold and silver coins in circulation can be expressed in mils, and therefore retained in use; but a new issue of copper coinage would be requisite, as the present farthing is larger than the mil by 4 per cent.

In such a system, the terms of value in money of account will follow in succession.

Thus, 4 = 4 mils.
54 = 5 cents. 4 mils.
654 = 6 florins 5 cents. 4 mils.
3654 = £3 6 florins 5 cents. 4 mils.
23654 = £23 6 florins 5 cents. 4 mils.

From this it is seen how completely reduction of money would be obviated by this system, since the unit's place is *mils*, the ten's *cents*, the hundreth's *florins* and the thousanth's *pounds*. 3654 mils may also be written as 36 florins 5 cents 4 mils, or 365 cents 4 mils, or £3.654.

153. *To shew the identity of simple and compound arithmetical processes in the decimal or* ten *system.*

Multiply 1635 by 5, and find the price of 5 cwt. of sugar, at £1 6 fl. 3 c. 5 mils per cwt.

	£	fl.	c.	m.		mils.			£	fl.	c.	m.		mils.
1635	1	6	3	5	=	1635	=	£1·635	1	6	3	5	=	1635
5				5		5		5	1	6	3	5	=	1635
——	——	——	——	——		——		——	1	6	3	5	=	1635
8175	£8	1	7	5		8175 mils.		£8·175	1	6	3	6	=	1635
									1	6	3	5	=	1635
									——	——	——	——		——
									£8	1	7	5	=	8175

154. If the decimal system were also applied in the gradations of

weights* and measures, then all operations would be performed as in the four simple rules, and the rules of compound addition, subtraction, multiplication, division, together with reduction, practice, duodecimals, and the rules of mental arithmetic would all be rendered unnecessary, besides much of the labour throughout the other rules being lessened.

EXAMPLE.

Express £6 18s. 9d. in terms of the decimal coinage.

£	s.	d.		£	fl.	c.	m.
6	0	0	=	6	0	0	0
0	18	0	=	0	9	0	0
0	0	6	=	0	0	2	5
0	0	3	=	0	0	1	2·5
£6	18	9	=	£6	9	3	7·5

EXERCISES 54.

1. Express £24 7s. 6d. in terms of decimal coinage.
2. ,, £18 16s. 9d. ,, ,,
3. ,, £98 17s. 10½d. ,, ,,

EXAMPLE.

Express £5 7 fl. 2 c. 5 m. in £. s. d.

£	fl.	c.	m.		£	s.	d.
5	0	0	0	=	5	0	0
0	7	0	0	=	0	14	0
0	0	2	5	=	0	0	6
£5	7	2	5	=	£5	14	6

4. Express £15 4 fl. 5 c. in £ s. d.
5. ,, £23 8 fl. 7 c. 5 m. in £ s. d.
6. ,, £11 9 fl. 3 c. 5 m. in £ s. d.

MISCELLANEOUS QUESTIONS.

EXERCISES 55.

1. What is the income tax on the rent of 95 a. 2 r. 27 p. of land, at £3 13s. 10½d. per acre, at 14d. in the £.?

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* For a comprehensive notion of the application of the decimal system to time, weights and measures, see a pamphlet on that subject, by Dover Statter, Esq., Liverpool.

2. What is the income tax, at 5d. in the £, on the rent of 310·216 acres of land, when ·25 of 45 acres are let for ·432 of £375?
3. If 3·5 pieces of road are each 18 m. 3 f. 36 p. in length, when measured by a chain 9 in. short, what is the expense of keeping them in repair at £1 11s. 8d. the true mile?
4. Find the purchase money of $13\frac{7}{8}$ plots of land, each containing 17 a. 2 r. 35 sq. p. 7 sq. ft. 126 sq. in., at £16 13s. 4d. per acre.
5. Sold $179\frac{3}{4}$ gallons of gin at 11s. $8\frac{1}{2}$d. per gallon, and $69\frac{11}{16}$ gallons at 11s. $5\frac{1}{2}$ per gallon; what is the average selling price of one gallon?
6. Bought $85\frac{3}{4}$ gallons of rum at 13s. $10\frac{1}{2}$d. per gal., and $78\frac{2}{3}$ at 12s. 8d. per gal.; what is the average cost of 1 gal.?
7. What income tax, at 16d. in the £, should be paid on the rental of 143 a. 2 r. $32\frac{1}{2}$ p. of land, at £4 12s. 6d. per acre?
8. How many pieces of gold of 2 lb. 7 oz. 17 dwt. 18 gr. each, can be bought for £3809 18s. $2\frac{31}{640}$d., when £47 18s. 6d. will purchase 1 lb. of the same?
9. If the weight of $26\frac{13}{16}$ packages of soda be 93 lb. 9 oz. 7 dr. 1 sc. $19\frac{2}{15}$ gr., what is the weight of 1 package?
10. Bought $9\frac{3}{8}$ barrels of pork, each weighing 9 cwt. 2 qr. 21 lb., which is found to be short weight by 4 lb. in every cwt.; what is the price at £2 16s. 6d. per true cwt.?
11. Bought $3\frac{1}{4}$ pieces of gold, each weighing 5 lb. 9 oz. 18 dwt., at £3 17s. $10\frac{1}{2}$d. per ounce; but being afterwards found to be short weight, (every lb. weighing but $10\frac{1}{2}$ oz.) what was the real cost?
12. How many loads of wheat, each containing 15 qr. 3 bush. 2 pk., at £2 18s. 6d. per quarter, can be bought for £849 9s. $5\frac{53}{128}$d.?
13. Purchased $2\frac{1\frac{1}{2}}{2}$ tierces of beef, each weighing $2\frac{2}{3}$ cwt. + $7\frac{5}{8}$ qr. + $18\frac{3}{8}$ lb., at $3\frac{1}{2}$ times $5\frac{2}{2\frac{1}{3}}$ lb. for 33s. 3d., and sold it to gain 5 per cent; what is the cost and gain?
14. How many parcels of pure nitre, each weighing 85 lb. 7 oz. 5 dr. 2 sc. 18 gr., can be bought for £24 19s. $7\frac{63}{256}$d. at 1s. 3d. per lb.?
15. Sold 87 a. 3 r. 35 p. of land at £7 10s. per acre on the 5th of August, 1859, the sum to be paid with interest on the 1st of January following; what sum was received?
16. What will ($3\frac{5}{8} + 7\frac{5}{12} + 8\frac{13}{16} - 3\frac{11}{24}$) yards come to, if ($1\frac{1}{2} + 2\frac{2}{3} + 1\frac{5}{6} - 2\frac{5}{6}$) yards cost ($\frac{2}{3} + \frac{3}{4} + 2\frac{3}{8} - 1\frac{7}{12}$) guineas?

17. Sold $2\frac{1}{2}$ bags of wheat, each 63 qr. 5 bush. 2 pk., at £2 18s. 6d. per quarter, and sold it to clear 25 guineas; what was it sold at the quarter?

18. What expense is incurred in $17\frac{2}{3}$ periods of time, each period consisting of $5\frac{2}{3}$ days, $3\frac{4}{5}$ hours, $17\frac{7}{8}$ minutes, at 1s. $1\frac{1}{2}$d. per hour?

19. Out of a purse containing $57\frac{4}{5}$ guineas + £$13\frac{4}{7}$ + $16\frac{7}{9}$ florins + $13\frac{7}{8}$ crowns, I pay $18\frac{5}{8}$ guineas + £$13\frac{1}{5}$ + $17\frac{4}{5}$ florins, and with the difference I purchased tea at 3s. 8d. per lb.; how many lb. of tea did I buy?

20. Bought 73 cwt. 2 qr. 21 lb. of tobacco at £7 13s. 6d. per cwt., and sold 23 cwt. 1 qr. 18 lb. at 1s. $7\frac{1}{2}$d. per lb.; how must the remainder be sold per cwt. to clear 15 per cent. on the whole?

21. Bought $79\frac{17}{196}$ barrels of flour at £1 12s. 8d. per barrel, and $91\frac{81}{196}$ barrels at £1 15s. per barrel; at what average must the flour be sold per barrel, to clear 10 per cent. on the whole?

22. What cost $13\frac{2}{5}$ pieces of cloth, each containing $7\frac{8}{15} + 16\frac{3}{4}$ yards, when $4\frac{2}{3} - 1\frac{3}{4}$ yards cost $\frac{3}{5}$ of 17s. 6d.?

23. Simplify the expression $\frac{3}{8}$ of £$1\frac{3}{5} + \dfrac{2\frac{1}{2} \times \cdot075}{\frac{2}{3} \text{ of } \cdot875}$ of £$2 + \dfrac{16\cdot2}{\cdot27}$ s.

24*. The total quantity of water used in the town of Liverpool during the week ending the 23rd of July 1859, was 96054854 gallons; how many tons of oxygen and of hydrogen did it contain, when 9 lb. of water will yield 8 lb. of oxygen and 1lb of hydrogen?

25. The expense incurred in making a railway amounted to £1,540,000; what must be the yearly receipts to pay expenses amounting to 30 per cent. of these receipts, and yield the shareholders $5\frac{1}{2}$ per cent.?

26. Find the several sums in the 3, $3\frac{1}{4}$, and $3\frac{1}{2}$ per cent. stocks, that will produce 5 per cent. on £100 purchased in them respectively.

27. Multiply the difference between $\frac{3}{5}$ of $4\frac{2}{3} \times 2\frac{2}{9}$ and $1\frac{1}{3}$ of $2\frac{1}{2}$ of $3\frac{1}{3}$, by $3\frac{1}{3} \times \dfrac{2\frac{1}{2}}{3\frac{1}{2}} \times \dfrac{1}{1\frac{1}{3}}$

28. If A. can do a piece of work in 3 days, B. in 4 days, and C. in 6 days, in what time will they do it when working together?

---

* Of this supply {69544248 gallons were from Rivington Pike.
{26510606 „ from the Wells.

Total 96054854 gallons.

How many days (supposing it practicable) would it take a man to walk from Liverpool, which is in latitude 53° 25′, to the equator, at the rate of 30 miles a day, a degree being $69\frac{1}{10}$ English miles?

What is the square root of $\frac{1687}{2187}$?

In England and Wales there are 8000 miles of railway, whose making cost, at an average, £38779 per mile; what must be the annual receipts to pay expenses amounting to 35 per cent. of the gross receipts, and to pay the shareholders 4 per cent.?

What is the cost of $5\frac{3}{4}$ packages of saltpetre, each weighing 6 cwt. 2 qr. 12 lb.; draft 14 lb. per cwt.; tare 4 lb. per 104 lb., and tret 2 lb. per 3 cwt; at £5 12s. 8d. per cwt. net?

The Italian war of 1859 cost Austria 200,000,000 florins; what is the interest of this sum in English money, at $3\frac{1}{2}$ per cent., if 9 florins, 50 kreutzers equal £1 sterling?

The amount assessed for income tax on land, houses, &c., was, in 1850, £94,217,995, and in 1858, £109,978,265; what time would this octennial increase of property take to pay off the National Debt of £804,445,483?

Divide 671787 a. 3 r. 30 sq. p. $7\frac{1}{2}$ sq. yd. by 4, 3, 5, 6, 7, 9, 10, successively.

Find the cost of 18 hogsheads of tallow, each weighing 5 cwt. 1 qr. 17 lb.; draft $10\frac{1}{2}$ lb. per cwt.; tare 2 lb. per 3 cwt.; at £1 17s. 8d. per cwt. net.

The total quantity of gold exported from the gold-fields of Victoria, up to the 1st of January, 1859 (a period of eight years), was 17609742 ounces; what is its value at £3 17s. $10\frac{1}{2}$d. per ounce?

The number of persons having deposits in savings' banks were, in 1860, 1383358, and the amount due to them £33,927,080; what is the average sum due to each, and the annual interest on the entire at $3\frac{1}{3}$ per cent.?

Suppose a boat launched into a pond of water to cause a rise of 1250 cubic feet; what is the weight of the boat?

The driving-wheel of a locomotive engine is 7 ft. in height, or 22 ft. in circumference; how many revolutions will it make in a second, when the engine travels at the rate of 60 miles an hour?

Both hands of a watch are together between 4 and 5 o'clock in the evening; what was the exact time then?

Reduce $\frac{(2{\cdot}05)^2 \times 2{\cdot}24}{{\cdot}0041}$ to a simple form.

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]
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Exports.

		1853.	1857.
6.	Cotton Manufacture .	£25,825,249	£30,782,831
	„ Twist and Yarn .	6,895,653	8,700,580
	Iron-bar and Rails . .	5,555,711	5,962,780
	Hardware and Cutlery .	8,665,051	4,016,230
	Haberdashery and Millinery	4,156,494	3,893,613
	Linen Manufactures . .	4,758,442	4,516,876
	Woollen Manufactures .	10,172,182	10,703,376
	Coals	1,507,950	3,019,228
	Woollen Yarn . . .	1,456,786	2,961,800
	Machinery	1,985,536	8,883,669
	Iron (Sheets and Plates) .	1,820,851	2,616,402
	Leather Manufactures .	1,082,966	1,666,894
	Linen Yarn	1,154,977	1,667,953
	Apparel and Slops . .	1,898,466	1,670,766
	Iron (Pig)	1,056,310	1,609,115
	Beer and Ale . . .	1,291,357	1,592,267
	Tin Plates	1,181,069	1,500,116
	Earthenware and Porcelain	1,338,370	1,692,236
	Wool	423,860	1,089,499

7. Value of the imports of the United Kingdom for 1858.

From British Possesions	£38,875,610
„ United States	34,281,255
„ France	13,911,401
„ Russia	11,746,003
„ South America	11,274,242
„ Germany	8,641,455
„ All other Countries . . .	45,505,887

Divide ·015 by ·0005, and 313·24 by 24997·5.

How many yards of paper ⅞ yard wide, will cover a room 40¾ ft. long, 32⅔ ft. broad, and 18½ ft. high; and what is the cost at 2s. 10½d. per dozen yards?

Reduce 5s. 4½d., 3s. 6d., 8s. 4½d., 12s. 8¼d., 18s. 11¾d., each to the decimal of a £.

EXERCISES 56.

At what rate per cent. will £651 13s. 4d. amount to £748 12s. 0½d. in 8½ years?

What is the diameter of the earth (supposing it a perfect globe) when the circumference is 24899 miles?

3. In a circle, whose radius is 85 yards, what is the length of an arc of 60°?
4. How many yards of paper ⅝ yard wide, will cover a room 39⅔ yards long, 19⅜ yards wide, and 22⅝ feet high; and what is the cost, at 2s. 7½d. per dozen yards?
5. How many revolutions must a locomotive engine's driving-wheel 8 ft. high, make in a minute to go 30 miles an hour?
6. Find the price of (47·388 + 81·4632 + 9·75 yards) ÷ ·04 at the rate of ·45 of 3.8 yards, for ·088 of £3·625.
7. How many yards of paper, 2½ ft. wide, will cover a room 35 ft. 6 in. long, 20 ft. 8 in. wide, and 16 ft. high, containing also two fire-places, each 7 ft. 8 in. by 4 ft. 6 in., five windows, each 5 ft. 8 in. by 3 ft. 9 in., and a door 6 ft. 6 in. by 3 ft. 9 in.?
8. What must be the diameter of a wheel to turn 1000 times in going over a mile?
9. A person who has £2854 in the 3 per cents. at 80⅛, sells out, and with the money purchases in the 4 per cents. at 98; what difference does it make in his income?
10. The base of the largest Egyptian pyramid is a square, the side of which is 693 ft.; required the number of acres it covers.
11. The length of a street is 468 ft. 9 in., and its breadth 33 ft. 4 in.; required the cost of paving the same at 8½d. per square yard.
12. Required the length of a cord to tie a horse so that he may graze only on 2 acres of ground.
13. Sold £3875 in the 3 per cents. at 89⅛, and with the same money purchased in the 3½ per cents. at 98⅜; what is the difference of interest?
14. Reduce $\frac{88134765625}{1000000000000}$ to its lowest terms.
15. Mr. Robert Watt bought of Thomas Tait,

550	yards	of cloth, at	2s.	8¾d.	per yard,
654	,,	,,	2s.	8⅓d.	,,
780	,,	,,	13s.	4½d.	,,
660	,,	,,	7s.	8¾d.	,,
950	,,	,,	7s.	8⅙d.	,,
460	,,	,,	13s.	4¼d.	,,
468	,,	,,	7s.	8½d.	,,
860	,,	,,	3s.	11¼d.	,,

Find the average price of 1 yard.

16. What is the weight of a cast-iron bomb-shell, whose inside diameter is 9 inches; the thickness of the metal being 1½ in., and the specific gravity, 7425 ounces?

17. A rectangular bowling-green 500 feet long and 300 feet broad, is to be raised 1 foot higher, by earth dug from a surrounding ditch, 9 feet broad; to what depth must the ditch be cut?

18. By selling cloth at 15s. 6d. per yard, I gained 5 per cent.; what would I gain or lose per cent. by selling the same at 15s., with 6 months' credit?

19. What is the specific gravity of a cubic foot of timber which sinks 6 inches in water, when a cubic foot of water weighs 1000 ounces?

20. The pressure of the atmosphere is fifteen pounds on every square inch of the earth's surface, and in every 100 lb. of air there are 22 lb. of oxygen to 78 lb. of nitrogen; what is the weight of each in the entire atmosphere if the earth's diameter be 7960 miles?

21. A kiln, of cylindrical shape, whose height is 23 feet, and diameter 18 feet, is filled with limestone, whose specific gravity is 2950, and which is converted into lime by fire. Required the weight of the carbonic acid expelled, when 50 lb. of limestone (carbonate of lime) yields 28 lb. of lime, and 22 lb. of carbonic acid?

22. Find the value of $\frac{5\frac{4}{5} - 2\frac{1}{8}}{3\frac{3}{4} + \frac{9}{20}}$ of $\frac{4\frac{1}{4} + 5\frac{19}{25}}{4\frac{1}{20}}$ of $\frac{2\frac{3}{5} + 1\frac{2}{3}}{7\frac{19}{24} - 2\frac{1}{4}}$ cwt. of pork at £$5\frac{3}{8}$ for $1\frac{2}{3}$ cwt.

23. Divide 14924 a. 0 r. 13 s. p. $29\frac{3}{4}$ sq. yds. 8 sq. ft. by 2, 3, 5, 6, 7, and 4, successively?

24. What is the diameter of a carriage-wheel, which turns 1640 times in a distance of $4\frac{1}{2}$ miles?

25. What quantity of oxygen, and of hydrogen, is there in the water of the earth, supposing water to cover $\frac{3}{5}$ of the earth's surface, and its average depth to be half a mile; the earth's diameter being taken at 7920 miles, and 9 lb. of water yielding 8 lb. of oxygen and 1 lb. of hydrogen?

26. Shipped to Bombay, 150 pieces of linen, each $25\frac{1}{2}$ yards, which is sold at 1 rupee 12 annas per yard; what do I gain when the prime cost of the linen is 2s. $7\frac{1}{2}$d. per yard; freight being charged at $3\frac{1}{2}$d. per piece; commission on the sale at 5 per cent., and the exchange being 2s. $1\frac{1}{2}$d. per rupee?

27. When the income-tax was 16d. in the £1, what was that person's income who had to pay £53 17s. 3d. income-tax?

28. What must be the difference of income in the transfer of £4500 stock from the $3\frac{1}{4}$ per cent. at $84\frac{1}{8}$ to the 4 per cent. at $95\frac{7}{8}$?

29. Suppose a railway, 126 feet broad, be constructed from the pole to the equator, how many acres will it take, and what is its cost at 3s. 9d. per square yard—a degree containing $69\frac{1}{10}$ miles?

30. What income tax at 16d. in the £ did a person pay on 69 a. 3 r. 27 p. of land, valued at £3 13s. 9d. per acre, and on the interest of £2835 13s. 9d. in the 3½ per cent. funds?

31. What would be the difference of income in transferring £2500 from the 3 per cent. at 92⅛, to the 4 per cent., at 109⅞?

32. At what rate must 3 per cent. consols be purchased so that after paying ⅛ for brokerage, and income tax at 10d. in the £1, the purchaser may realise 3 per cent.?

33. Purchased an estate for £4619 10s. 7½d. yielding a rental of £154 13s. 7½d.; how many years purchase is it, and what rate per cent. does the purchase-money realise?

34. A gentleman in England has an income from France of 6356 francs 35 centimes, exchanged at 25f. 40 cents. for £1 sterling; and from Portugal 1697 milreas 250 reas, at 5s. 4d. per milrea; what is the income tax upon the whole sum at 16d. in the £?

35. Convert [illegible] into a decimal.

36. Find a 4th proportional to ·45, ·8, and ·367.

37. How many hours a day must 171 men work to do in 91 days what it takes 143 men to do in 133 days of 9 hours each?

38. What is the present value of £765 15s. 4·545d. due 5 years hence, at 3 per cent. compound interest.

39. What degree of temperature will 48° in Fahrenheit's be in the centigrade?

40. A owes B £900, to be paid as follows:—£400 at the end of the first year, £300 at the end of the second, and £200 at the end of the third year; what is the true discount that ought to be allowed to B. for discharging the debt at once?

41. What principal put to interest for six years, will amount to £[illegible] [illegible]s. 4¾·363d., at 5 per cent. compound interest?

42. Bought 36 pieces of cloth at £30 12s. per piece, and sold 15 pieces at £35 10s. per piece; at what price must the remainder be sold to clear 24 per cent. on the whole?

43. What is the weight of air in a room whose dimensions are 17 ft. 6 in. long, 14 ft. 8 in. broad, and 10 ft. 4 in. high, if a cubic foot of air weigh 1·22 ounces?

44. What principal put to interest for 9 years at 5 per cent. compound interest, will realise £1241 1s. 8·017467875d.?

45. A sugar wareroom, 45 ft. 8 in. long and 35 ft. 6 in. broad, is filled with grape sugar to the height of 21 ft. Now, if a cubic foot of sugar weigh 8 lb., and a mass of 15 lb. yields 8 lb. of oxygen, 1 lb. of hydrogen, and 6 lb. of carbon; how many lbs. of oxygen, hydrogen, and carbon, respectively, will the entire mass contain?

46. Find the value of 3·25 — 2·765 + 3·125 × 8 — ·607095 ÷ ·027.

47. Add 8s. 6d. + 13s. 9d. + ·875s. + 10s. 6d. + ·564d., and at this amount for one cwt. of cheese, find the value of 973·85 cwt.

48. Bought $21\frac{3}{4}$ packs of cloth, each containing $23\frac{3}{8}$ pieces of $35\frac{5}{8}$ yards each, at 8s. 9d. per yard, and paid for it in flour at £1 16s. 6d. per barrel; how many barrels were given?

49. Bought $16\frac{3}{4}$ bags of wheat, each containing 9 qr. 5 bushels 2 p., at £3 5s. 4d. per quarter, and sold 65 qr. 3 bus. 2 p. at £2 18s. 4d. per qr.; how must the remainder be sold per quarter to clear £8 on the whole?

50. Find a mean proportional between ·004 and 72250.

EXERCISES 57.

1. Sold sugar at $5\frac{1}{2}$d. per lb., and lost by the sale $\frac{7}{8}$ per cent.; what is the gain or loss per cent. when the same is sold for $6\frac{1}{4}$d. per lb.?

2. Find a fourth proportional to the quantities 1·3, ·0104, and 2·375.

3. What will be the price of two packs of wool, the first weighing $94\frac{5}{16}$ stones, and the second, $305\frac{7}{8}$ stones; but $4\frac{2}{3}$ stones of No. 2, are only worth $2\frac{1}{4}$ stones of the first, and the best quality is sold at 10s. $6\frac{2}{3}$d. per stone?

4. What sum must be put to interest for 1 year and 292 days at 5 per cent to realise £65 12s. 6d. interest?

5. What is the rent of a field 18·75 chains long, and 14·45 chains broad, at £3 11s. 6d. per acre?

6. Add together 289·7587 ÷ ·6842, ·000576 ÷ 24, ·08250704 ÷ ·8256, and ·0178986144 ÷ ·00464.

7. If ·0041 lb. of soap cost ·375d., what cost ·01581247 cwt.?

8. Bought 37 yards 3 qr. 2 nails of cloth at 6s. 3d. per yard, and 55 yds. 2 qr. 2 nl., at 6s. $10\frac{1}{2}$d. per yard; what is the average price of a yard?

9. Find the price of ·04 of a plot of land containing 478·63 + 89·005 + ·00125 + 5437·25 acres, at 1s. $10\frac{1}{2}$d. for every 5 square yards.

10. Required the length of an arc of 17° 50′, the circle having a radius of 356 miles.

11. How many yards of paper $\frac{7}{8}$ yards wide will cover a room $40\frac{3}{4}$ feet long, $32\frac{2}{3}$ feet wide, and $18\frac{1}{2}$ feet high, and what is its cost at 2s. $10\frac{1}{2}$d. per dozen yards?

12. At what rate per cent., simple interest, will £651 13s. 4d. amount to £748 12s. $0\frac{1}{2}$d. in $3\frac{1}{2}$ years?

13. If the pressure of the atmosphere be 14 lb 10 oz. on every square inch, what is its pressure on an acre of land?

14. Reduce $\frac{3}{5}$ of 2s. $4\frac{1}{2}$d. to the fraction of half-a-crown.

15. Reduce $\frac{4}{5}$ of 1s. $7\frac{1}{2}$d. to the fraction of a florin.

16. Find the amount of $\cdot 3\dot{2}\dot{7}$ of 16s. 6d. + $\cdot 8\dot{3}$ of a guinea + $5\cdot\dot{1}6\dot{4}$ of 2s. + $\cdot 36\dot{3}4\dot{2}$ of a crown.

17. A widow lady has £2854 in the 3 per cents. at $80\frac{1}{8}$. and transfers it into the 4 per cents. at 98; what difference does it make in her income?

18. Find the number of postage stamps, each $\frac{4}{5}$ by $\frac{3}{5}$ inches, that will cover a circle whose circumference is 45 yards.

19. Find the value of $\left\{\frac{11\frac{3}{4} - 10\frac{1}{2}}{11\frac{3}{4} + 10\frac{1}{2}} \div \frac{10\frac{5}{8} + 11\frac{1}{6}}{10\frac{5}{8} - 9\frac{1}{6}}\right\} \times \frac{\frac{3}{7} + \frac{2}{11}}{\frac{3}{7} - \frac{2}{11}}$.

20. A cistern 3 ft. 3 in. deep, 4 ft. 2 in. long, and 2 ft. 8 in. wide is to be lined with lead; if 9 lbs. are taken to the square foot, what will be the expense at £2 9s. per cwt.?

21. A railway 84 feet broad contains 2138 a. 0 r. 29 sq. p. $2\frac{3}{4}$ sq. yds; what is its length?

22. To what depth is a tank, 1780 ft. long, 1645 ft. broad, reduced every week by supplying the town of Liverpool with 95 million gallons of water weekly; and what is the cost per gal. when the annual water rate amounts to £84,000, and the year consists of 52 weeks?

23. How many days would it take a person to walk round the earth (supposing it possible), if he walked $3\frac{1}{2}$ miles per hour, and travelled 10 hours a-day, a degree consisting of $69\frac{1}{10}$ English miles?

24. If £3570 in the 3 per cents. at $91\frac{1}{8}$, be transferred to the $3\frac{1}{2}$ per cents. at $94\frac{3}{8}$, what is the advantage?

25. Find the price of 3·5 pieces of cloth, each measuring (3·5 + 47·25 + 2·0073 + 927·01 + 1·5) yards, at ·09 times 7 yards for ·25 of 11s. $4\frac{1}{2}$d.

26. Find the purchase money of $13\frac{3}{4}$ plots of land, each containing 17 a. 2 r. 35 sq. p. 30 sq. yds. 7 sq. ft. 126 sq. in., at £16 13s. 4d. per acre.

27. What is the product of the sum, difference and quotient, of $6\frac{3}{8}$ and $3\frac{2}{5}$?

28. If an iron bar, 6 feet long, 5 in. broad, and $1\frac{1}{4}$ in. thick, weigh 180 lb., what will be the weight of a bar 7 feet long, $1\frac{1}{2}$ in. broad, and $2\frac{1}{4}$ in. thick?

29. Convert 74 g. 30′ 14″ French measure into degrees English?*

30. Convert 54° 35′ 6″ into French measure?

31. Suppose a mass of pure gold, one of pure silver, and one of gold and silver mixed, each mass weighing 9 ounces, be immersed in water, and the gold is found to displace ·45 oz., the silver, ·8 oz., and the mixture of gold and silver ·6 oz. of water; how much of the compound is gold, and how much silver?

32. Find a mean proportional between ·0375 and ·384.

33. In 1851 the population of Ireland was 6552385, and in 1861, 5764543; what was the decrease per cent. in the interval?

34. The population of Ireland in 1841 was 8175124, and in 1861, 5764543; what was the decrease per cent. in these 20 years?

35. A railway company declares a half-yearly dividend of 12s. per share of £30; what is that per cent. per annum?

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* Laplace, with some other French mathematicians, divided the quadrant of a circle into 100 equal parts called *grades*, each grade into 100 *minutes*, and each minute into 100 *seconds*, which were denoted by the marks g. ', ''. The grade division is now abandoned for the sexagesimal, because the latter can be divided into a greater number of aliquot parts.

APPENDIX.

DIFFERENT SCALES OF NOTATION.

155. Numbers increase in a ten-fold ratio, because there are *nine digits* used in the system. Had there been but six digits instead of nine, then numbers would increase in a seven-fold ratio, and seven would be written 10; eight would be written 11; nine would be written 12; and fourteen would be written 20; &c. And 5364 in the septenary scale $= 5 \times 7^3 + 3 \times 7^2 + 6 \times 7 + 4 = 1908$ in the ordinary scale.

*To transform any number from the decimal scale into any other scale.**

Rule.—Divide the given number by the radix of the scale to which it is to be reduced; divide the quotient again by this radix; deal with the next quotient in a similar way, and thus proceed until the quotient is 0; the last remainder, followed by the other remainders in succession, will be the digits of the required scale, the first one taking the unit's place.

EXAMPLE.

Transform 2305 into the septenary scale.

$$\begin{array}{r l}
7\,)\,\underline{2305} & \\
7\,)\,\underline{329} & \ldots 2 \\
7\,)\,\underline{47} & \ldots 0 \\
\underline{6} & \ldots 5 \\
0 & \ldots 6
\end{array}$$

Whence 6502 in the scale 7 = 2305.

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* Since 5364 in the septenary scale $= 5 \times 7^3 + 3 \times 7^2 + 6 \times 7 + 4.$

$\frac{7}{38} = 5 \times 7 + 3.$

$\frac{7}{272} = 5 \times 7^2 + 3 \times 7 + 6.$

$\frac{7}{1908} = 5 \times 7^3 + 3 \times 7^2 + 6 \times 7 + 4.$

In producing the result 1908, the digits 3, 6 and 4 were respectively added to

EXERCISES.

1. Transform 347 in the decimal scale to the octenary scale.
2. " 6873 " to the nonary scale.
3. " 1373 " to the duodenary scale.
4. " 2987 " to the senary scale.

EXAMINATION QUESTIONS.

How many different characters are used in expressing numbers?

What two kinds of unity are there?

Define notation and numeration.

Define abstract and concrete numbers.

Define prime and composite numbers.

What is meant by numbers being prime to each other?

In multiplication of whole numbers, which is the largest, the multiplicand, multiplier, or product?

In division of whole numbers, which is the largest, the dividend, divisor, or quotient?

State the difference between multiplication and addition.

Point out the difference between division and subtraction.

State the property of the number 9,* and the use that is made of it in testing solutions in simple multiplication and division.

Define reduction, descending and ascending.

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the products which arose in the successive multiplications by 7. Now it is obvious that if 1908 be divided by 7, and the resulting quotient be divided by 7, and the next quotient be divided in a similar way, &c., these digits will be reproduced as remainders in succession, the first remainder occupying the place of units, the second remainder the next place, &c. Whence the rule for finding the several digits corresponding to any scale.

* If the sum of the digits of a number be divisible by 9, the number itself will be divisible by 9, and if there be a remainder in the former, the same remainder will be in the latter. Thus the remainder, in dividing 672 by 9, will be the same as in dividing $6 + 7 + 2$ by 9.

For $672 = 6 \times 100 + 7 \times 10 + 2 = 6 \times 99 + 6 + 7 \times 9 + 7 + 2 =$
$$6 \times 99 + 7 \times 9 + 6 + 7 + 2.$$
Now in dividing the last equality by 9, the remainders are obviously $6 + 7 + 2$, the sum of the digits of 672. This quality of the number 9 is only a particular case of the general proposition, viz., that if the sum of the digits in a number to any scale be divisible by the radix, less 1, the original number is also divisible by it, and if there be remainders, they shall be equal.—See Barlow's *Theory of Numbers*, Prop. 4 (p. 227.)

Define multiple, common multiple, and least common multiple.

Define measure, common measure, and greatest common measure.

Define ratio; when do ratios form a proportion?

What is the object of direct proportion?

To 3, 8, and 21 find a 4th proportional.

In a proportion, what other product is the product of the means equal to?

Define a proper, an improper, a mixed, a complex, and a compound fraction.

What condition must fractions be in before being added or subtracted?

How are fractions reduced to a common denominator?

How may a whole number be expressed in the form of a fraction?

If a fraction be multiplied by (1), what is the product?

Is the product of a fraction by a quantity less than (1), less than the multiplicand?

Is the product of two proper fractions less than either? Explain why.

May the terms of a fraction be expressed by several numbers without the value being altered?

State the quotient of a fraction divided by (1).

If a fraction be divided by a quantity less than (1), will the quotient be greater than the dividend? Give the reason why.

If the greater of two fractions be divided by the less, the quotient will be more than unity; explain why.

State the process of converting a vulgar fraction into a decimal, and the reason of the process.

State what fractions produce terminating decimals, and what produce non-terminating.[*] Explain the reason.

Show why, if the same number be added to the terms of a proper fraction, the new fraction will be greater than the former,[†]

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[*] Since 2 and 5 are the only factors 10 has, and 10 or some power of 10 must be divisible by the denominator to realise a terminating decimal; then if the denominator be not one of the factors, 2 and 5, or powers of one of them, or the product of the multiplicand of different powers of 2 and 5, the decimal will never terminate.

[†] Suppose $\frac{3}{7}$ to be the fraction, and a the number to be added, then the frac-

when brought to a common denominator will stand thus $\left\{\begin{matrix} \frac{3}{7} = \frac{21 + 3a}{49 + 7a} \\ \frac{3 + a}{7 + a} = \frac{21 + 7a}{49 + 7a} \end{matrix}\right\}$.

Now since $21 + 7a$ is greater than $21 + 3a$, $\frac{3 + a}{7 + a}$ is greater than $\frac{3}{7}$; that but if added to an improper fraction, the new fraction will be less?

State the rules for placing the "decimal point" in addition, subtraction, multiplication, and division.

What are the advantages arising from the use of "decimal fractions" instead of "vulgar fractions" and what the disadvantages.[*]

What is the difference between interest and discount?—Which of the two is the greater?

What is the difference between simple and compound interest?

State the class of "proportional questions" that can be wrought by "practice."

Point out the difference between Exchange and Barter.

What is meant by "different scales of notation?"

How do you convert English miles into French mètres?

What is the meaning of the expression "purchased in the 3 per cent. consols at $95\frac{3}{8}$?"

Explain the method of pointing off whole numbers and decimals in extracting the square root.

State how you convert pounds troy into pounds avoirdupois.

Distinguish between the rule of three direct, inverse, and compound

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is, in the new fraction, the quantity *a* is multiplied by the denominator, whereas in the original fraction it is multiplied by the numerator; consequently the new will be greater or less than the old according as the denominator of the old is greater or less than its numerator.

[*] The advantages consist in the facility which decimal fractions have over vulgar fractions in the various fractional operations, and in their being compared by mere inspection, which cannot be done with vulgar fractions until reduced to a common denominator.

The disadvantages consist in the results produced by the use of non-terminating decimals not being exactly correct.

A GLOSSARY OF ARITHMETICAL AND BUSINESS TERMS.

ABSTRACT NUMBER. *See* Art. 12 (p. 5).

ACCOUNT, CURRENT.—A statement of transactions between two parties in *Dr.* and *Cr.* form, in which interest is charged on all sums paid, and allowed on all sums received.

AGIO.—The premium borne by metallic over paper money, or standard over current.

ALIQUOT PART. *See* Art. 101 (p. 140).

ANTECEDENT.—The first term of a ratio.

AREA. *See* Art. 135 (p. 252).

ASSETS.—A general term for cash and property in contradistinction to liabilities.

ASSIGNEE.—One to whom the management of an insolvent's affairs is confided for the benefit of the creditors.

ASSURANCE, LIFE.—An agreement to secure a certain sum of money at the death of an individual upon paying a stipulated single or annual sum.

ATOMIC THEORY.—A theory admitted to be true, which is a recorded statement in chemistry of the proportions by weight in which the small particles or atoms of the elementary bodies combine to form compounds.

BALANCE OF TRADE.—The difference in value between the exports and imports of a country.

——— SHEET.—A statement of the assets and liabilities of a concern.

BANCO.—A continental term for bank money.

BILL OF EXCHANGE.—A note ordering the payment of a certain sum of money at a specified time to a person therein named.

BAROMETER.—An instrument used for marking the variations of weight in the atmosphere.

BONUS.—An extra dividend paid to the members or shareholders of a company.

BUDGET.—The annual estimates of the public revenue and expenditure laid before the House of Commons.

BULLION.—Uncoined gold and silver, but very frequently used to denote metals both in a coined and uncoined state.

CALENDAR.—The distribution of the year into days, weeks and months, with a notice of the festivals occurring.

CANCELLING. *Note*, Art. 19 (p. 20).

CHAIN.—An instrument consisting of 100 links, and equal to 4 perches in length, used in the measurement of land.

CIRCULAR NOTE.—A banker's bill made payable at a number of places for the convenience of travellers.

CHEMICAL UNION.—The term used when two or more bodies unite in forming a new substance differing from either.

CIRCULATION OF A BANK.—Its issued notes.

CIRCULATING DECIMALS. Art. 88 (p. 118).

CIVIL TIME.—Astronomical time accommodated to civil uses, and distinguished into years, months and days.

COIN.—Metallic money stamped by government.

COMPOSITE NUMBER.—A number which will be produced by the multiplication of two or more numbers.

CONCRETE NUMBER. Art. 13 (p. 5).

CONSEQUENT.—The second term of a ratio.

CONTRABAND.—A term for articles prohibited from being exported or imported.

———— **OF WAR.**—Such articles as neutrals are not permitted to convey to belligerent powers in time of war.

CONSOLS. (*Consolidated Funds*).—A Stock Exchange term for that portion of the National Debt consolidated under the 3 per cent. annuities, amounting to about £400,000,000.

CONTINUAL PROPORTIONALS.—A series of quantities in which the ratio between every two adjacent terms are equal, as in geometrical progression.

COUPONS.—Small slips appended to bonds warranting dividend, which are severally cut off for presentation, according as each payment becomes due.

CUBE. *Note*, Art. 38 (p. 29).

CURRENCY.—The coin, notes, and other media by which commodities are interchanged.

CUSTOMS.—Duties levied on goods exported or imported.

DAYS OF GRACE.—Days allowed beyond the expressed time of a bill becoming due before payment can be legally demanded.

DEBENTURE.—A certificate of mortgage or loan on railways or public works.

DECIMAL SYSTEM.—The system of notation in which the unit is made to increase or decrease by ten.

DIGIT.—Any one of the nine characters, 1, 2, 3, 4, 5, 6, 7, 8, 9.

DISCOUNT, AT.—A term denoting that stocks or shares are below their standard value.

DIVIDEND.—The proportional part of the profits or interest of a public company accruing to each shareholder.

DRAFT.—A term applied to both bills and cheques.

——— A small allowance made on the weight of certain goods. Art. 104 (p. 154).

DUODECIMALS.—Numerical operations, in which the unit is made to increase or decrease by twelve.

DUPLICATE RATIO.—The square of the ratio, or the ratio of the squares, of two quantities.

EASTER.—A festival in commemoration of our Saviour's resurrection, ordained by the Council of Nice, in the year 325, to be held on the Sunday which falls upon, or immediately after, the full moon happening next after the 21st of March.

EVOLUTION.—The extraction of the roots of numbers or quantities.

EXCHEQUER BILLS.—Promissory notes issued by Government and bearing interest.

EXCISE.—A tax on articles produced or manufactured at home.

EXPORTS.—Goods sent out of the country.

FACTOR.—One who transacts business for another.

———.—A multiplier which contributes to make a product.

FIGURE.—Any one of the numerical characters.

FOLIO.—A sheet made into two leaves.

——— (*in Book-keeping*).—Two pages numbered alike.

FREIGHT.—The term for goods carried by ship; also the money paid for the conveyance of goods by ship.

FRACTIONS. Art. 66 (p. 90.)

FUNDS.—A term denoting the funded debt of the country which comprises nearly the whole of the National Debt, amounting in 1860 to £771,897,831.

GALLON.—A measure containing 277·274 cubic inches.

GOOD-WILL.—The advantage accruing to a concern from its situation and established trade.

GRAIN.—A weight, 7000 of which make a pound avoirdupois, and 5760, a pound troy.

GRADIENT.—The deviation of a road or railway from a level to an inclined plane.

GRAVITY.—That force by which bodies are drawn towards the centre of the earth.

——— SPECIFIC.—The relative weight of any body compared with an equal bulk of water which is assumed as a standard.

Hydrostatical Paradox. — A truth in hydrostatics adverse to appearance; namely, that any quantity of water, however small, may be made to balance and support any quantity, or any weight, however large.

Hypothenuse.—That side of a right-angled triangle which is opposite to the right angle.

Hypothesis (in Geometry).—A principle admitted as true, and upon which the proof of the proposition is to be founded.

Imports.—Goods brought into a country from abroad.

Indorse.—To write one's signature on the back of a document.

Induction.—The process of drawing a general conclusion from a number of facts or observations collected on any particular subject.

Insolvent.—One whose resources are insufficient to pay his debts.

Integer.—A whole number, as distinguished from a fraction.

Invoice.—An inventory of the quantity and price of goods sold or consigned.

Involution.—The raising a quantity to a given power, by multiplying it by itself.

I. O. U.—A memorandum acknowledging a debt.

Lemma.—One proposition which serves to prepare the way for the perception of another.

Lloyd's.—The name given to a set of rooms in London, used by the Society of Underwriters or Marine Insurers as their offices.

Leap-Year.—A year in which a day is added to 365 days.

Measure.—Any certain quantity assumed as unity, with which other like quantities are compared.

Manifest.—The specification of a ship's cargo and passengers.

Mint.—The place where money is coined.

Mortgage.—A pledge of land or property for money borrowed.

National Debt. — The entire debt of the nation amounting to about £804,000,000.

New Style.—The Gregorian method of reckoning time, which corresponds more exactly with the period of the earth's revolution round the sun. *See* Art. 42 (p. 32).

Notation. Art. 5 (p. 3).

Number.—One or more things of the same kind.

Numeration. Art 6 (p. 3).

Old Style.—The Julian manner of reckoning time, in which the mean year consists of 365¼ days. *See* (Art. 42).

Par of Exchange. Art. 119 (p. 203).

——, at.—A term denoting that stocks, shares, &c., are at their standard value, as when £100 funded or railway stock will sell for £100 in the market.

Per Cent.—Contracted for *per centum*, *i.e.*, by the hundred, as interest at 5 *per cent.*

Perimeter.—The outward boundary of any thing.

Perpetuity.—An annuity that is to continue for ever.

Plant.—A trade term, designating machinery or furniture for carrying on a business.

Policy.—A stamped form embodying the contract of insurance.

Power.—That result which arises from the successive multiplication of any quantity into itself.

Premium.—Money paid for the insurance of life or property.

———, **at.**—A term denoting that stocks and shares are above their standard value.

Prime Numbers.—Numbers that have no common divisor.

Principal. Art. 106 (p. 159).

Progression (*Arithmetical*).—A series of numbers, increasing or decreasing, having the same difference.

——— (*Geometrical*).—A series of terms having a common ratio.

Proposition.—Something proposed to the mind for consideration; or a judgment of the mind expressed in words.

Pyrometer.—An instrument for measuring the degrees of heat in furnaces and intense fires.

Quadrature.—The finding a square equal to the area of a given figure.

Quantity.—A general term applied to everything that can be estimated or measured.

Ratio. Art. 57 (p. 74).

Riciprocal of a Number.—Unity divided by that number.

Rectification of Curves.—The finding a straight line equal to a given curve.

Rentes.—A term denoting the public funds of France.

Reserve Fund.—A fund set aside for meeting any untoward contingency.

Revenue.—The annual receipts of a country from taxes of every kind.

Root.—The root of a given quantity is such a quantity as, when multiplied into itself a certain number of times, will produce that given quantity.

Rule of Three. *Vide* Proportion, Art. 58 (p. 74).

Schedule.—A statement of the affairs of a bankrupt delivered to the Judge of a Bankruptcy Court.

SCRIP.—A certificate of money paid for shares in public companies.

SINKING FUND.—A fund set apart for special appropriation.

SPECIE.—Coined money of any kind.

SQUARE.—*Note*, Art. 7 (p. 29).

SQUARING THE CIRCLE.—The finding a rectilineal figure, such as a square, equal to the area of any given circle;—a problem now generally considered impossible.

STANDARD.—A measure of acknowledged authority by which others are adjusted. See Art. 32, 33, and 34 (pp. 27, 28).

STERLING.—A term denoting English money: a word of uncertain origin, but supposed to be derived from *Easterling*, money coined in the time of the Heptarchy by the Easterlings, or East Saxons.

STEAM ENGINE.—A machine which derives its moving power from the elasticity and condensation of steam.

SUBMULTIPLE.—A number or quantity contained in another number or quantity, a certain number of times exactly.

SUBDUPLICATE RATIO.—The ratio of the square roots of two quantities.

SUBTRIPLICATE RATIO.—The ratio of the cube roots of two quantities.

SUPPLIES.—The annual grants voted by parliament for the public service.

TARE. Art. 105 (p. 154).

TARIFF.—A list of duties on imports and exports. *See* Bills of Parcels (p. 70).

THERMOMETER.—An instrument for measuring the different degrees of heat in bodies.

TONTINES.—A system (so called from the inventor, Lorenzo Tonti) of raising loans by granting a life annuity to the contributors, with benefit of survivorship, so that they are shared among those living, until at last the entire falls to one.

TONNAGE.—The number of tons burden that a ship will carry.

TRET.—An allowance made for waste or dust in goods; a deduction now obsolete.

TRIPLICATE RATIO.—The ratio of the cubes of two quantities.

TRUCK SYSTEM.—Paying workmen in goods instead of money.

UNDER-WRITER.—One who insures ships, cargoes and merchandise from losses at sea, so called because he writes his name under the policy of insurance.

USANCE.—The usage, as regards time, of drawing foreign bills of exchange between certain place.

Vinculum.—A mark or line drawn over a quantity to denote its oneness.

Voucher.—A document to prove that some payment has been made, or some business transacted.

Weight.—That quality in bodies by which they are made to tend downwards; the result of which is measured by either of two sorts of weights called troy and avoidupois weight. *See* Art. 33 (p. 27).

Yard.—The standard measure of length. *See* Art. 32 (p. 27).

Year.—The average time taken by the Sun in passing from any point in the heavens to its return to the same point again.

Zero.—The cypher, or 0; also the point from which the scale of a thermometer is graduated. In Fahrenheit's, zero is 32° below the freezing point of water.

ANSWERS TO THE EXERCISES.

EXERCISES 1—(p. 8).

1. 225010.
2. 187170559.
3. 349770691.
4. 2571548.
5. 11342336.
6. 25436448423936.
7. 2715738.
8. 72218988.
9. 68241216.
10. 65689573.
11. 66879716.
12. 163735803.
13. 101371828.
14. 130440427.
15. 3898070.
16. 5360 ft. 74854 tons.
17. 5945500.
18. 229226929.
19. 274174052.

EXERCISES 2—(p. 11).

1. £3977772.
2. £1190143.
3. £1930385.
4. £23004.
5. £58079626.
6. £3920843.
7. £1415247.
8. £1118678.
9. £2378858.
10. £14606974.
11. £2590899.
12. £8005431.
13. £1549327.
14. £1100003.
15. £2304335.
16. £2823237.
17. £1572811.
18. £2008719.
19. £1099518.
20. £387428.
21. £883916.
22. 6607148 lb.
23. 14174076 lb.
24. 3142642 qrs.
25. 388230872 lb.
26. 299589 cwt.
27. 249290 tons.
28. 326483 cwt.
29. 120172 cwt.
30. 363582 cwt.
31. 3235721 cwt.
32. 2127644 lb.
33. 4130721 cwt.
34. 156303 cwt.
35. 22285500 lb.
36. 23591546 lb.
37. 2792930 gal.
38. 61024962 lbs.

EXERCISES 3—(p. 14).

Answers to these exercises are the dividends in exercises 4.

EXERCISES 4—(p. 19).

Answers to these exercises are the multiplicands in exercises 3.

A. (p. 22.)

Are the dividends of B.

B. (p. 25.)

Are the multiplicands of A.

EXERCISES 5—(pp. 38 to 42).

Answers to A are the corresponding questions in B, and the converse holds in reference to B.

EXERCISES 6—(p. 42.)

1. 46080.
2. £56.
3. 14469.
4. 20176.
5. £2 7s. $9\frac{1}{2}$d.
6. £394 16s.
7. 94.
8. £3 13s. 9d.
9. £2 13s. $8\frac{5}{32}$d.
10. 10080.
11. 5.
12. 175g. 2s. 11d.
13. 114.
14. 680.
15. $62\frac{1}{2}$ lb.
16. 24 tons, 8 cwt. 2 qr. $19\frac{1}{4}$ lb.
17. 10945440 yards.
18. 254364484239360000 sq. in.
19. 2312.
20. 7.
21. 4.
22. £355 17s. 6d.
23. 21120.
24. 48840.
25. 49000.
26. 239.
27. 4690.
28. 9212.
29. 141.
30. 201.
31. 12401.
32. 15 a. 3 r. 30 sq. p. $22\frac{1}{2}$ sq. yd. 4 sq. ft. 64 sq. in.
33. 159 a. 1 r. 27 sq. p. $17\frac{1}{4}$ sq. yd. 8 sq. ft. 64 sq. in.
34. 100000000 sq. in.

EXERCISES 7—(p. 45).

1. 3691347519 lb. 11 oz. 7 dwt. 4 gr.
2. 4097581851 lb. 3 oz. 15 dwt. 8 gr.
3. 3427095 tons 11 cwt. 3 qr. 11 lb. 3 oz. 6 dr.
4. 4135785 tons 2 cwt. 3 qr. 7 lb. 9 oz. 10 dr.
5. 40475652 lb. 3 oz. 0 dr. 2 sc. 12 gr.
6. 34378192 lb. 9 oz. 7 dr. 2 sc. 13 gr.
7. 3759635 m. 1 fur. 35 p. 3 yd. 1 ft. 2 in.
8. 3871235 m. 3 fur. 15 p. 5 yd. 2 ft. 4 in.
9. 3775189266 yd. 0 qr. 2 nl. 2 in.
10. 4135746844 yd. 0 qr. 0 nil. 2 in.
11. 430975 a. 3 r. 35 sq. p. 12 sq. yd. 6 sq. ft. 108 sq. in.
12. 443686 a. 0 r. 37 sq. p. 8 sq, yd. 6 sq. ft. 126 sq. in.
13. 44629197 c. yd. 8 c. ft. 784 c. in.
14. 37948412 c. yd. 24 c. ft. 1174 c. in.
15. 3709278 qr. 6 b. 2 p. 1 gl. 2 qt.
16. 4975093 qr. 5 b. 3 p. 1 gl. 3 qt.
17. 3513479 yr. 110 days 11 hr. 47 min. 16 sec.

18. 4654219 yr. 126 days 13 hr. 19 min. 34 sec.
19. 3913597 circles 118° 19′ 34″.
20. 4537909 circles 203° 47′ 38″.
21. £854000.
22. £106240 6s. 4d.
23. 71104127 0s. 6d.

EXERCISES 8—(p. 49).

1. £4417241 5s. 9¾d.
2. £121735 16s. 8½d.
3. 91726 lb. 2 oz. 17 dwt. 6 gr.
4. 181713 lb. 8 oz. 5 dwt. 21 gr.
5. 23347 tons 14 cwt. 3 qr. 22 lb. 5 oz. 13 dr.
6. 50548 tons 15 cwt. 3 qr. 5 lb. 13 oz. 5 dr.
7. 466085 lb. 5 oz. 6 dr. 1 sc. 7 gr.
8. 310360 lb. 9 oz. 2 dr. 1 sc. 15 gr.
9. 62176 m. 2 fur. 35 poles 1 yd. 1 ft. 7 in.
10. 25547 m. 6 fur. 15 poles 4½ yd. 1 ft. 10 in.
11. 13794979 yd. 1 qr. 2 nl. 1¼ in.
12. 27851241 yd. 3 qr. 2 nl. 1 in.
13. 3815 a. 2 r. 25 sq. p. 28¼ sq. yd. 2 sq. ft. 96 sq. in.
14. 1729 a. 2 r. 17 sq. p. 20¼ sq. yd. 5 sq. ft. 44 sq. in.
15. 4127037 c. yd. 8 c. ft. 1148 c. in.
16. 4658498 c. yd. 19 c. ft. 526 c. in.
17. 17091 qr. 7 bush. 2 pk. 0 gal. 3 qt.
18. 29166 qr. 3 bush. 1 pk. 0 gal. 3 qt. 1 pt.
19. 22060 yr. 47 days 17 hr. 40 min. 32 sec.
20. 28247 yr. 247 days 8 hr. 31 min. 42 sec.
21. 3510 circles 268° 25′ 42″.
22. 42864 circles 194° 35′ 36″.

EXERCISES 9—(p. 55).

1. £5 7s. 2d.
2. £11 4s. 3d.
3. £19 11s. 6d.
4. £29 13s. 11½d.
5. £23 19s. 3d.
6. £41 12s. 1½d.
7. £37 17s. 10d.
8. £25 9s. 5¼d.
9. £16 16s. 5½d.
10. £240 7s. 6d.
11. £583 12s. 6d.
12. £5221 2s. 6d.
13. £3661 17s. 6d.
14. £30310 17s. 6d.
15. £10922 12s. 6d.
16. £10513 2s. 6d.
17. £6780 7s. 6d.
18. £16924 13s. 9d.
19. £57757 10s.
20. £32212 13s. 9d.
21. £241034 1s. 3d.
22. £31362 3s. 9d.

23. £768941 5s.
24. £928273 10s.
25. £324194 1s. 3d.
26. £334589 1s. 3d.
27. £12314826 11s. 3d.
28. 199577 tons 8 cwt. 3 qr.
29. 2591 tons 1 cwt. 2 qr. 24 lb. 8 oz.
30. 49885 tons 11 cwt. 19 lb. 4 oz.
31. 145695 lb. 4 oz. 10 dwt.
32. 110448 lb. 7 oz. 10 dwt.
33. 83709 yr. 352 d. 1 hr. 48 min.
34. 43575 yr. 34 d. 9 hr. 18 min.
35. 23600 miles 7 fur. 13 p. 3 yd. 1 ft. 6 in.
36. 13826 miles 5 fur. 14 p. 3 yd.
37. 28052 a. 1 r. 14 sq. p. 19 sq. yd. 2 sq. ft. 72 sq. in.
38. 64790 a. 1 r. 30 sq. p. 14 sq. yd. 6 sq. ft. 72 sq. in.

EXERCISES 10—(p. 61).

1. £9 16s. 10d.
2. £4 18s. 10d.
3. £3 19s. 5d.
4. £3 7s. 6d.
5. £2 13s. 5½d.
6. £16 13s. 9d.
7. £2 9s. 2½d.
8. £5 11s. 3d.
9. £2 13s. 8½d.
10. £3 7s. 9d.
11. £2 8s. 7d.
12. £1 18s. 5½d.
13. £2 16s. 7½d.
14. £2 18s. 7½d.
15. £1 13s. 5½d.
16. £3 7s. 9d.
17. £2 16s. 10½d.
18. £3 15s. 5½d.
19. £2 13s. 5½d.
20. £2 15s. 5¾d.
21. £2 15s. 9¾d.
22. £1 13s. 10½d.
23. £3 18s. 5¾d.
24. £1 5s. 9¾d.
25. £2 17s. 9½d.
26. £1 13s. 10½d.
27. £2 13s. 7¾d.
28. £2 13s. 10½d.
29. £2 11s. 11¾d.
30. £3 18s. 11¾d.
31. £2 14s. 8¾d.
32. £2 15s. 8¾d.
33. 17 tons 19 cwt. 2 qr. 25 lb. 13 oz. 13 drs.
34. 18 tons 13 cwt. 3 qr. 25 lb. 13 oz. 9 drs.
35. 25 cwt. 3 qr. 15 lb. 10 oz.
36. 39 a. 3 r. 33 sq. p. 23 sq. yd. 7 sq. ft. 59 sq. in.

EXERCISES 11—(p. 66).

1. £49 17s. 8d.
2. £250 7s. 11d.
3. £1072 18s. 11½d.
4. £2 15s. 9d.
5. £1 15s. 9d.
6. £1327 19s. 2d.
7. £3150 9s. 3d.
8. £1 13s. 10½d.
9. £5502 5s. 3d.
10. £2 18s. 5½d.
11. £1169.
12. £2132 18s. 8¼d.
13. £2 11s. 6d.
14. £171 17s. 1½d.
15. £612 14s. 10¾d.
16. £245 6s. 3¼d.

17. £1 18s. $7\frac{1}{2}$d.
18. 8s. $4\frac{1}{2}$d.
19. 8s. $7\frac{1}{2}$d.
20. £1471 19s. $10\frac{1}{4}$d.
21. 18s. 7d.
22. 1s. $5\frac{3}{4}$d.
23. £91 7s. $7\frac{1}{4}$d.
24. £185 10s. $7\frac{3}{4}$d.
25. £1 7s. 8d.
26. £3 19s. 4d.
27. 1s $5\frac{1}{2}$d.
28. £1094 1s. $6\frac{3}{4}$d.
29. £601 2s. $4\frac{3}{4}$d.
30. £509 11s. $0\frac{1}{2}$d.
31. £2 1s. 6d.
32. £2 18s. 4d.
33. £3 5s. 4d.
34. £1 14s. 2d.
35. £1423 19s. $7\frac{1}{2}$d.
36. £62 4s. 6d.
37. £1 7s. 4d.
38. 6987 yards.
39. 547 cwt.
40. 854 cwt.
41. 6059 yards.
42. £3 14s. $8\frac{1}{2}$d.
43. £690 14s. $11\frac{1}{4}$d.
44. £717 4s. $10\frac{1}{2}$d.
45. £16 8s. $4\frac{1}{2}$d.
46. 18 m. 5 fur. 33 poles 4 yds. 2 ft. 7 in.
47. 38 a. 3 r. 33 sq. p. 25 sq. yd. 7 sq. ft. 59 sq. in.
48. 1 acre.
49. 5 m. 5 fur. 33 poles 4 yd. 2 ft. 11 in.
50. 2133 a. 0 r. 8 sq. p. 29 sq. yd. 6 sq. ft.
51. 3 m. 7 fur. 33 poles 3 yd. 2 ft. 7 in.
52. 15 a. 3 r. 30 sq. p. $22\frac{1}{2}$ sq. yd. 4 sq. ft. 64 sq. in.
53. 18 m. 5 fur. 36 poles.
54. 244 a. 0 r. 34 sq. p. $1\frac{1}{2}$ sq. yd. 3 sq. ft. 72 sq. in.
55. 6 butts 2 barrels. 33 gal. 3 qt.
56. 33 lb. 2 oz. 0 dwt. $2\frac{1}{2}$ gr.
57. £1 9s. 8d.
58. 5 lb. 12 oz. 10 dwt.
59. 9 cwt. 2 qr. 21 lb.
60. 1161 qr. 5 bushel $1\frac{1}{2}$ pecks.
61. 9 lb. 4 oz. 3 dr. 2 scr. 12 grs.
62. 37 sq. yd. 7 sq. ft. 96 sq. in.
63. 11 h. 48 m. 8 s. A.M.
64. £1 15s. $5\frac{1021885}{11291713}$d.
65. 12 moons, and 11 days of the 13th moon.
66. 2 h. 2 m. A.M.
67. 5 h. 6 m. 32 s. A.M., or 6 m. 32 s. past 5 o'clock in the morning.
68. £185 19s. $0\frac{12}{121}$d.
69. 1 m. 7 fur.
70. £1 4s. $0\frac{3}{4}$d.
71. 40604 tons 2 cwt. 3 qr. 12 lb.
72. 4s. 4d.
73. 346 thousand.
74. 11 h. 22 m. A.M.

EXERCISES 12—(p. 70).

1. £947 16s.
2. £16 13s. $4\frac{1}{2}$d.
3. £49 3s. 10d.
4. £98 14s. 5d.
5. £354 18s. $5\frac{1}{2}$d.
6. £17 9s.
7. £142 12s. $0\frac{1}{2}$d.
8. £238 13s.
9. £197 4s. $3\frac{1}{4}$d.
10. £1161 5s. $6\frac{3}{4}$d.
11. £9 14s. 9d.
12. £242 7s. $2\frac{3}{4}$d.

13. £388 10s. 7½d.
14. £179 0s. 10¾d.
15. £89 10s. 9½d.
16. £1514 9s. 11¾d.
17. £51913 9s. 7¼d.
18. £570 0s. $5\frac{37}{64}$d.
19. £5 11s. 2d.
20. £2148 19s. 0¼d.

EXERCISES 13—(p. 79).

1. 164.
2. £162.
3. £460.
4. 195 yards.
5. £15 9s. 2d.
6. £18 13s. 4d.
7. £38 10s.
8. £348 16s. 9d.
9. £34 12s. 4d.
10. 24.
11. 25.
12. 140.
13. £1216 13s. 4d.
14. 10.
15. 3 lb. 4 oz.

EXERCISES 14—(p. 82).

1. £260.
2. £238 12s. 8d.
3. £54 17s. 3d.
4. 111 yards.
5. £467.
6. £147 5s.
7. £1 5s. 4½d.
8. £343 9s. 3d.
9. 75 acres 2 r. 10 p.
10. £20 1s. 7½d.
11. £278 13s. 9d.
12. £27 15s. 4d.
13. £14 13s. 4d.
14. £3 9s. 2d.
15. £426 6s.
16. £1 5s. 4½d.
17. £40 16s. 8d.
18. £12 16s. 8d.
19. £2 14s. 8d.
20. £74 13s. 4d.
21. £30 8s.
22. £1 7s. 4d.
23. £25 9s. 0¾d.
24. 6059.
25 £87 18s. 10¼d.
26. £8 19s. 4d.
27. £12 10s. 0½d.
28. £4 4s.
29. 75 days.
30. £64 1s. 3d.
31. £2619 15s. $2\frac{6}{7}$d.
32. £40 10s. 7¼d.
33. £21.
34. £574 16s. 1¾d.
35. 118 days.
36. £795 0s. 1⅛d.
37. £25.
38. £39 10s. 10d.
39. £3240.
40. £12 5s. 7½d.
41. 2s. 10½d
42. 25 cwt. 3 qr. 15 lb. 10 oz.
43. £3 19s. 4½d.
44. £1 7s.
45. £260 13s. 1½d.
46. £1 16s. 8d.
47. { Valuation, £293 2s. 1d.
    Poor-rate, £29 6s. 2½d.
48. £61 11s. 10½d.
49. £121 10s.
50. £31 5s. 4d.
51. £2 6s. 8d.
52. £7333 6s. 8d.
53. £21 1s. 4d.

54. £149 6s. 8d.
55. £13 17s. 6d.
56. £348 15s.
57. 7 cwt. 0 qr. 6 lb.
58. 1 cwt. 1 qr. 24 lb.
59. 16s. 3d.
60. 25 tons 11 cwt. 2 qr. 12 lb.
61. £3 0s. 5$\frac{11}{841}$d.
62. £32500.
63. £357 9s. 4$\frac{1}{2}$d.
64. £3 18s. 9d.
65. £93 14s. 3$\frac{18}{20}$d.
66. 108672$\frac{32}{35}$ ounces.
67. 17s. 6d.
68. £1 3s. 2d.
69. 56 cwt. 1 qr. 14 lb.
70. 216 lb. 14 dwts. 18 grains.
71. 16 lb. 6 oz. 15 dwt. 13$\frac{1}{2}$ gr.
72. 350 cwt. 1 qr. 6$\frac{3}{8}$ lb.
73. 1842 sq. yd. 8 sq. ft. 30 sq. in.
74. £767.
75. £7853 15s.
76. 3570.
77. 875.
78. 18662685.

EXERCISES 15—(p. 88).

1. £4 10s.
2. 9 bushels.
3. 17s. 6d.
4. 14400.
5. 6.
6. 14.
7. 400.
8. 11$\frac{1}{5}$ days.
9. 180 acres.
10. 2371$\frac{1}{2}$.
11. £65 12s. 6d.
12. 80.
13. 3150.
14. 144.
15. £147 16s. 6d.
16. 14$\frac{2}{5}$ days.
17. £50.
18. 124$\frac{62}{73}$.

EXERCISES 16—(pp. 93 to 96).

1. 819.
2. 120.
3. 1080.
4. 840.
5. 2520.
6. 504.
7. 51.
8. 24.
9. 13.
10. 2.
11. 4.
12. 21.
13. 9.
14. 33.
15. 14,$\frac{13}{14}$.
16. 73,$\frac{13}{40}$.
17. 146,$\frac{43}{50}$.
18. 56,$\frac{21}{32}$.
19. 15,$\frac{19}{72}$.
20. 17739,$\frac{23}{353}$.
21. 12,$\frac{461}{576}$.
22. 5832,$\frac{127}{128}$.
23. 5,$\frac{421}{448}$.
24. 49,$\frac{19}{32}$.
25. 9,$\frac{457}{2292}$.
26. 471,$\frac{13}{16}$.
27. 112,$\frac{891}{2894}$.
28. 36,$\frac{76}{229}$.
29. 21,$\frac{6449}{9936}$.
30. 65,$\frac{80}{251}$.
31. 942,$\frac{13}{16}$.
32. 9,$\frac{457}{2292}$.
33. 228,$\frac{151}{324}$.
34. 84,$\frac{389}{1152}$.
35. 576,$\frac{611}{1089}$.
36. 42,$\frac{257}{464}$.
37. 104;$\frac{3293}{5376}$.
38. 32,$\frac{198637}{366061}$.
39. 8052,$\frac{21}{26}$.
40. 4619,$\frac{29}{119}$.
41. 138272,$\frac{139}{203}$.

EXERCISES 17—(pp. 96 to 100).

A. 1. 7$\frac{5}{9}$. 2. 5$\frac{3}{7}$. 3. 17$\frac{4}{17}$. 4. 4$\frac{5}{13}$. 5. 6$\frac{2}{11}$.
B. 1. $\frac{35}{6}$. 2. $\frac{141}{8}$. 3. $\frac{119}{12}$. 4. $\frac{1049}{13}$. 5. $\frac{133}{9}$. 6. $\frac{170}{11}$.

C. 1. $\frac{77}{216}$. 2. $10\frac{1}{8}$. 3. $1\frac{3}{4}$. 4. $127\frac{7}{9}$. 5. $36\frac{41}{45}$.

D. 1. $\frac{31}{35}$. 2. $\frac{16}{21}$. 3. $\frac{4}{7}$. 4. $17\frac{5}{8}$. 5. $1\frac{7}{20}$. 6. $\frac{15}{22}$.

E. 1. $\frac{70}{80}$, $\frac{75}{80}$, and $\frac{72}{80}$. 2. $\frac{30}{40}$, $\frac{35}{40}$, and $\frac{16}{40}$.

F. 1. £$\frac{3}{16}$. 2. $\frac{1}{8}$ cwt. 3. $\frac{11}{20}$ ton. 4. $\frac{13}{32}$ acre. 5. £$\frac{4}{9}$.
6. $\frac{7}{12}$ guinea. 7. $\frac{4}{15}$ lb. 8. £$\frac{13}{320}$. 9. $\frac{2}{5}$ mile.

G. 1. 8s. 9d. 2. 8s. 9d. 3. 1 r. 20 sq. p.
4. 17 cwt. 2 qr. 5. 7 oz. 10 dwt. 6. 3 hhd. 31 gals. 2 qt.
7. 4 fur. 32 p. 8. 22 min. 30 sec. 9. 5 oz.
10. 10s. 6d. 11. 45 gals. 12. 1qr. 4 lb.

EXERCISES 18—(p. 102).

1. 14.
2. 15.
3. 9.
4. $20\frac{1}{4}$.
5. $2\frac{11}{45}$.
6. $2\frac{47}{80}$.
7. $4\frac{31}{720}$.
8. $26\frac{7}{12}$.
9. $2\frac{3}{8}$.
10. $18\frac{97}{160}$.
11. $18\frac{19}{56}$.
12. $13\frac{157}{832}$.
13. $109\frac{11}{24}$.
14. $15\frac{9}{28}$.
15. $75\frac{19}{24}$.
16. $23\frac{103}{1920}$
17. $12\frac{345}{448}$.
18. $47\frac{31}{120}$.
19. £9 1s. $10\frac{1}{2}$d.
20. $50\frac{43}{224}$.
21. £1 9s. $6\frac{123}{128}$.
22. 4cwt. 2qr. $26\frac{13}{24}$lb.
23. 13cwt. 0qr. $2\frac{85}{96}$lb.
24. $3\frac{75}{112}$.
25. $47\frac{31}{120}$.
26. $208\frac{5}{48}$.
27. $15\frac{19}{24}$.
28. $12\frac{127}{144}$.
29. $8\frac{23}{48}$.
30. £14 7s. $10\frac{7}{12}$d.
31. $16\frac{23}{60}$.
32. £4 8s. $6\frac{15}{64}$d.
33. £165 3s. $8\frac{4223}{5840}$.
34. $60\frac{231}{320}$.
35. $21\frac{99}{1024}$.
36 4 cwt. 3 qr. 8 lb. 10 oz. $11\frac{1}{5}$ dr.
37. $931\frac{11}{16}$.
38. £153 18s. $4\frac{13}{22}$d.
39. $14\frac{7}{12}$.
40. $22\frac{203}{240}$.
41. $9\frac{17}{220}$.
42. $22\frac{23}{24}$.
43. 8s. $7\frac{6877}{10168}$d.
44. £51 0s. $5\frac{18608}{29200}$.
45. 4cwt. 2qr. $26\frac{13}{24}$lb.
46. £6 10s. $3\frac{1}{60}$d.
47. £17 4s. $0\frac{37}{42}$.
48. $12\frac{9355}{10296}$.

EXERCISES 19—(p. 104).

1. $1\frac{5}{8}$.
2. $2\frac{1}{2}$.
3. $14\frac{23}{35}$.
4. $3\frac{3}{16}$.
5. $2\frac{17}{24}$.
6. $5\frac{7}{64}$.
7. $3\frac{17}{24}$.
8. $26\frac{27}{416}$.
9. $6\frac{4026}{9125}$.
10. $15\frac{11}{16}$ lb.
11. $4\frac{55}{56}$.
12. $4\frac{243}{280}$.
13. 11s. $4\frac{71}{112}$d.
14. $5\frac{1}{80}$.
15. £4 6s. $5\frac{7}{64}$d.
16. $15\frac{11}{12}$.
17. $91\frac{25555}{29232}$.
18. $\frac{1}{154}$.
19. $29\frac{7}{8}$.
20. 7s. $3\frac{249}{320}$d.
21. $17\frac{5}{12}$.
22. $15\frac{11}{12}$.
23. $9\frac{4}{15}$.
24. $58\frac{53}{72}$.
25. £27 10s. $7\frac{31}{80}$d.
26. $3\frac{1}{8}$.
27. $90\frac{125}{144}$.
28. $14\frac{1}{8}$.
29. $10\frac{47}{144}$.
30. $7\frac{1}{16}$.
31. £5 8s. $9\frac{45}{56}$.
32. £66 11s. $4\frac{71}{112}$d.

EXERCISES 20—(p. 105).

1. 6.
2. 9.
3. $10\frac{1}{2}$.
4. 15.
5. 14.
6. $12\frac{1}{4}$.

.
.
$\frac{1}{10}$.
.
$\frac{7}{8}$.
.
.
$\frac{131}{187}$.
07$\frac{13}{18}$.
$\frac{4}{5}$.
58$\frac{5}{32}$.
6$\frac{7}{8}$.
$\frac{13}{24}$.
$\frac{27}{28}$.
$\frac{221}{152}$.
50.
47853097.

26. 6895487689287.
27. £89735 13s. 5$\frac{1}{2}$d.
28. £69357 13s. 7d.
29. 7895 tons 13 cwt.
3 qr. 17 lb. 13 oz.
9 dr.
30. 1489$\frac{4}{5}$.
33. 47$\frac{1}{2}$.
33. 166$\frac{2}{3}$.
35. 7895 yrs. 213 d.
21 h. 39 m. 43 s.
34. 7893 lb. 9 oz.
17 dwt. 21 gr.
35. £38765 18s. 9$\frac{1}{2}$d.
36. 68793 17s. 9$\frac{3}{4}$d.
37. 401472$\frac{13}{36}$
38. 150$\frac{871}{4500}$.
39. 80$\frac{1}{15}$.
40. 106$\frac{47}{60}$.

41. 17728$\frac{1617}{7680}$.
42. 309$\frac{17}{36}$ yards.
43. 2125$\frac{31}{36}$ oz.
44. 265$\frac{73}{96}$ lb.
45. 13cwt. 0qr. 2$\frac{88}{96}$lb.
46. 189$\frac{1}{30}$.
47. 32$\frac{1}{12}$.
48. 309$\frac{17}{36}$.
49. 931$\frac{11}{16}$.
50. £85 13s. 8$\frac{3}{8}$d.
51. £68 5s. 1$\frac{1}{32}$.
52. 498$\frac{5}{24}$.
53. £83 7s. 10$\frac{5}{16}$.
54. 92$\frac{61}{72}$.
55. $\frac{1}{8}$.
56. £7090 0s. 8$\frac{3}{4}$d.
57. 81$\frac{5}{8}$.
58. £242 8s. 6$\frac{1}{16}$d.

EXERCISES 21—(p. 108).

.
$\frac{1}{2}$.
.
.
.
.
9288$\frac{3881}{4032}$.
$\frac{6247}{45360}$.
982$\frac{4187}{5040}$.
69$\frac{43668}{60480}$.
1$\frac{37763}{90720}$d.
.

21. $\frac{113}{165}$.
22. 6$\frac{3967}{6012}$.
23. 202$\frac{1787}{3780}$.
24. 19090$\frac{5}{8}$.
25. 5s. 3$\frac{1}{2}$d.
26. 5s. 1$\frac{3}{4}$d.
27. 4s. 2d.
28. 9s. 9d.
29. 5$\frac{151}{375}$.
30. 5985666396$\frac{1088}{1152}$.
31. £3 19s. 1$\frac{26483}{45360}$d.
32. £6 2s. 3$\frac{10063}{11340}$d.
33. 10 tons 19 cwt.
1 qr. 8 lb. 6 oz.
2$\frac{568}{720}$ dwt..
34. 1$\frac{2679}{6545}$.
35. 56$\frac{8}{9}$.
36. 2$\frac{19}{32}$.
37. 8$\frac{1}{10}$.
38. 6$\frac{19}{60}$.

39. 317 d. 16 h. 2 m.
40$\frac{3863}{5072}$. sec.
40. 6 lb. 3 oz. 3 dwt.
14$\frac{62}{1260}$ gr.
41. 22209288$\frac{3881}{4032}$.
42. £2 11s. 3$\frac{2231}{30240}$.
43. 29$\frac{7}{8}$.
44. £2 5s. 5$\frac{237487}{241152}$d.
45. 6$\frac{2355}{20592}$.
46. 604$\frac{1287}{1800}$.
47. 3s. 3$\frac{6449}{9938}$d.
48. 8s. 7$\frac{6577}{10166}$d.
49. 73 c. yds. 19 c. ft.
999 c. in.
50. 3s. 8$\frac{1}{2}$d.
51. 4s. 8$\frac{1}{2}$d.
52. 5s. 3$\frac{457}{2292}$d.
53. 7s. 3$\frac{1}{2}$d.
54. 161$\frac{1159}{1320}$.
55. 3$\frac{221}{448}$.

56. £3 0s. 11$\frac{3312}{51840}$d. 57. $\frac{47603}{84208}$. 58. 16$\frac{2}{3}$.
59. 36$\frac{41}{45}$. 60. $\frac{1}{28}$.

EXERCISES 22—(p. 111).

1. £184 3s. 4d.
2. £516 19s. 4$\frac{65}{66}$d.
3. £160 0s. 6$\frac{803}{1512}$d.
4. 76 yd. 3 qr. 0$\frac{20}{21}$ nl.
5. £123 1s. 6$\frac{31}{50}$d.
6. £83 5s. 11$\frac{163}{180}$d.
7. £149 3s. 7$\frac{38}{229}$d,
8. £1438 15s. 10$\frac{2}{45}$d.
9. 381$\frac{3}{16}$ yards.
10. 9$\frac{3}{68}$ lb.
11. £636 16s. 10$\frac{2}{3}$d.
12. $\frac{3}{8}$.
13. £168 18s. 7$\frac{61}{285}$d.
14. £17 6s. 8d.
15. £94 2s. 2$\frac{23}{32}$d.
16. 150$\frac{671}{4500}$.
17. £74 4s. 0$\frac{37}{42}$d.
18. £166 13s. 11$\frac{7}{55}$d.
19. £44 10s. 9$\frac{393}{560}$d.
20. £912 2s. 5$\frac{1}{11}$d.
21. 3s. 2$\frac{25}{57}$d.
22. £107 4s. 7$\frac{2945}{7488}$d.
23. 150$\frac{671}{4500}$.
24. 14s. 5$\frac{457}{724}$d.
25. 99$\frac{97}{128}$.
26. 72$\frac{2143}{2304}$.
27. 43$\frac{1111}{1296}$.
28. £21 13s. 11$\frac{1}{4}$d.
29. £3 17s. 0$\frac{13568}{27027}$d.
30. 43 cwt. 0 qr. 10$\frac{1006}{2691}$ lb.
31. 7s. 9$\frac{15}{4238}$d.
32. 4s. 5$\frac{866}{1819}$d.
33. £1582 9s. 3$\frac{45}{64}$d.
34. £119 2s. 4$\frac{252}{261}$d.
35. £55957 11s. 5$\frac{1}{4}$d.
36. £170 5s. 6d.
37. { 376$\frac{1}{5}$ lb. of flour.
    222$\frac{3}{10}$ of water.
38. 1$\frac{2785167}{10064817}$s.
39. { 3 r. 24 p. meadow land.
    1 a. 2 r. 30 p. arable.
    Rent, £17 10s. 0$\frac{3}{4}$d.
40. £1268 4s. 7$\frac{1}{2}$d.
41. £83 5s. 11$\frac{163}{180}$d.
42. £115 18s. 6$\frac{87}{112}$d.
43. £22 12s. 11$\frac{124}{385}$d.
44. £4254 6s. 9$\frac{2}{3}$d.
45. £2476 8s. 3$\frac{461}{576}$d.
46. £10$\frac{80}{251}$.
47. 9$\frac{747}{5083}$,
48. £12 3s. 1$\frac{281}{417}$d.
49. £7507 1s. 6$\frac{3}{34}$d.
50. 5$\frac{2}{3}$ pieces.
51. 469$\frac{7}{12}$ bushels.
52. 3$\frac{283}{385}$.
53. £35 10s. 7$\frac{9077}{16137}$d.
54. 1.
55. 1s. 9$\frac{433}{2492}$d.
56. 8s. 9$\frac{1503}{4244}$d.
57. £1089 15s. 3$\frac{3}{4}$d.
58. 47$\frac{1115}{1900}$.
59. 45$\frac{77}{80}$.
60. £11 10s. 8$\frac{389}{1152}$d.
61. { £167 2s. entire cost.
    17s. 9$\frac{675}{1001}$d. average price of 1 gallon.
62. { Entire cost £594 13s.
    Average price of 1 yard 8s. 8$\frac{179}{227}$d.
63. { £339 9s. 11$\frac{13}{16}$d. entire cost.
    5s. 3$\frac{457}{2292}$d. average cost of a yard.
64. £1 3s. 11$\frac{91}{352}$d.
65. £22 12s. 11$\frac{124}{385}$d.
66. £1003 5s. 1$\frac{47}{85}$d.

EXERCISES 23—(p. 116).

1. $3\frac{65}{100}$.
2. $\frac{375}{10000}$.
3. $56\cdot\frac{78125}{100000}$.
4. $493\frac{35}{10000}$.
5. 1·35.
6. 11·005.
7. 25·0375.
8. 74·56985.
9. 2817·355.
10. ·40708.

EXERCISES 24—(p. 119).

The answers to these exercises are the exercises (25).

EXERCISES 25—(p. 120 to 124).

The answers to these are the exercises (24).

EXERCISES 26—(p. 125).

A.

1. £·3125.
2. £·1875.
3. ·125 cwt.
4. ·55 tons.
5. ·1875 yard.
6. ·40625 acre.
7. £·628125.
8. £·790625.
9. £·01875.
10. £·014583.
11. £$\cdot18\dot{3}$
12. ·6875 cwt.
13. ·09375 cwt.
14. $\cdot837594\dot{6}\dot{9}$.

B. (p. 126).

1. 14s.
2. 3s. 6d.
3. 5s. $4\frac{1}{2}$d.
4. 7s. 6d.
5. 2s. $0\frac{3}{4}$d.
6. 9 cwt. 2 qr.
7. 18s. $7\frac{1}{2}$d.
8. 1s. 6d.
9. 2 fur. 24 p.
10. 16 min. 48 sec.
11. 3 quarts 1 pint.
12. 13 hr. 30 min.
13. 7 oz. 10 dwt.
14. 1 nail.
15. 12 perches.
16. 3s. $7\frac{1}{2}$d.
17. 129·6 cubic inches.
18. 91 d. 7 hr. 30 m.
19. 5 fur. 3 p. 3·5 yd.
20. 2 a. 2 r. 12 sq. p. 13 sq. yd. 4 sq. in.
21. 11s. 8d.
22. 4 fur. 11 p. 5 yd. 0 ft. 10 in.
23. 32 min. 30 sec.

EXERCISES 27—(p. 128).

1. 78·46447.
2. 67·5625.
3. 30·6532546875.
4. 102·16471425.
5. 104 7759.
6. 78·46447.
7. £67 10s. 6·348288d.
8. 99·4165625 qrs.
9. 6·9945.
10. 25·0765.
11. 9·3355 yards.
12. 12·0125 lb.
13. £328 3s. 7·194d.
14. 110 cwt. 2 qr. 20·415 lb.
15. £145 4s. 0·165d.
16. 19 cwt. 2 qr. 22·792 lb.
17. £1·6836.
18. ·29516875 qr.
19. 99·4165625 qr.
20. $17\cdot939285714\dot{2}$ cwt.
21. 159·49975 cwt.
22. 638·37.
23. £·46553
24. 11 shillings.

EXERCISES 28—(p. 129).

1. 7·9509.
2. 14·19.
3. 12·74.
4. 4·86825 cwt.
5. 21·07.
6. £3 5s. 2·87021d.
7. £1 9s. 0·1275d.
8. 12·779.
9. 54·07015.
10. 48·62715.
11. 10·2625.
12. £3 5s. 2·87021.

EXERCISES 29—(p. 130).

1. 1033·40625.
2. ·975.
3. 190·5.
4. 25·95.
5. 36528·28125.
6. ·020493.
7. 4475·99992.
8. ·0405375.
9. ·000049938.
10. 651·96875.
11. ·00032.
12. ·00000625.
13. 5·184.
14. ·0144.
15. 47·460641.
16. ·001728.
17. 1511·57885625.
18. 1·16985856.
19. ·002197.
20. 9914·94
21. 51·99009375.
22. 1093·9623125.
23. 459464·17125.
24. 19091·226.
25. 17209·815.
26. 16065·84.
27. £100 2s. 10·5d.
28. 742 cwt. 1 qr. 8·8886 lb.
29. 629 cwt. 0 qr. 6·746875 lb.
30. 19577612·787890625.
31. 208065·95355.
32. 77025·44630859375.
33. 60·86705625.
34. 12825·75.
35. £49 4s. 2·8125d.
36. 76·2416.
37. 46·607 cwt.
38. £117 7s. 11·3136d.
39. 2·86875.
40. £1797 1s. 7·89d.
41. 328·968234375.
42. £207 8s. 5·8125d.
43. 365·2712875.
44. £3465 5s. 9·4176d.
45. £34 19s. 0·198d.
46. £53 6s. 4·875d.
47. 75·24825.
48. 130 cwt. 3 qr. 12·6875 lb.
49. 16519·03125.
50. 2495222·8125.
51. 2931885·7115625.
52. £299 9s. 4·8764797995.
53. 2131·033.
54. 98375·1112.

EXERCISES 30—(p. 133).

1. 7282.
2. 961.
3. 1503·25.
4. 35·7725.
5. 12·74.
6. 437·619.
7. 49013·325.
8. 313·52578125.

9. 3864.
10. 973·85.
11. 1·645.
12. 18·3225.
13. 18.3225.
14. 73·785.
15. 5s. 7·5d.
16. £3 13s. 6d.
17. 19 cwt. 2 qr. 22·792 lb.
18. 3s. 7·5d.
19. 19 cwt. 2 qr. 22·792 lb.
20. £2 1s. 3d.
21. £1 17s. 7·56d.
22. 973·85 cwt.
23. £1 13s. 9d.
24. £3 18s. 9d.
25. £3 13s. 9d.
26. 89·85 barrels.
27. 9 lb. 7 oz. 15 dwts.
28. 107·5 gallons.
29. 81 cwt. 2 qr. 24·5 lb.
30. 56·625 bushels.
31. 9 a. 3 r. 25 p.
32. 57·875 cwt.
33. 125 oz. 10 dwt. 18 gr.
34. 1782·820$\dot{6}$849315$\dot{0}$.
35. 389·45190$\dot{1}$260504201680672
2689075630252100840336 1
34453781$\dot{5}$.
36. 1155·01 yard.
37. 239·6875 bushels.
38. 125·$\dot{7}$8735632183908045977
011494252$\dot{8}$.
39. £1 8s. 7½d.
40. 303·525 times.
41. 4971·701$\dot{7}$945205$\dot{4}$.
42. 16s. 10·5d.
43. 35889·32$\dot{1}$4285$\dot{7}$.
44. 6·37675.
45. 3·112.
46. 9·2976552.

EXERCISES 31—(p. 138).

1. 27 cwt. 1 qr. 16·4 lb.
2. 19 a. 2 r. 15 sq. p.
3. £309 11s. 9·28125d.
4. £30 5s. 2·993088d.
5. £207 8s. 5 8125d.
6. 23 cwt. 1 qr. 21 lb.
7. 159293·316.
8. £456 11s. 7·$\dot{6}$3$\dot{6}$d.
9. £120 1s. 3·132d.
10. £123 1s. 4·9824375d.
11. £2 19s. 3·61d.
12. £227 4s. 11·4d.
13. 77025·44630859375.
14. £1639 11s. 5·7264d.
15. £2328 17s. 9·49375d.
16. 2052·12 lb.
17. 8s. 2·379744375d.
18. £366 15s. 5·41276875d.
19. £1425 17s. 11·5d.
20. £171 15s. 3·4622$\dot{1}$4285$\dot{7}$d.
21. £336 8s. 10·81007d.
22. 6·9$\dot{7}$d.
23. £3 5s. 2·87021d.
24. 365 yd. 2 qr. 3 nl.
25. 5s.
26. £149 10s. 9·32$\dot{1}$4285$\dot{7}$d
27. £20 14s. 3·701$\dot{7}$945205$\dot{4}$d.
28. £317 15s. 5·9296875d.
29. £465 6s. 2·59875d.
30. £389·45190$\dot{1}$2605042016806
7226890756302521008403 3
61344537815$\dot{}$.
31. £300 5s. 7·59525d.
32. £80 19s. 9·24308644362792d.
33. 67·5625 yards.
34. 22·15455$\dot{5}$7142$\dot{8}$ qrs.
35. £82 0s. 11·205588d.
36. £10396 15s. 2·8125d.

37. £41 7s. 1·2765625.
38. £12216 3s. 9·7115625.
39. 6·75 pieces.
40. 27 cwt. 1 qr. 16·4 lb.
41. £646·4000$\dot{3}$.

EXERCISES 32—(p. 145).

1. £126 8s.
2. £99 5s.
3. £160 6s.
4. £130 19s.
5. £523 16s.
6. £142 13s. 4d.
7. £123 3s. 4d.
8. £253 16s.
9. £381 10s.
10. £48 2s. 6d.
11. £53 5s.
12. £57 1s. 4d.
13. £30 11s. 3d.
14. £33 18s. 6d.
15. £11 5s.
16. £10 19s.
17. £7 17s. 6d.
18. £3 9s. 8d.
19. £1 9s. $10\frac{1}{2}$d.
20. £3 10s. $1\frac{1}{2}$d.
21. £1 0s. $6\frac{3}{4}$d.
22. £4 7s. 1d.
23. £4 4s. 1d.
24. £4 19s. $10\frac{3}{4}$d.
25. £8 5s. $11\frac{1}{4}$d.
26. £7 0s. $7\frac{1}{2}$d.
27. £9 11s. 7d.
28. £53 11s.
29. £46 5s. 1d.
30. £46 1s. $11\frac{3}{4}$d.
31. £37 17s. $1\frac{1}{2}$d.
32. £54 13s. $1\frac{1}{2}$d.
33. £33 10s. $7\frac{1}{2}$d.
34. £42 11s. $9\frac{3}{4}$d.
35. £107 13s. 6d.
36. £91 14s. 3d.
37. £86 18s. 9d.
38. £253 9s. 2d.
39. £99 13s. $9\frac{3}{4}$d.
40. £428 2s. 6d.
41. £656 13s. 4d.
42. £756 17s. 6d.
43. £585 15s.
44. £171 11s. 3d.
45. £1808 3s. 4d.
46. £2686 2s. 6d.
47. £1150 19s. 4d.
48. £914 9s. 4d.
49. £5471 13s. 4d.
50. £1558 7s. 6d.
51. £2514 13s. 4d.
52. £2203 17s. 6d.
53. £3702 4s. 4d.
54. £3372 5s.

EXERCISES 33—(p. 149).

1. £258 4s. $6\frac{1}{16}$d.
2. £137 8s. 4d.
3. £1375 16s. 8d.
4. £111 17s. $11\frac{3}{4}$d.
5. £93 10s. 8d.
6. £434 12s. 9d.
7. £358 17s. $3\frac{15}{64}$d.
8. £222 14s. $8\frac{13}{16}$d.
9. £129 3s. 6d.
10. £307 2s. $4\frac{1}{4}$d.

EXERCISES 34—(p. 151).

1. £554 6s. 3d.
2. £233 3s. $0\frac{1}{4}$d.
3. £423 9s. $0\frac{3}{4}$d.
4. £509 11s. $0\frac{1}{2}$d.
5. £21 1s. 4d.
6. £139 1s. 4d.
7. £286 7s. 8d.
8. £30 8s.
9. £207 7s. 8d.
10. £128 6s. 5d.
11. £526 4s. 3d.
12. £485 17s. $1\frac{1}{8}$d.
13. £87 18s. $10\frac{1}{4}$d.
14. £310 7s. $1\frac{1}{2}$d.
15. £12 10s. $0\frac{1}{2}$d.
16. £94 2s. $0\frac{3}{4}$d.
17. £111 17s. $11\frac{3}{4}$d.
18. £174 9s. $2\frac{1}{2}$d.
19. £922 4s. $10\frac{1}{2}$d.
20. £284 9s. 9d.
21. £3 12s. $4\frac{1}{2}$d.
22. £5 12s. $1\frac{1}{4}$d.

23. £20 1s. 4d.
24. £1113 12s. $7\frac{1}{2}$d.
25. £9907 1s. $0\frac{1}{2}$d.
26. £106 2s. $7\frac{1}{4}$d.
27. £33737 11s. $1\frac{23}{32}$d.
28. £2048 6s. $4\frac{7}{8}$d.
29. £752 7s. $7\frac{9}{16}$d.
30. £772 16s. $8\frac{5}{8}$d.
31. £334 16s. $4\frac{33}{80}$d.
32. £583 1s. $9\frac{75}{112}$d.
33. £940 6s. $9\frac{111}{224}$d.
34. £698 1s. $7\frac{43}{160}$d.
35. £2341 7s. $2\frac{5}{8}$d.
36. £231 18s. $2\frac{801}{1792}$d.
37. £42716 11s. $2\frac{758}{768}$d.
38. £14414 14s. $3\frac{15}{16}$d.
39. 7119 8s. $6\frac{255}{512}$d.
40. £17 10s. $2\frac{13}{64}$d.
41. £274 9 flor. 4 c. 1·5484375 m.
42 £18435 2s. $11\frac{473}{2304}$d.
43. £148 11s. 6·984375d.
44. £316 2s. 8·15625d.
45. £186 17s. $4\frac{1}{8}$d.
46. £1549 14s. $7\frac{111}{128}$d.
47. £1867 6s. $8\frac{9}{16}$d.
48. £11 18s. $10\frac{1043}{2560}$d.
49. £6 9s. $0\frac{3921}{20480}$d.
50. £247 8s. $1\frac{41}{1792}$d.
51. £342 12s. $8\frac{1}{8}$d.
52. £849 9s. $5\frac{53}{128}$d.
53. £5 17s. $2\frac{1}{24}$d.
54. £583 1s. $9\frac{75}{112}$d.
55. £24 19s. $7\frac{53}{288}$d.
56. £2487 18s. $7\frac{1}{2}$d.
57. £2400 6s. $9\frac{4}{219}$d.

EXERCISES 35—(p. 157).

1. £122 13s. 4d.
2. £4891 3s. $5\frac{1}{4}$d.
3. £702 11s. $6\frac{3}{4}$d.
4. £189 15s.
5. £1145 0s. $2\frac{5}{8}$d.
6. £1086 16s. $4\frac{7}{8}$d
7. £1366 5s. $11\frac{87}{112}$d.
8. £1426 11s. $10\frac{1}{32}$d.
9. £238 19s. $4\frac{2887}{2912}$d.
10. £117 7s. 11d. +
11. £178 16s. $5\frac{139}{128}$d.
12. £68 11s. $2\frac{10}{21}$d.
13. £1024 14s. $5\frac{403}{448}$d.
14. £262 18s. $0\frac{1}{4}$d.
15. £101 3s. $0\frac{21}{104}$d.
16. £89 17s. $8\frac{1307}{1864}$d.
17. 2 cwt. 3 qr. 16 lb.

EXERCISES 63—(p. 162).

1. £125.
2. £29 10s. 6d.
3. £166 5s. 0d.
4. £36 16s. $10\frac{1}{2}$d.
5. £22 10s. 9d.
6. £600 18s. 9d.
7. £57 9s. 9d.
8. £1180 10s. $11\frac{1}{4}$d.
9. £108 13s. 6d.
10. £153 19s. $8\frac{1}{4}$d.
11. £181 6s. 8d.
12. £365 15s. $8\frac{3}{4}$d.
13. £3722 12s. 6d.
14. £174 18s. 9d.
15. £1236 13s. $1\frac{1}{2}$d.
16. £12 18s. $2\frac{30}{73}$d.
17. £15 5s. $1\frac{11}{73}$d.
18. £18 14s. $2\frac{487}{3850}$d.
19. £53 4s. $10\frac{3239}{29200}$d
20. £76 4s. $1\frac{1687}{5840}$d.

21. £44 5s. $4\frac{4561}{73000}$d.
22. £17 2s. $8\frac{352809}{1752000}$d.
23. £20 6s. $4\frac{4309}{58400}$d.
24. £226 18s. 5·295d.
25. 5 per cent.
26. $7\frac{1}{2}$ per cent.
27. 8 per cent. and $12\frac{1}{2}$ years.
28. £10 13s. 3·36d.
29. 18s. $8\frac{86891}{147000}$d.
30. 7 years.
31. 6 per cent.
32. 225 days.
33. 46 months.
34. For 1 year 213 days, due Oct. 3rd, 1859.
35. 240 days.
36. £3079 13s. 9d.
37. 8 per cent.
38. £13 14s. $7\frac{307121}{367700}$d.
39. £20 14s. $3{\cdot}70\dot{1}79452205\dot{4}$d.
40. £21 19s. $8\frac{60539}{146000}$d.
41. £129 8s. $8\frac{4687}{7300}$d.
42. £385 16s. 3d.
43. 256 days.
44. 2 years 219 days.
45. £18 8s. $6\frac{13681}{73000}$d.
46. 1 year 174 days, or till Aug. 31, 1859.
47. £11 8s. $8\frac{34}{74}$d. balance of interest against Hartley.
48. £165 3s. $8\frac{4523}{5840}$d.
49. £273 17s. $3\frac{8181}{9125}$d.
50. 4 per cent. and 25 years.
51. £3450 9s. $2\frac{5}{8}$d.
52. 20 years.
53. 25 years.
54. £1140 12s. 6d.
55. £378 12s. 6d.
56. $6\frac{2}{3}$ per cent.
57. £1971.
58. 5 per cent.
59. 268 days.
60. £7333 6s. 8d.
61. £74 7s. $6\frac{1581}{29200}$d.
62. 19s. $2\frac{2}{5}$d. interest in favour of Davis & Co.
63. Principal £125
Interest £8 6s. $8\frac{40}{73}$d.
64. Principal £3150
Interest £68 13s. $0\frac{184}{365}$d.
65. £31438 11s. $5\frac{1}{7}$d.

EXERCISES 37—(p. 171).

1. £121 13s. $3\frac{1}{2}$d.
2. £165 11s. 5·2640625d.
3. £97 9s. $10\frac{1}{2}$d.
4. £94 12s. $11\frac{1}{2}$d.
5. £136 0s. 9·1815d.
6. £584 18s. 7·0272d.
7. £108 6s. 6·348288d.
8. £8050 8s. 0·474320625d.
9. £38 14s. 6·6532546875.

EXERCISES 38A—(p. 175).

1. £1 3s. 4d.
2. £6 5s.
3. £19 15s.

EXERCISES 38—(p. 177).

1. £2294 1s. $5\frac{12}{253}$d.
2. $\begin{cases}\text{True value £476 3s. } 9\frac{5}{7}\text{d.}\\ \text{Bank value £475.}\end{cases}$
3. £13 14s. $3\frac{363}{463}$d.
4. $\begin{cases}\text{True value £23 6s. } \frac{24}{103}\text{d.}\\ \text{Bank value £24.}\end{cases}$

5. True discount £2 9s. 6d.
   Bank discount £2 9s. $7\frac{1738}{1825}$d.
6. £162 0s. $4\frac{236}{421}$d.
7. True discount £9 4s. $6\frac{21132}{22302}$d.
   Bank discount £9 7s. $11\frac{221}{365}$d.
8. True discount £547 1s. $5\frac{10418}{10998}$d.
   Bank discount £547 1s. $3\frac{1548}{3650}$d.
9. True value £154 14s. $3\frac{2649}{3721}$d.
   Bank value £154 13s. $1\frac{1208}{1825}$d.
10. True value £114 4s. $11\frac{5444}{13109}$d.
    Bank value £136 15s. $5\frac{491}{730}$d.
11. True value £934 10s. $10\frac{3779}{7567}$d.
    Bank value £933 5s. $10\frac{1651}{3650}$d.
12. True discount £25 11s. $9\frac{8}{9}$d.
    Bank discount £26 10s. $2\frac{3463}{7300}$d.
13. True present worth £257 0s. $7\frac{1}{4}$d. + $\frac{271}{421}$ far.
    Bank present worth £256 2s. $5\frac{77}{200}$d.
14. True value £29 17s. $8\frac{3187}{3737}$d.
    Bank value £30 11s. $11\frac{3011}{3650}$d.
15. True value £18 19s. $5\frac{7311}{7443}$d.
    Bank value £19 6s. $11\frac{139}{730}$d.
16. £4 11s. $1\frac{281}{328}$d.

EXERCISES 38A—(p. 178.)

1. £61 7s. 10d. 2. £640. 3. £78 7s. $0\frac{1}{2}$d.

EXERCISES 39—(p. 180).

1. £7 13s. $10\frac{1}{2}$d.
2. £16 12s. $7\frac{1}{2}$d.
3. £81 5s.
4. £6275.
5. £22 0s. $6\frac{21}{80}$d.
6. £3 9s. $7\frac{5199}{12800}$d.
7. Purchase money £1661 19s. $6\frac{333}{490}$d.
   Commission £39 9s. $5\frac{107267}{392000}$d.
8. £1 7s. $8\frac{8093}{51200}$d.
9. £2 7s. $9\frac{1001}{6400}$d.
10. £1 5s. $1\frac{253379}{409600}$d.
11. £2 17s. $\frac{357}{12800}$d.
12. £3 18s. $2\frac{259}{260}$d.
13. £3 0s. $7\frac{2643}{12800}$d.
14. £41 7s. $5\frac{67}{160}$d.
15. 8s. $6\frac{5057}{6400}$d.

EXERCISES 40—(p. 183).

1. £2080.
2. Sum purchased £334913 16s. $9\frac{67}{383}$d.
   Interest £11721 19s. $8\frac{92}{383}$d.
3. £5 2s. $3\frac{171}{800}$d.
4. £18 5s. $7\frac{1}{2}$d.

5. £1556266$\frac{2}{3}$ the sum secured in the Funds.
   £50578$\frac{2}{3}$ the interest it would yield for the better.
6. £4 12s. 6d. for the better.
7. £550 10s.
8. £5595.
9. £7 17s. 9$\frac{1781}{3925}$d. advantage.
10. £6 5s. 10$\frac{131}{200}$d. for the better.

EXERCISES 41—(p. 187.)

1. £23 12s. 6d.
2. £50 12s. 6d.
3. £475.
4. £19 7s. 5$\frac{2031}{10240}$d.
5. £145.
6. £54.
7. £72.
8. £101 10s.

EXERCISES 42—(p. 191)

1. 16s. 10$\frac{1}{2}$d.
2. 13s. 6$\frac{1}{2}$d.
3. £1 17s. 4d.
4. £33 17s. 8d.
5. £2 3s. 6$\frac{3}{4}$d.
6. £2 9s. 7$\frac{1}{2}$d.
7. £308 6s. 2$\frac{1}{4}$d.
8. £68 14s. 7d.
9. £44 5s. 5d.
10. £246 14s. 10$\frac{3}{4}$d.
11. £4 4s.
12. £7 3s. 7$\frac{1}{4}$d.
13. £13 2s. 4$\frac{1}{2}$d.
14. £61 19s. 10$\frac{5}{8}$d.
15. £57 11s. 1d.
16. £14 7s. 4d.
17. £10 0s. 8d.
18. £6 14s. 3$\frac{1}{4}$d.
19. £12 2s. 3$\frac{3}{14}$d.
20. Gain £266 13s. 11$\frac{7}{8}$d.
    Sell. price £13 10s. 8d. per cwt.
21. £1 10s. 11$\frac{5544}{37089}$d.
22. 20 per cent.
23. £3 5s. 2·87021d.
24. 13$\frac{7}{11}$ per cent.
25. 12$\frac{1}{2}$ per cent.
26. 10 per cent.
27. £61 16s. 1·47675d.
28. £1 18s. 7$\frac{26702}{29095}$d.
29. Entire selling price £300 12s. 6$\frac{1}{8}$d.
    Selling price of 1 yard 4s. 3$\frac{6409}{11192}$d.
30. 17s. 10$\frac{41}{55}$d.
31. Average cost of 1 lb. 2s. 10·949d.
    Average selling price 2s. 11·822725d.
32. £2050 0s. 8·996064d.
33. £1656 17s. 5·55066d.
34. Entire cost £60 8s. 4·5d.
    Averg. „ £3 18s. 7·12d.
35. £143 11s. 1·74387d.
36. £1196 9s. 10·262146875d.
37. £4 7s. 3$\frac{1}{2}$d.
38. 6s. 0$\frac{7815}{18004}$d.
39. £2 16s. 9d.
40. £6 8s. 4d.
41. £2 7s. 5$\frac{6109}{7435}$d.
42. Entire sell. price £556 3s. 8$\frac{2183}{4480}$d.
    Selling price of 1 cwt. £5 1s. 5$\frac{1881481}{3438120}$d.
43. £11 9s. 6$\frac{1653}{2368}$d.
44. 3s. 7·75d.

45. Loss £25 14s. 6d.
„ per cent. £1$\frac{21871}{81029}$
46. Gain £36 16s. 1·725d.
„ per cent. 23·765
47. Gain £8 4s. 1·8875d.
„ per cent. 8·3928
48. Gain £17 14s. 6$\frac{153}{1120}$.
„ per cent. 19$\frac{18510277}{24102817}$.
49. £561 8s. 8$\frac{1}{16}$d. loss.
50. Loss £19 0s. 9·73125
„ per cent. 7·93423
51. £1 15s.
52. £15 16s. 0$\frac{2222}{6229}$d.
53. 18s. 1$\frac{1}{5}$d.
54. £2 11s. 6$\frac{891}{2894}$d.
55. Loss £23.
„ per cent. 8$\frac{1}{15}$.
56. Gain £117 9s. 1d.
„ per cent. 53$\frac{1}{3}$.
57. 800 lb.
58. 8s. 4$\frac{4}{7}$d.
59. Gain £77 19s. 9$\frac{111}{196}$d.
„ per cent. 22$\frac{1}{2}$.
60. £269 5s. 0·3689250336d.
61. £453 15s. 10·4625d.
62. £523 9s. 5·92359375d.
63. £539 6s. 4·871484375d.
64. £77 18s. 7·91015625d.
65. Gain £35 1s. 5$\frac{1}{2}$d.
„ per cent. 10$\frac{80}{251}$.
66. 8s. 7$\frac{6577}{10166}$d.
67. Cost £189 15s. 6$\frac{99}{256}$d.
Loss £7 11s. 9$\frac{219}{224}$d.
68. £4 6s. 5$\frac{7}{64}$d. loss.
69. Selling price £11228 10s
0·6375d.
Gain £831 14s. 9·825d.
70. Selling Price £13132 8s.
1·1399296875d.
Gain 9164s. 3·4283671875d
71. Gain £1241 2s. 6·21d.
Cost per yard 6s. 3$\frac{3}{4}$d.
72. Gain £28 15s. 6$\frac{1}{2}$d.
„ per cent. 8$\frac{35141}{38773}$.
73. Gain £107 13s. 7$\frac{11}{16}$d.
„ per cent. 10$\frac{126124}{331963}$.
74. 98$\frac{641}{1040}$ stone.
75. £7 8s. 5$\frac{2}{7}$d.
76. £486 6s. 6$\frac{13}{84}$d.
77. £93 7s. 3$\frac{243}{320}$d.
78. 12$\frac{1}{2}$.
79. £4 18s. 11$\frac{11}{19}$d.
80. £256 15s. 10$\frac{4}{5}$d.
81. £3 13s. 5$\frac{31}{39}$d.

EXERCISES 43—(p. 199).

1. 1980.
2. 12s. 7$\frac{1}{4}$d.
3. 136 pairs.
4. 436 yd. 2 qr. 1$\frac{5}{7}$ nl.
5. 1s. 4d.
6. 844$\frac{301}{2736}$ stone.
7. 66$\frac{6751}{30996}$ lb.
8. 18157$\frac{4053}{4450}$ lb.
9. 577$\frac{89}{136}$ yards.
10. 103$\frac{839}{927}$ gallons.
11. 233$\frac{19}{72}$ gallons.
12. 45$\frac{4}{77}$ yards.

EXERCISES 44—(p. 201).

1. A.'s share £12.
B.'s share £20.
C.'s share £28.
2. A.'s share £375.
B.'s share £250.
C.'s share £125.
3. 99$\frac{5}{9}$ lb. of oxygen, and
12$\frac{4}{9}$ lb. of hydrogen.
4. A. £218 15s.
B. £281 5s.
C. £375, and 12s. 6d. in the £.
5. £152 18s. 4$\frac{4}{11}$d.

6. 56 tons.

7. 840 acres; the eldest gets 280 a., the second 210 a., and the third 168 a.

8. $112\frac{1}{2}$ lbs. of copper.
$37\frac{1}{2}$ lbs. of tin.

9. A.'s share £1400.
B.'s share £1050.
C.'s share £1750.

10. A.'s share £810.
B.'s share £540.

11. 24 cwt. of carbon.
$29\frac{1}{3}$ cwt. of oxygen.
$3\frac{2}{3}$ cwt. of hydrogen.

12. A.'s share £493 6s. 8d.
B.'s share £466 13s. 4d.

13. A.'s share $86\frac{6}{29}$ acres, rent £53 17s. $7\frac{1}{29}$d.
B.'s share $60\frac{10}{29}$ acres, rent £37 14s. $3\frac{21}{29}$d.
C.'s share $37\frac{27}{29}$ acres, rent £23 14s. $1\frac{18}{29}$d.
D.'s share $15\frac{16}{29}$ acres, rent £9 13s. $11\frac{17}{29}$d.

EXERCISES 45—(p. 208).

1. 6356 f. 35 c.
2. £350.
3. 17564 f. $62\frac{1}{2}$ c.
4. £96 4s.
5. 7555 fl. 2 kr.
6. £845 6s. 8d.
7. 3163 thal. 25 gros.
8. £480 10s.
9. 5790 fl. 15 kr.
10. 6978 fl. 30 kr.
11. £1447 11s. 3d.
12. £1722 16s. $4\frac{1}{8}$d.
13. 5722 marks 8 s.
14. £540 6s. 8d.
15. 4489 fl. 20 c.
16. £1051 10s.
17. £378 10s.
18. 10928 lir. 40 c.
19. £627 8s.
20. 1838 sc. 2 p. 5 baj.
21. £738 10s.
22. 2043 riksdalers.
23. £720 10s.
24. 876 rigs. dols. 3 mrks.
25. 4853 rigs. dols. 4 mrks. 8 shil.
26. £290 6s. $9\frac{3}{4}$d.
27. 9237 drach. 75 lepta.
28. £369 10s.
29. 1883 dols. 25 cents.
30. 2752 dols. 75 cents.
31. £1162 10s.
32. £1173 10s.
33. £99 16s. $10\frac{1}{2}$d.
34. 540 roubles.
35. 875 roubles 50 copecks.
36. £256 10s.
37. £410 7s. $9\frac{3}{4}$d.
38. 450 milreas 750 reas.
39. 354 milreas 500 reas.
40. £315 10s. 6d.
41. £245 18s. $8\frac{1}{4}$d.
42. 375 hard dols, 5 reals vellon.
43. 476 hard dols, 5 reals.
44. £150 13s. 6d.
45. £179 12s. $3\frac{3}{4}$d.
46. 8452 rupees.
47. 6938 rupees.
48. 5947 rupees 12 annas 6 pices,
49. £2059 14s. $4\frac{1}{2}$d.
50. £1189 11s. $1\frac{1}{2}$d.
51. 875 taels.
52. 684 taels 7 ma. 5 cand.
53. £142 3s. 9d.
54. £641 19s. $0\frac{3}{4}$d.

EXERCISES 45A—(p. 212).

1. 9345 fr. 80 cen.
2. £32.
3. 25 fr. 60 cen.
4. £327 3s. $9\frac{17}{27}$d.
5. 25 fr. 40 cen.
6. 33668 dol. $52\frac{36}{107}$ cent.
7. Pi. 3507 D. 10 reals.
8. 17820 fr. $53\frac{7}{16}$ c.
9. £3480 4s. 4d.
10. £3480 4s. 4d.
11. £221 0s. $5\frac{36}{1217}$d.
12. £383 12s. $6\frac{160}{479}$d.
13. 25 fr. 60 cent.
14. £1180 5s. $1\frac{1}{2}$d.
15. 10927 fr. $82\frac{1726}{6144}$ cent.
16. $7301\frac{43103}{60480}$ dol.
17. 35323 fr. $25\frac{4208}{7168}$ c.
18. £586 10s. $6\frac{3}{4}$d.
19. £1291 14s. $7\frac{1}{2}$d.
20. £354 0s. 10d.
21. 931 mil. $656\frac{16}{253}$ reas.
22. 8151 dol. 60 c.
23. Sum £2011 2s. $2\frac{2}{3}$d.
    Interest £60 6s. 8d.
24. £234 10s. $0\frac{167}{240}$d.
25. £106 11s. $11\frac{1}{4}$d.
26. 230770996 fr. 50 c.
27. 119722719 fr. 48 cents.
28. £20338983 1s. $0\frac{13}{59}$d.
29. 512542372 fr. $88\frac{8}{59}$ c.
30. Arbitrated Exchange 26 fr.
    10 c. per £1.
    Direct value 16129 francs.
    Indirect „ $16573\frac{1}{2}$ „

EXERCISES 46.

A (p. 217).

1. £1302 14s. 5·1072d.
2. £663 1s. 6d.

B (p. 218).

1. £1264.
2. £3489 8s. $4\frac{1}{2}$d.
3. £5556 4s. 6d.

EXERCISES 47.

A (p. 223, 224, 225).

1. £573 15s. $10\frac{1}{2}$d.
2. £901 16s. 3d.
3. £2 10s.
4. £409 10s. 6d.
5. £97 2s. $9\frac{3}{4}$d.

B (p. 230).

1. $8\frac{1}{3}$.
2. 10·236.

EXERCISES 48—(p. 233).

1. £2 13s. $2\frac{2}{65}$d.
2. $11\frac{41}{144}$.
3. £3 5s. 4d.
4. $16\frac{1}{4}$.
5. £4 4s.
6. $19\frac{1}{21}$.

EXERCISES 49.

A (p. 239).

1. 1296.
2. 8·0656.
3. 279841.
4. 121550625.
5. 147745544378906 25.
6. 63361600.
7. 25436448423936.
8. 6426225.
9. 8555625.
10. 62599744.

11. 23004973·3225.
12. 5292287975689366890062 5.
13. 617737·837369.
14. 3816000357173290·92842161.

B (p. 242).

1. 36.
2. 284.
3. 5·29.
4. 11025.
5. 7960.
6. 253·5.
7. 2925.
8. 7912.
9. 121·550625.
10. 5043456.
11. 4796·35.
12. 1·21550625.
13. 23004973·3225.
14. 785·963.
15. 61773783·7369.
16. ·2.
17. ·7071068 +.
18. ·2236068 +.
19. ·8944272 +.
20. ·09476.
21. ·012.
22. ·037947 +.
23. 1·2649 +.
24. ·4.
25. 12649 +.
26. 23·002.
27. 785·968.
28. 7633·822 +.
29. 13779·106 +.
30. 923·0232.
31. ·316227 +.
32. ·1.
33. ·0316227 +.
34. $\frac{2}{115}$.
35. 647 yards.

C. (p. 244).

1. 72 feet.
2. 204 feet.
3. 138 feet.
4. 8 feet.
5. 435 yards.

EXERCISES 50.

A. (p. 245).

1. 46656.
2. 91125.
3. 306125748.
4. 595884873375.
5. 999700029999.
6. 19059838576128.
7. 155·999772913250161.

B. (p. 251).

1. 16.
2. 84.
3. 26·5.
4. ·9999.
5. 2183.
6. 79·12.
7. 4·30886.
8. 584763928.
9. 970299.
10. 26712.
11. 5·383212612087283 9.
12. 2·154434.
13. 358.
14. 479635.

C. (p. 251).

1. $\frac{3}{8}$.
2. $\frac{4}{7}$.
3. ·87358.
4. ·956465625.
5. .82207.
6. ·882587.
7. 36 inches.
8. 12 inches.
9. £1 11s. $7\frac{11}{16}$d.

EXERCISES 51 (p. 254).

1. 16 sq. ft. 10 parts 6 sq. in.
2. 18 sq. ft. 10 parts 8 sq. in.
3. 7 sq. ft. 1 part 4 sq. in.
4. 31 sq. ft. 7 parts 3 sq. in.
5. 14 sq. ft. 10 parts 6 sq. in.
6. 9 sq. ft. 4 parts 6 sq. in.
7. 132 sq. ft. 6 parts 9 sq. in.
8. 217 sq. ft. 7 parts.
9. 107 sq. ft. 5 parts 2 sq. in.
10. 41 sq. ft. 3 parts.
11. 125 sq. ft. 11 parts 3 sq. in.
12. 25 sq. ft. 11 parts 2 sq. in.
    0. 2 sq. sec.
13. 13 sq. ft. 6 parts 6 sq. in.
14. 49 sq. ft. 6 parts 2 sq. in.
15. 14 sq. ft. 1 part 2 sq. in.
16. 127 sq. ft. 10 parts 6 sq. in.
17. 64.
18. 195.
19. 85.
20. £35 12s. 6d.
21. 56.
22. 5000 yards.
    Cost £1432 5s. 10d.
23. 2138 a. 0 r. 29 sq. p. $2\frac{3}{4}$ sq. yd.
24. $16\frac{1}{2}$ miles.
25. £61 9s. $8\frac{17}{18}$d.
26. $103\frac{1}{4}$ yards.
27. 112 yards.
28. £792 9s. 2d.

EXERCISES 52.

A. (p. 255).

1. 41 sq. ft. 1 part.
2. 8 a. 0 r. 4·2816 sq. p.
3. £96 17s. $2\frac{7}{16}$d.
4. 8 a. 2 r. 10 sq. p. $29\frac{1}{2}$ sq. yds.
5. 5 a. 0 r. 33 sq. p.
6. 240 feet.
7. 76338·22 + square yards.
8. 15310 sq. yds. 1·06 sq. ft.
9. 5541·7824 yards.
10. 4765489 feet.
11. 754·7694 square yards.
12. 40828·2336 square yards.
13. 16.

B. (p. 257).

1. $181\frac{1}{3}$ square yards.
2. 25 cubic feet.
3. 3675 cubic feet.
4. $2031\frac{1}{4}$ lbs.
5. 31·2851676 cubic feet.
6. 14·966 cubic feet.
7. £10 4s. $5\frac{1}{16}$d.
8. 106·029 cubic inches.
9. 91·1064 cubic feet.
10. 2300·3125 lbs.
11. 1465744·896 square miles.
12. ·594 + in.
13. £7 12s. 3·43825d.
14. 15·4 yards.
15. 5276681625·6 cubic miles.
16. 259333411782·8608 cub. mls.

EXERCISES 53—(p. 260).

1. 3s. 6d.
2. 15s. $7\frac{1}{2}$d.
3. 10·25 ounces.
4. 4 gallons of each.
5. 30 lb. of each.
6. 5.
7. 4 ounces of alloy.
8. 24 lb. at 3s., 8 lb. at 5s., and 24 lb. at 6s. per lb.
9. 40 lb. at 2s. 4d., 16 lb. at 2s. 6d., 8 lb. at 3s., and 16 lb. at 3s. 6d. per lb.

10. 60 gals. at 10s., 30 gals. at 13s. and 30 gals. at 15s. per gal.
11. 12 oz. of 19, and 12 oz. of 20 carats fine.
12. 2.
13. { 5 oz. of 13 carats fine.
    5 oz. of 14 " "
    5 oz. of 15 " "
    20 oz. of 21 " "
14. { 5 gals. of water.
    15 gals. of gin.

EXERCISES 54—(p. 276).

1. £24 8 fl. 7 c. 5 mils.
2. £18 8 fl. 3 c. 7·5 mils.
3. £98 8 fl. 9 c. 3·75 mils.
4. £15 9s.
5. £23 17s. 6d.
6. £11 18s. 6d.

EXERCISES 55—(p. 276)

1. £20 12s. $6\frac{6429}{12800}$d.
2. £93 1s. 3·552d.
3. £102 13s. $1\frac{481}{704}$d.
4. £4070 4s. $3\frac{611}{1089}$d.
5. 11s. $4\frac{469}{7982}$d.
6. 13s. $3\frac{26628}{47352}$d.
7. £44 6s. $2\frac{1}{32}$d.
8. $29\frac{11}{12}$ pieces.
9. 3 lb. 5 oz. 7 dr. 2 sc. 17 grs.
10. £247 8s. $1\frac{41}{1792}$d.
11. £773 19s. $10\frac{101}{640}$d.
12. $18\frac{13}{16}$ loads.
13. { Cost £47 12s. $3\frac{257}{464}$d.
    Gain £2 7s. $7\frac{701}{1856}$d.
14. $4\frac{2}{3}$.
15. £673 5s. $7\frac{221}{292}$d.
16. £10 13s. $11\frac{151}{324}$d.
17. £3 1s. $9\frac{579}{1019}$d.
18. £139 4s. $9\frac{21}{64}$d.
19. $241\frac{229}{1540}$ lb.
20. £8 13s. $11\frac{13283}{18770}$d.
21. £1 17s. $3\frac{774}{1085}$d.
22. £65 5s. $1\frac{23}{25}$d.
23. £4 13s. $5\cdot\dot{1}4285\dot{7}$d.
24. { Oxygen 381170 tons 1 cwt. $12\frac{4}{9}$ lb.
    Hydrogen 47646 tons 5 cwt. $15\frac{5}{9}$ lbs.
25. £121000.
26. { £60 in the 3 per cents.
    £65 " " $3\frac{1}{4}$ "
    £70 " " $3\frac{1}{2}$ "
27. $8\frac{46}{63}$.
28. $1\frac{1}{3}$ days.
29. $123\frac{131}{3600}$ days.
30. $\frac{23}{27}$.
31. £19091200.
32. £178 19s. $9\frac{3393}{5376}$d.
33. £711864 8s. $1\frac{37}{69}$d.
34. $408\frac{2686862}{7880135}$ years.
35. 2 a. 3 r. 33 sq. p. 27 sq. yd 8 sq. ft. 99 sq. in.
36. £164 19s. $3\frac{16077}{25088}$d.
37. £68567932 8s. 3d.
38. { £24 10s. $6\frac{54012}{1383358}$d.
    £1130902 13s. 4d. interest
39. 34 tons 17 cwt. 2 qr. 5 lb.
40. 4.
41. $21\frac{9}{11}$ minutes past 4.
42. 2296.
43. £115 7s. $0\frac{11}{56}$d.
44. { A. £105.
    B. £135.
    C. £240 7s. 6d.
    D. £210.
    E. £505 17s. 6d.
    F. £750.
    15s. = sum paid in the £.

45. £122307098 imports of 1853.
    £147650097 imports of 1857.
46. £77227280 exports of 1853.
    £93546231 „ of 1857.
47. £163735803.
48. 30 and $\cdot 012\dot{5}30\dot{8}$.
49. $344\frac{178}{189}$ yards.
    £4 2s. $7\frac{535}{756}$d. cost.
50. £·26875.
    £·175.
    £·16875.
    £·634375.
    £$\cdot 948958\dot{3}$.

EXERCISES 56—(p. 281).

1. $4\frac{1}{4}$ per cent.
2. $792\dot{5}\cdot 579322638146167\dot{}$ miles
3. 89·012 yds.
4. $1437\frac{134}{135}$ yards.
   £15 14s. $6\frac{263}{360}$d. cost.
5. $1\dot{0}5\cdot 04201680672268907563025210100700280103781512\dot{6}$
6. £$646\cdot 400\dot{3}$.
7. 213 yds. 1 in.
8. $\dot{1}\cdot 680672268907563025210084033613445378151260504\dot{2}0$ ft.
9. £7 17s. $9\frac{1781}{3925}$d. greater.
10. 11 a. 0 r. 4 sq. p.
11. £61 9s. $8\frac{17}{18}$d.
12. 55·5088 + yards.
13. £6 5s. 10·655d.
14. $\frac{361}{4096}$.
15. 7s. $3\frac{6112}{10764}$d.
16. 140·47 + lb.
17. 10·18744 feet.
18. $\frac{1191}{1271}$ per cent. loss.
19. 500 ounccs.
20. Oxygen 1177262424180817 tons 18 cwt. 1 qr. $16\frac{4}{5}$ lb.
    Nitrogen 4173930413004718 tons 1 cwt. 2 qr. $11\frac{1}{5}$ lb.
21. 474808·4649 lb. of carbonic acid.
    604301·6826 lb. of lime left.
22. £5 10s. $0\frac{446}{475}$d.
23. 2 a. 3 r. 33 sq. p. 23 sq. yd. 5 sq. ft. 73 sq. in.
24. 55·33 + inches.
25. Hydrogen 2697484308398352 tons.
    Oxygen 21579874467184281 6 tons.
26. £171 8s. $7\frac{19}{32}$d. gain.
27. £807 18s. 9d.

28. £11 5s. advantage annually.
29. { 94981 a. 0 r. 14 s. p. $16\frac{1}{2}$ s.yd.
    £17808 19s. $1\frac{1}{11}$d. cost.
30. £23 16s. 1·19125.
31. £8 12s. $8\frac{8}{11}$d.
32. $95\frac{17}{24}$ per cent.
33. 25 yrs. purchase and 4 per cent.
34. £46 17s. $1\frac{3}{5}$d.
35. ·0185546875.
36. ·65224.
37. 11 hours.
38. £600.
39. $8\frac{5}{8}$°.
40. £65 15s. $8\frac{211844}{5171567}$d.
41. £200.
42. £61 2s. $4\frac{304}{525}$d.
43. 202·23175 lb.
44. £800.
45. { $145256\frac{8}{15}$ lb. of oxygen.
    $18157\frac{1}{15}$ lb. of hydrogen.
    $108942\frac{6}{15}$ lb. of carbon.
46. 8.
47. £1639 11s. 5·7264d.
48. $4383\frac{1647}{2356}$ barrels.
49. £8 11s. $8\frac{3709}{6197}$d.
50. 17.

EXERCISES 57—(p. 285).

1. $12\frac{113}{178}$.
2. ·019.
3. £127 10s. $4\frac{69}{84}$d.
4. £729 3s. 4d.
5. £96 17s. $1\frac{97}{125}$d.
6. 427·6132.
7. 13s. 5·9814d.
8. 6. $7\frac{621}{1496}$d.
9. £21797 14s. 8·901d.
10. 110·80539 + miles.
11. { $28\frac{645}{1134}$ yards.
    £4 2s. $7\frac{535}{756}$d. cost.
12. $4\frac{1}{4}$.
13. 40954 tons 3 cwt. 2 qr. 8 lb.
14. $\frac{67}{100}$.
15. $\frac{13}{24}$.
16. £1 15s. $0\frac{908}{1665}$d.
17. £7 17s. $9\frac{1721}{3125}$d. greater.
18. 435088·125 +.
19. $\frac{1075}{7854}$.
20. £10 18s. $7\frac{11}{16}$d.
21. 210 miles.
22. { 5 ft. 2·47 in.
    $\frac{126}{30875}$d.
23. $710\frac{26}{35}$ days.
24. £14 4s. $3\frac{1}{3}$d.
25. £775 2s. 7·925609d.
26. £4070 4s. $8\frac{611}{1089}$d.
27. $62\frac{43327}{43520}$, or $16\frac{31383}{53000}$.
28. 81 lbs.
29. 66° 52′ 16·536″.
30. 60 g. 65`.
31. { $5\frac{1}{7}$ ounces of gold.
    $3\frac{6}{7}$ ounces of silver.
32. ·12.
33. 12·0237.
34. 29·48.
35. 4.

London; Printed by Hodson and Son, 22, Portugal Street, W.C.